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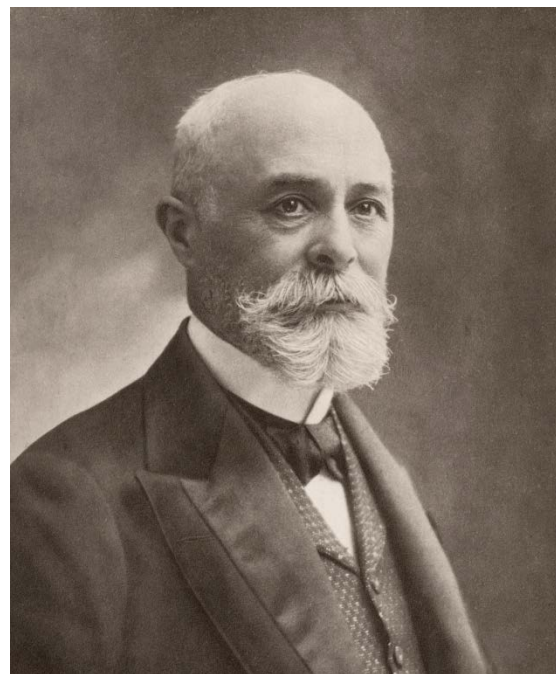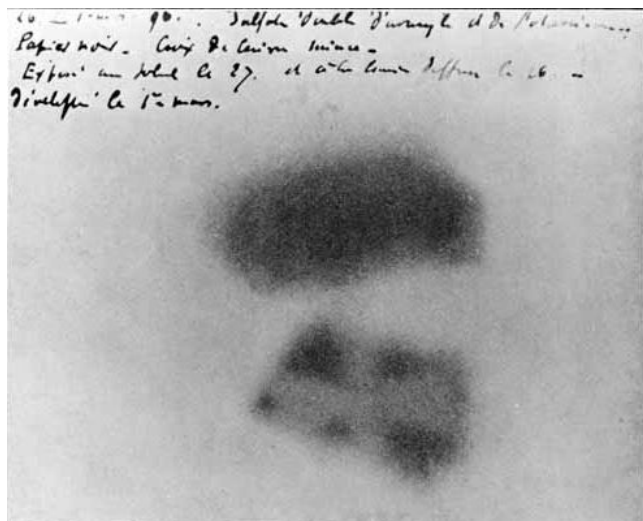web-page: https://web-docs.gsi.de/~wolle/   and click on
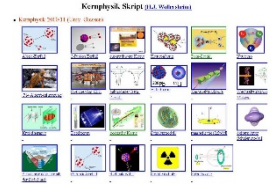
1. history
2. primary modes of radioactive decays
3. radioactive decay law
4. radioactive activity
5. radiation protection

Radioactive decay

Radioactive Decay: is the process by which a nucleus of an unstable atom decreases its total energy by spontaneously emitting radiation.

- Discovered in 1897 by the French scientist Henri Becquerel.
- He place various phosphorescent salts on photographic plates and noticed that only uranium salts exposed the plates.

Henri Becquerel

Marie Curie

Henri Becquerel's discovery inspired Marie and Pierre Curie to investigate the phenomena further.

- They found that the mineral pitchblende was more active than uranium, and concluded that it must contain different radioactive substances

- From this, they discovered polonium and radium, both of them more radioactive than uranium.

