Outline: Doppler effect

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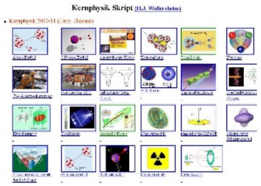
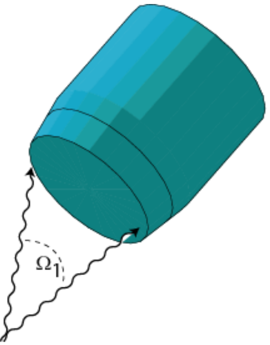
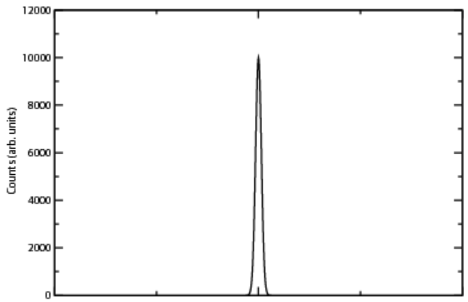


1. special relativity
2. Lorentz transformation
3. Doppler effect and broadening
4. γ- and electron detection

Efficiency versus resolution

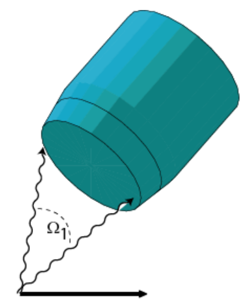


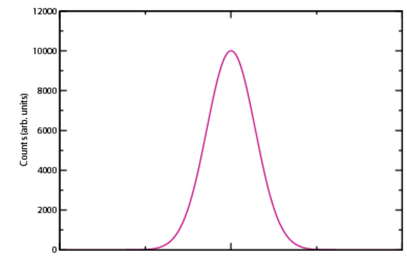


With a source at rest, the intrinsic resolution of the detector can be reached;

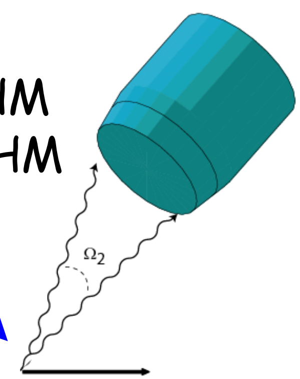
efficiency decreases with the increasing detector-source distance.

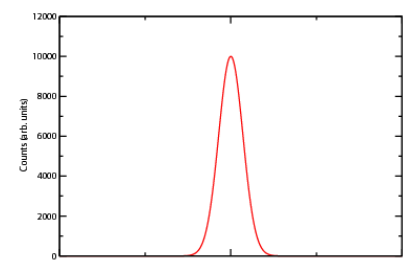
With a moving source also the effective energy resolution depends on the detector-source distance (Doppler effect)





Small *d*	⟺	Large Ω	⟺	High ε	⟺	Poor FWHM
Large *d*		Small Ω		Low ε		Good FWHM

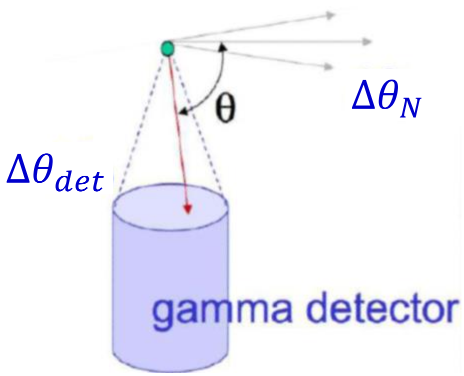




Energy resolution

The major factors affecting the final energy resolution (FWHM) at a particular energy are as follows:

$$\Delta E_{\gamma}^{final} = \left(\Delta E_{Int}^2 + \Delta\theta_{det}^2 + \Delta\theta_N^2 + \Delta v^2\right)^{1/2}$$