The Nobel Prize in Physics 1903

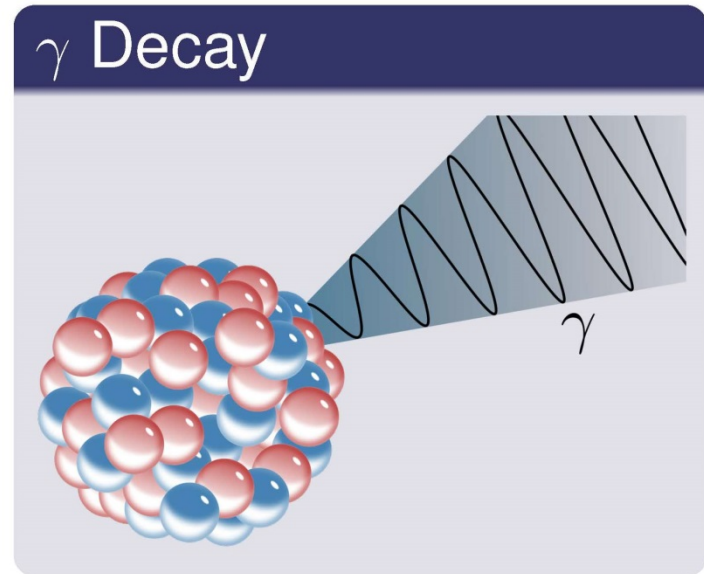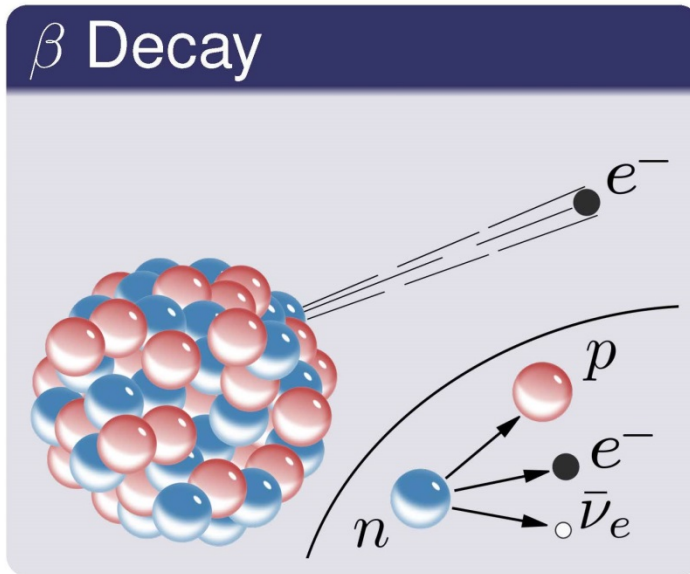**Antoine Henri Becquerel**

Prize share: 1/2

Pierre Curie

Prize share: 1/4

Marie Curie, née Sklodowska

Prize share: 1/4

α Helium nucleus

γ electromagnetic wave

β electron

α Decay

β Decay

γ Decay

chart of nuclides:

-representation of isotopes in the Z-N plane

-isotope: atom (nucleus) of an element with
different number of neutrons

proton number Z

neutron number N

different modes of
radioactive decay

The number of nuclei that decay is proportional to the number of radioactive nuclei $\mathbf{N}$ in the sample:

$$-\frac{dN}{dt} = \lambda \cdot N$$
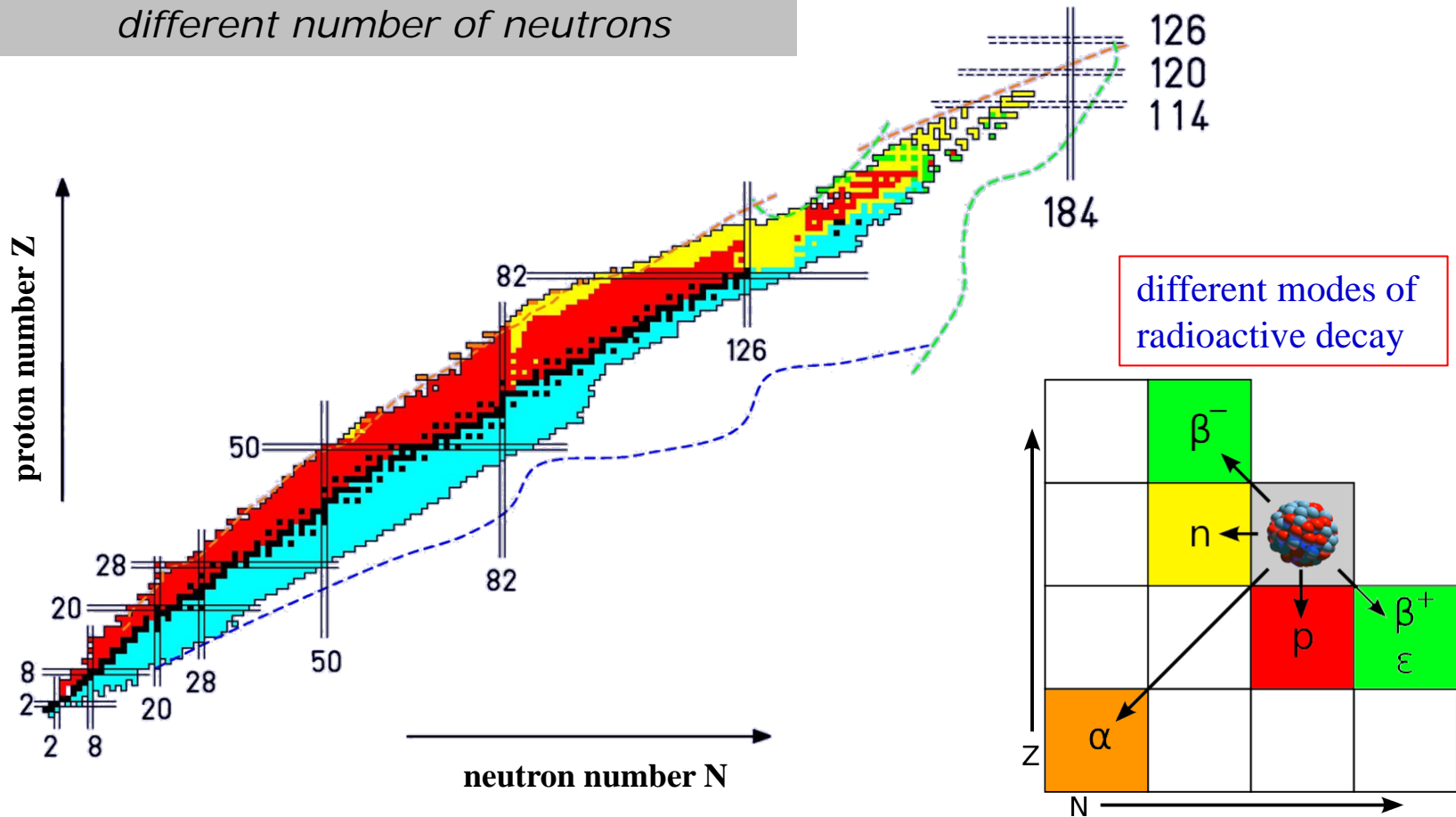
λ is the decay constant

- Radioactivity is an exponential decay process

$$N(t) = N_0 \cdot e^{-\lambda \cdot t}$$

- The sample activity or decay rate is given by

$$A = \lambda \cdot N$$

- The lifetime and the half-life is defined as

$$\tau = \frac{1}{\lambda} \qquad T_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \cdot \tau$$

Number of undecayed nuclei at time t

$N = N_0 e^{-\lambda t}$
$N_0 = 100$

N_0

$N_0/2$

$N_0/4$

$N_0/8$

$N_0/16$

$N_0/32$

$T_{1/2} \quad 2T_{1/2} \quad 3T_{1/2} \quad 4T_{1/2} \quad 5T_{1/2}$

Time (t) ⟶
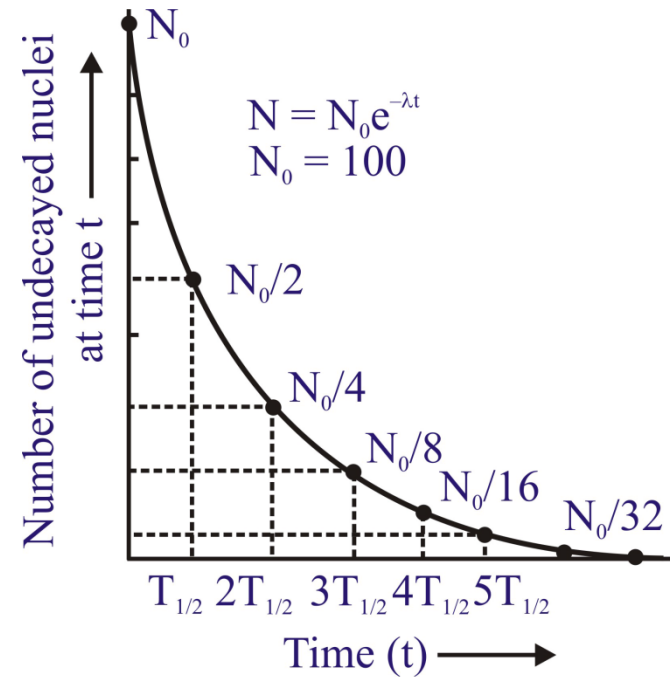
Decay curve for a radioactive element