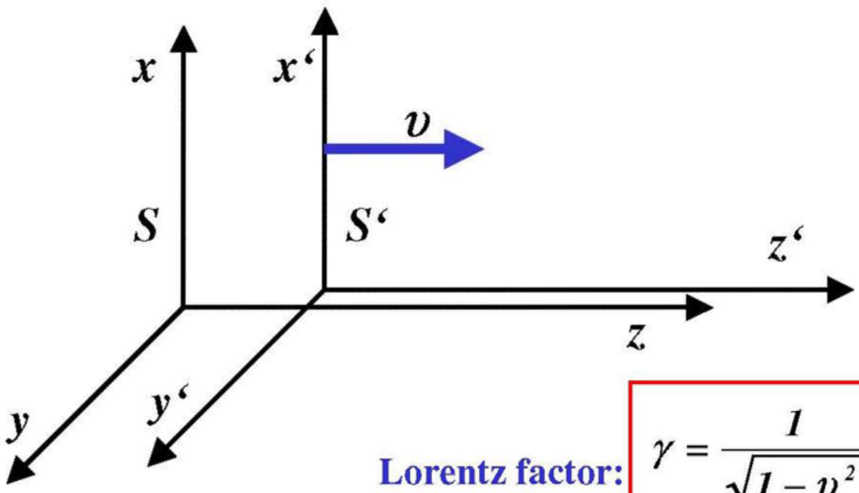
ΔE_{Int} – The intrinsic resolution of the detector system. It includes contributions from the detector itself and the electronic components used to process the signal.

$\Delta\theta_{det}$ – The Doppler broadening arising from the opening angle of the detectors

$\Delta\theta_N$ – The Doppler broadening arising from the angular spread of the recoils in the target

Δv – The Doppler broadening arising from the velocity (energy) variation of the excited nucleus

Lorentz transformation:



Lorentz factor: $\gamma = \dfrac{1}{\sqrt{1-v^2}}$

❑ **Consider the space-time point**

- **in a given frame S:** (t, x, y, z)
- **and in a (moving) frame S':** (t', x', y', z')

1) S' moves with a constant velocity v along z-axis

Space-time Lorentz transformation S←→S':

$S \Rightarrow S'$	$S' \Rightarrow S$
$x' = x$	$x = x'$
$y' = y$	$y = y'$
$z' = \gamma(z - vt)$	$z = \gamma(z' + vt)$
$t' = \gamma(t - vz)$	$t = \gamma(t' + vz)$

Note: **units c=1**

$$t' = \gamma\left(t - \frac{v}{c^2}z\right)$$

$$t = \gamma\left(t' + \frac{v}{c^2}z\right)$$

$$v_z = |\vec{v}| = v$$

❑ **Consider the 4-momentum:**

- **in a given frame S:** $p \equiv (E, p) = (E, p_x, p_y, p_z)$
- **in the (moving) frame S':** $p' \equiv (E', \vec{p}\,') = (E', p'_x, p'_y, p'_z)$

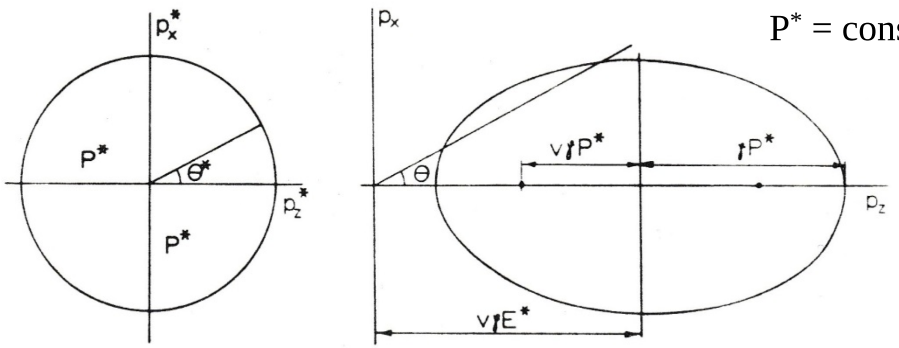
Lorentz transformation for 4-momentum S←→S':

$$p'_x = p_x, \quad p'_y = p_y$$

$$p'_z = \gamma(p_z - vE)$$

$$E' = \gamma(E - vp_z)$$

Lorentz transformation



P* = const.