$$T_{1/2} = 1.5, \ \tau = T_{1/2}/\ln(2) = 2.16$$
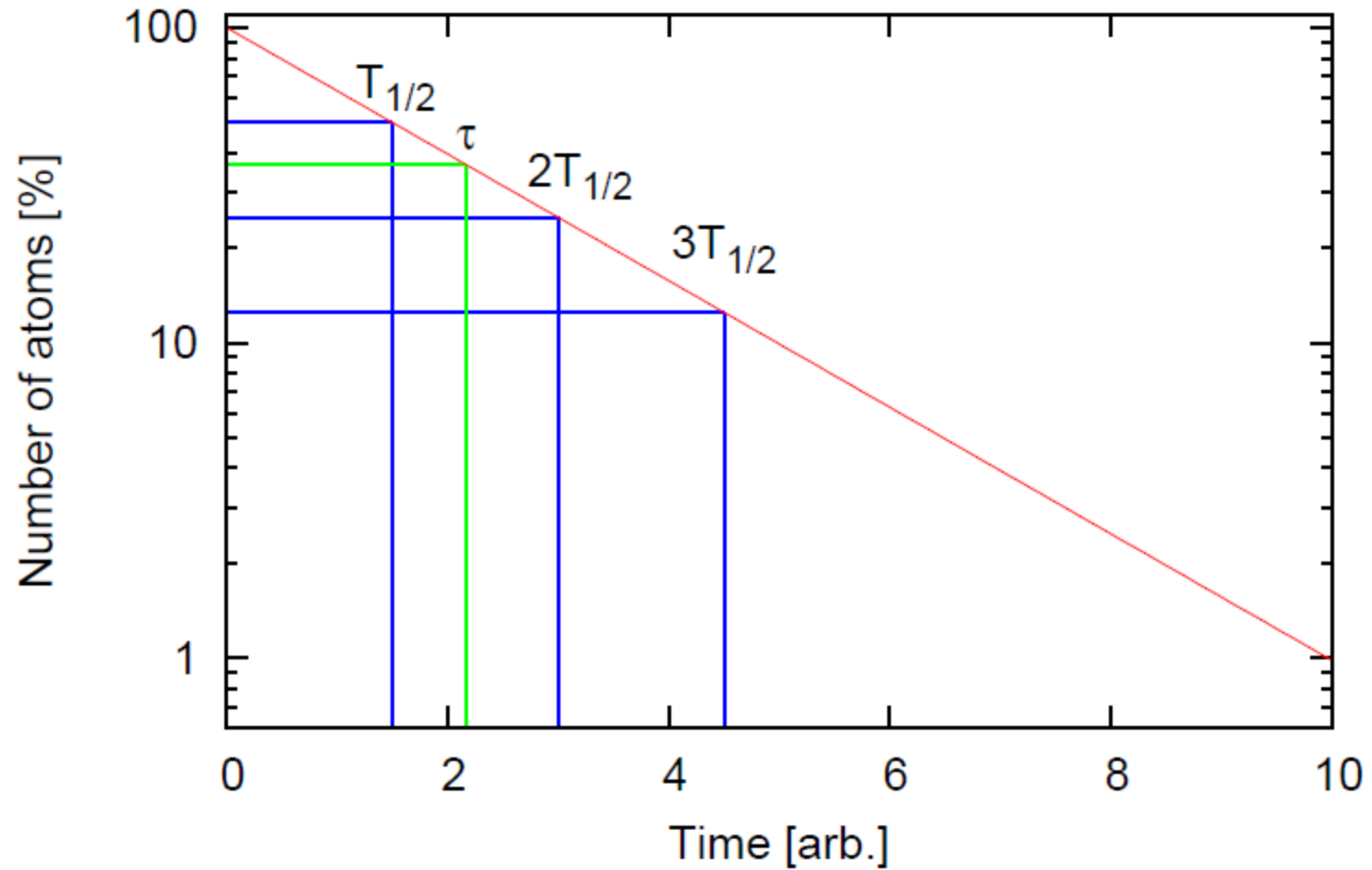
❖ The initial nucleus can decay into either of two products, e.g. in a sample of ^{212}Bi, 36% of the nuclei decay to ^{208}Tl and 64% to ^{212}Po.

Consider the case A → B and A → C in parallel. We have for all time t:

$$N = N_A + N_B + N_C$$

which is constant, since the total number of nuclides remain constant. Differentiating with respect to time:

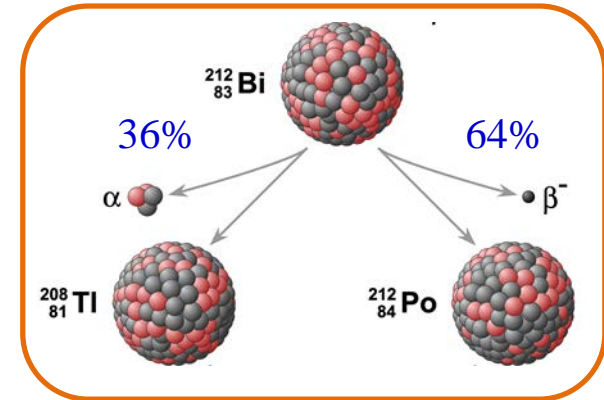$$\frac{dN_A}{dt} = -\lambda_1 \cdot N_A - \lambda_2 \cdot N_A = N_A(0) \cdot e^{-(\lambda_1 + \lambda_2) \cdot t} \qquad \lambda = \lambda_1 + \lambda_2$$

Definition of the branching ratio:

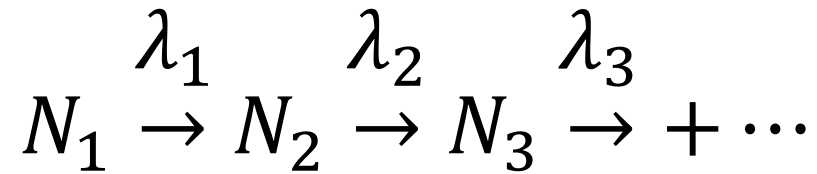$$f_1 = \frac{\lambda_1}{\lambda} \qquad f_2 = \frac{\lambda_2}{\lambda}$$

β-activity: $A_\beta(t) = N_A \cdot \lambda \cdot f_1 = N_A \cdot \lambda_1$

α-activity: $A_\alpha(t) = N_A \cdot \lambda \cdot f_2 = N_A \cdot \lambda_2$

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3 \xrightarrow{\lambda_3} + \cdots$$

Ultimately, the characterization of multiple decay chains boils down to solving a series of coupled differential equations:

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3$$

One can show that a general solution can be obtained as

$$N_1(t) = a_{11}e^{-\lambda_1 t}$$

$$N_2(t) = a_{21}e^{-\lambda_1 t} + a_{22}e^{-\lambda_2 t}$$

$$N_3(t) = a_{31}e^{-\lambda_1 t} + a_{32}e^{-\lambda_2 t} + a_{33}e^{-\lambda_3 t}$$

with initial conditions $\quad a_{11} = N_1(t = 0) \quad\quad \sum_{i=1}^{k} a_{ki} = 0$

For the particular case of $N_2(t)$ we obtain:

$$\boxed{\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2}$$

$$-\lambda_1 a_{21}e^{-\lambda_1 t} - \lambda_2 a_{22}e^{-\lambda_2 t} = \lambda_1 a_{11}e^{-\lambda_1 t} - \lambda_2 a_{21}e^{-\lambda_1 t} - \lambda_2 a_{22}e^{-\lambda_2 t}$$