rest system   laboratory system

total energy:

$$E^* = \gamma \cdot E - \gamma \cdot v \cdot P \cdot cos\theta$$

with

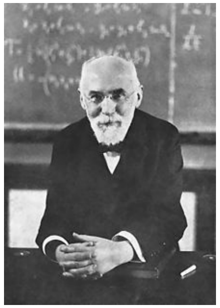
$$E = \sqrt{(mc^2)^2 + (Pc)^2}$$

E^*, P^* total energy and momentum in the rest system
E, P   total energy and momentum in the laboratory system

Doppler formula for zero-mass particle (photon):   E=Pc

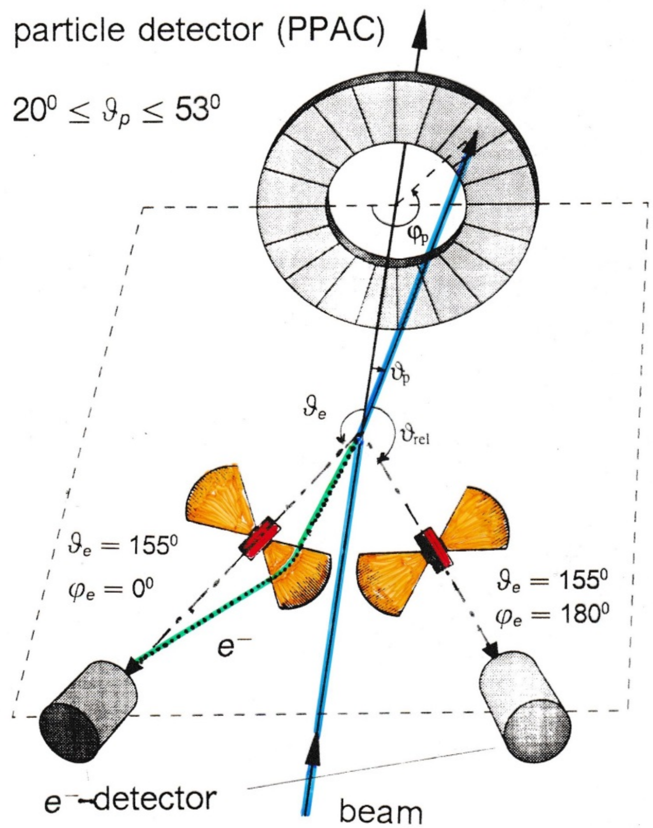
$$E^* = \gamma \cdot E - \gamma \cdot \beta \cdot E \cdot cos\theta$$

$$\boxed{E^* = \gamma \cdot E(1 - \beta \cdot cos\theta)}$$



Hendrik Lorentz

E. Byckling, K. Kajantie   J. Wiley & Sons   London



Doppler broadening

$\Delta\vartheta_e = 20^0$
target – Mini-Orange: 19 cm
Mini-Orange – Si detector: 6 cm

For projectile excitation:

$$T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\, c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)$$