$$a_{21} = \frac{\lambda_1}{\lambda_2 - \lambda_1}a_{11} \quad\quad a_{22} = \frac{\lambda_1}{\lambda_1 - \lambda_2}a_{11}$$
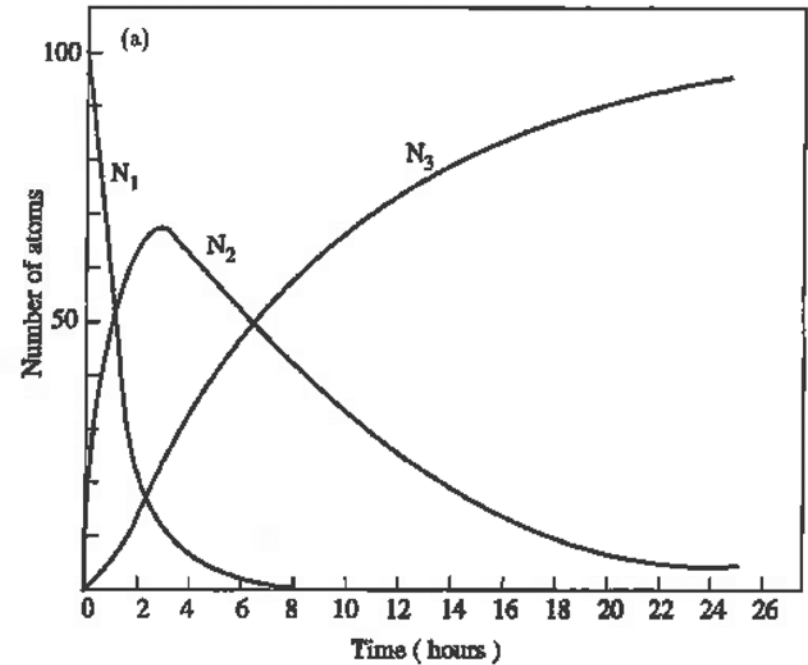
General solution:

$$a_{ki} = \frac{\lambda_{k-1}}{\lambda_k - \lambda_i}a_{k-1,i} \quad\quad\quad a_{ki} = \frac{\lambda_1 \lambda_2 \cdots \lambda_{k-1}}{(\lambda_1 - \lambda_i)(\lambda_2 - \lambda_i)\cdots(\lambda_k - \lambda_i)}a_{11}$$

The results for a radioactive series with T=1h, T=5h and T=∞

$$N_1(t) = N_1^0 e^{-\lambda_1 t}$$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 \left(e^{-1\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_3(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_1^0 \left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right)$$

If we extend our solution to k decays, we can generalize our solution
This is referred to as the Bateman equation.

$$N_k(t) = \sum_{i=1}^{k} \left[N_i(0) \cdot \left(\prod_{j=1}^{k-1} \lambda_j \right) \cdot \left(\sum_{J=1}^{k} \left(\frac{e^{-\lambda_j\, t}}{\prod_{p=i,p \neq j}^{k}(\lambda_p - \lambda_j)} \right) \right) \right]$$

❖ For lifetimes comparable or longer than the span of a human life there are no measurable changes in the activity of a sample which prohibits direct decay curve measurements. In these cases the decay rates λ are deduced from the ratio of the observed activity $\mathbf{A}$ to the absolute number of radioactive atoms $\mathbf{N}$ in a sample.

$$A = \lambda \cdot N = \lambda \cdot \frac{m}{\mu} \cdot N_A = \frac{1}{\tau} \cdot \frac{m}{\mu} \cdot N_A$$

with the molar mass μ and the Avogadro number N_A

❖ For example 1 MBq of tritium (^{3}H, $T_{1/2}$=12.33 [y]) corresponds to $5.59 \cdot 10^{14}$ atoms or 2.78 [ng] of mass.

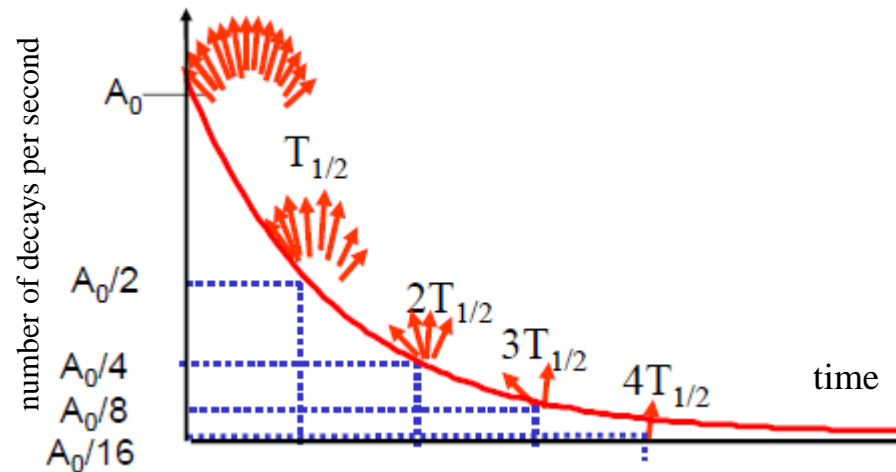❖ 1 MBq of ^{14}C ($T_{1/2}$=5730 [y]) corresponds to $2.6 \cdot 10^{17}$ atoms or 6 [µg] of mass.

$$A = \frac{ln2}{T_{1/2}} \cdot N \quad \rightarrow \quad N = \frac{A \cdot T_{1/2}}{ln2} = \frac{10^6 \cdot 12.33 \cdot 365 \cdot 24 \cdot 60 \cdot 60}{0.693} = 5.6 \cdot 10^{14} \; atoms$$

The number of radioactive nuclei is often unknown. One can only measure the number of decays per second (decay rate). This is the reason why the activity is defined as the number of decays per second:

$$A(t) = -\frac{\Delta N(t)}{\Delta t} = A_0 \cdot e^{-\lambda \cdot t}$$

1 Becquerel (Bq) = 1 decay/s

When at the beginning N(t=0) radioactive nuclei were present, then the activity A after the time t is given by

$$A(t) = -\frac{dN}{dt} = \lambda \cdot N_0 \cdot e^{-\lambda \cdot t} = \lambda \cdot N = \frac{N}{\tau} = A_0 \cdot e^{-\lambda \cdot t}$$

Activity at the beginning: A_0

Activity at time t: $A_1 = A_0 \cdot e^{-\lambda \cdot t} \quad \rightarrow \quad \dfrac{A_1}{A_0} = e^{-\lambda \cdot t}$

Activity at time 2t: $A_2 = A_0 \cdot e^{-\lambda \cdot 2t} = A_1 \cdot e^{-\lambda \cdot t} = A_1 \cdot \left(\dfrac{A_1}{A_0}\right)$

Activity at time 3t: $A_3 = A_0 \cdot e^{-\lambda \cdot 3t} = A_2 \cdot e^{-\lambda \cdot t} = A_2 \cdot \left(\dfrac{A_1}{A_0}\right) = A_1 \cdot \left(\dfrac{A_1}{A_0}\right)^2$

General expression: $A_{n+1} = A_n \cdot \left(\dfrac{A_1}{A_0}\right) = A_1 \cdot \left(\dfrac{A_1}{A_0}\right)^n$

Example: The activity of a sample was measured 1000 [Bq] one hour ago. The present activity is 900 [Bq]. What is the activity in one hour?

$$A_2 = A_1 \cdot \left(\frac{A_1}{A_0}\right) = 900 \cdot \left(\frac{900}{1000}\right) = 810$$

Consider a general quantum state: $\psi = c_1\psi_1 + c_2\psi_2$

where the probability $|c_1|^2$ is to be found in state 1, and $|c_2|^2$ is to be found in state 2.

- We assume a time-independent Hamiltonian that generates stationary states.
- For radioactive decay we need to adopt a perturbative approach
- Our potential is of the form $V + V'$, where V' is a very weak addition to out potential that allows for transitions between the two states.

Transition probability is therefore just the decay constant λ, and is given by Fermi's Golden Rule:

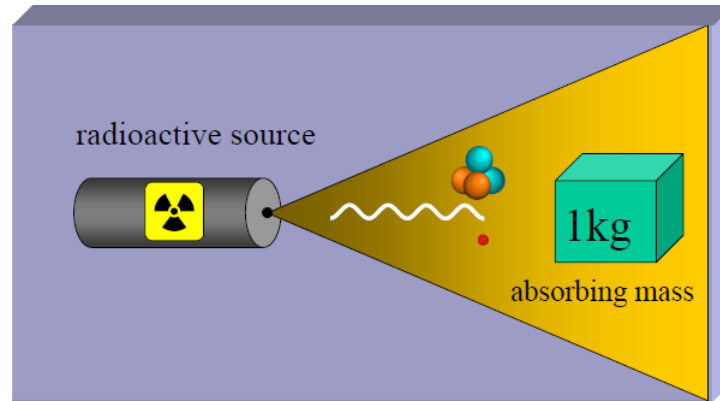$$\lambda = \frac{2\pi}{\hbar}\left|V'_{fi}\right|^2\rho(E_f)$$

- The transition matrix element that connects the initial and final states
  $V'_{fi} = \langle f|V'|i\rangle$