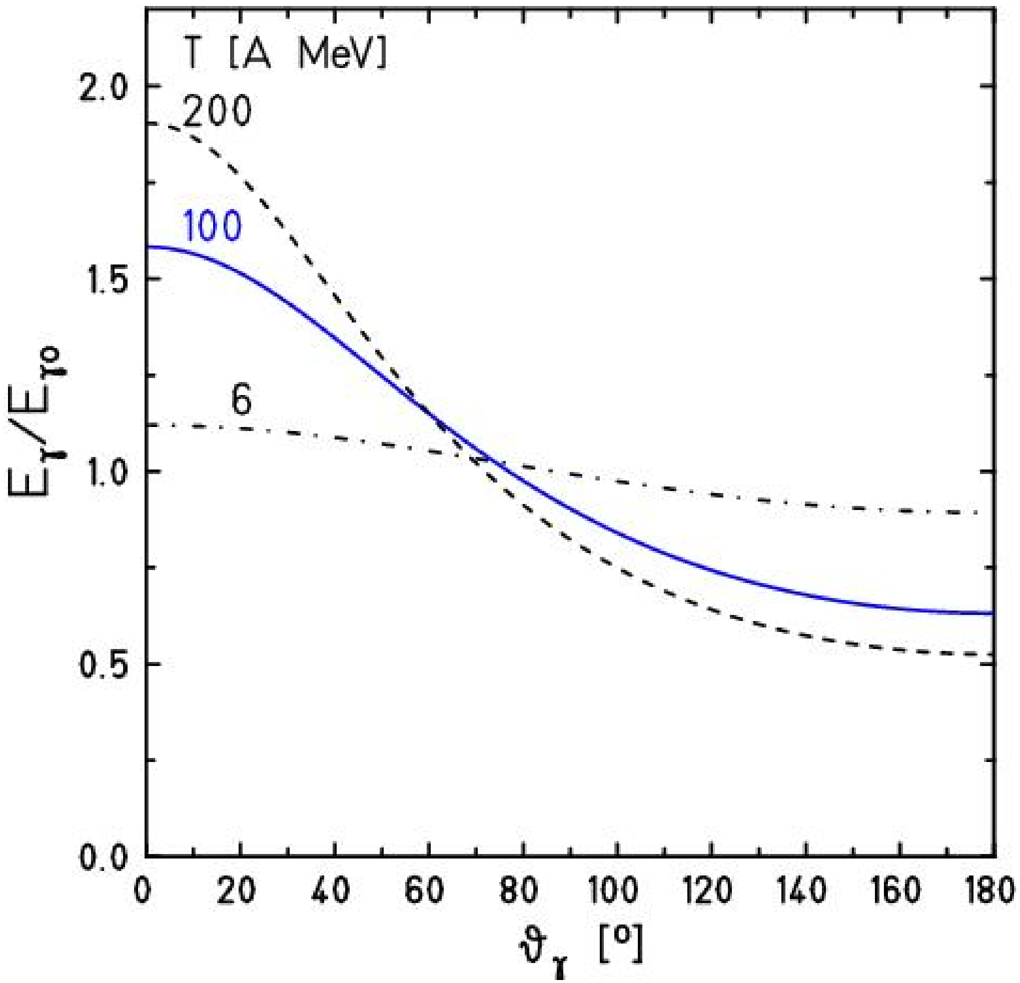
with

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

Doppler effect

$$\frac{E_{\gamma 0}}{E_\gamma} = \frac{1-\beta \cdot \cos \vartheta_\gamma^{lab}}{\sqrt{1-\beta^2}}$$

for $\vartheta_p \cong 0^0$



$$\frac{d\Omega_{rest}}{d\Omega_{lab}} = \left(\frac{E_\gamma}{E_{\gamma 0}}\right)^2$$

Doppler broadening and position resolution

$$E_{\gamma 0} = E_\gamma \frac{1-\beta\cdot\cos\vartheta_\gamma}{\sqrt{1-\beta^2}} \qquad \left(\beta, \vartheta_p = 0^0, \vartheta_\gamma \text{ and } E_\gamma \text{ in lab}-\text{frame}\right)$$

$$\left(\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}}\right)^2 = \left(\frac{\beta\cdot\sin\vartheta_\gamma}{1-\beta\cdot\cos\vartheta_\gamma}\right)^2 (\Delta\vartheta_\gamma)^2 + \left(\frac{\beta-\cos\vartheta_\gamma}{(1-\beta^2)\cdot(1-\beta\cdot\cos\vartheta_\gamma)}\right)^2 (\Delta\beta)^2 + \left(\frac{1}{E_\gamma}\right)^2 (\Delta E_\gamma)^2$$