- The density $\rho(E_f)$ of final states
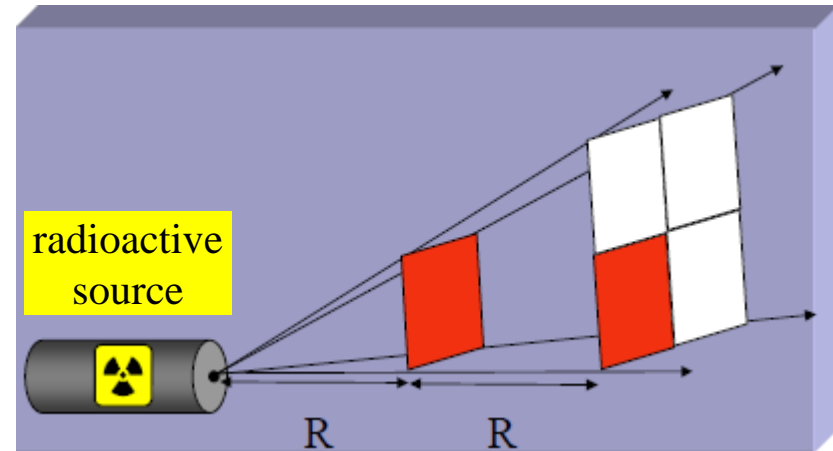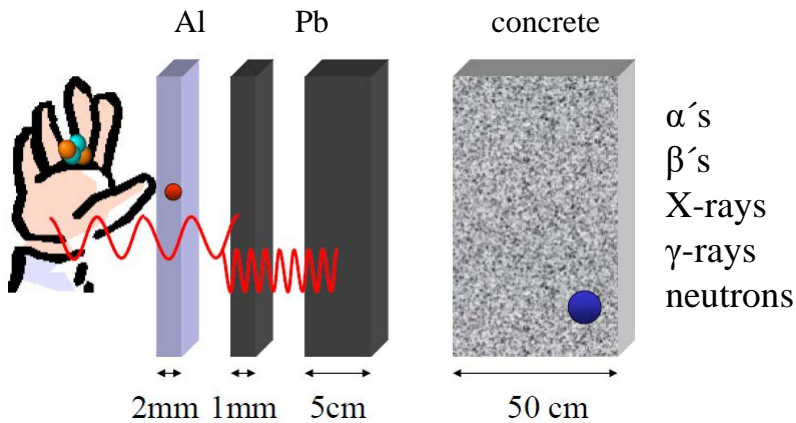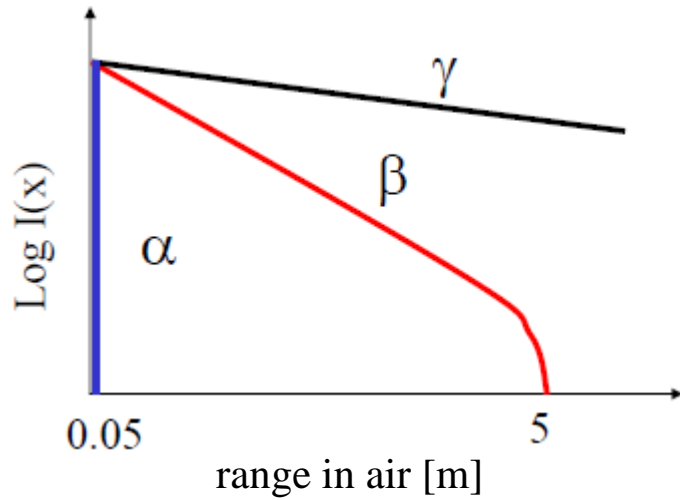
Radiation protection

$$\Omega = \frac{a^2}{4\pi \cdot R^2} = \frac{\pi \cdot r^2}{4\pi \cdot R^2}$$

$$absorbed\ dose = \frac{absorbed\ energy}{mass}$$

$$1\ \frac{Joule}{kg} = 1\ Gray\ (Gy)$$

The biological dose, sometimes also known as the dose equivalent is expressed in units of Sieverts [Sv]. This dose reflects the fact that the biological damage caused by a particle depends not only on the total energy deposited but also on the rate of energy loss per unit distance traversed by the particle.

equivalent dose $[Sv]$ = energy dose · quality factor Q

radiation	quality factor Q
X-ray, γ, β	1
thermal neutrons	2.3
fast neutrons	10
α-particles, heavy ions	20