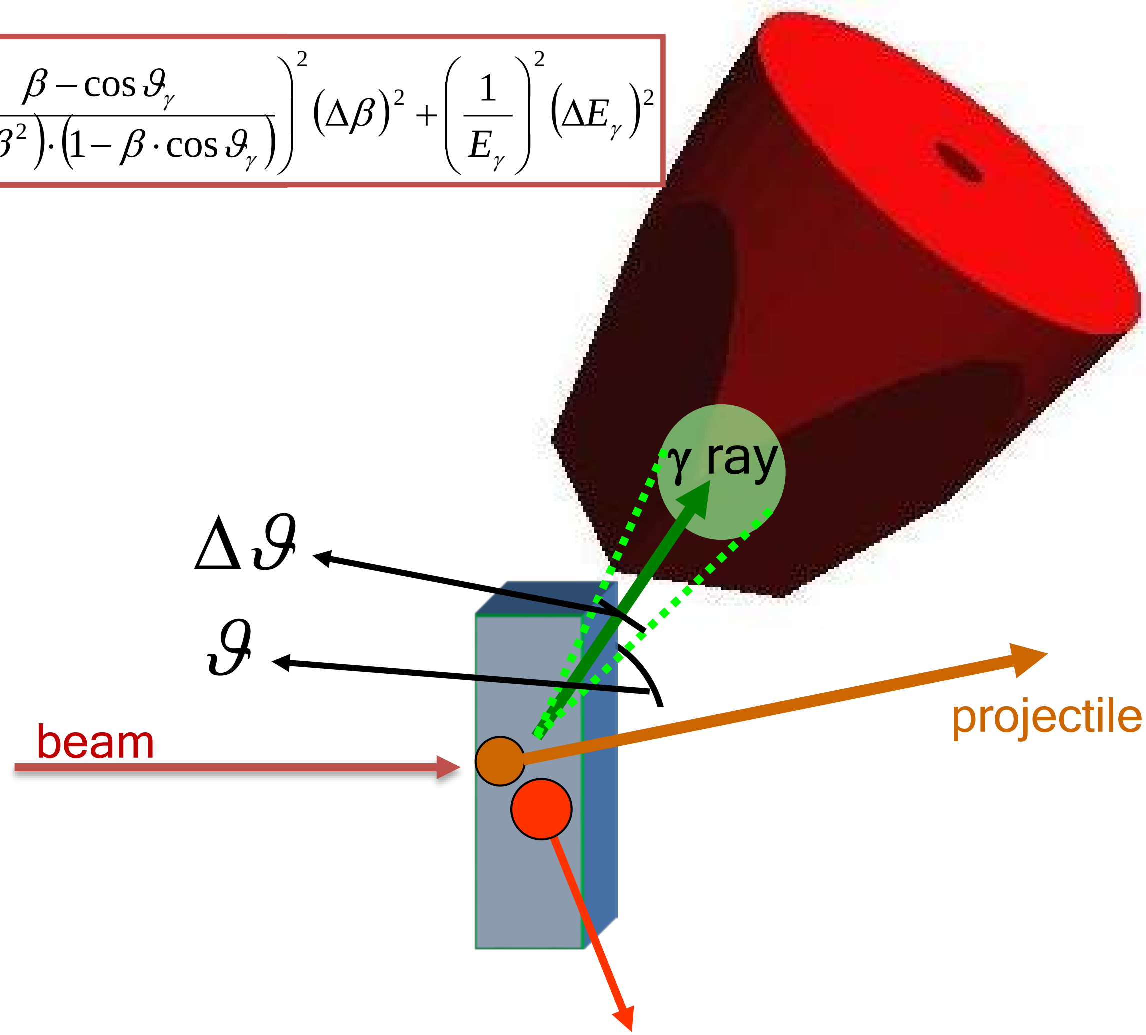
Position resolution

↓

Angular resolution

↓

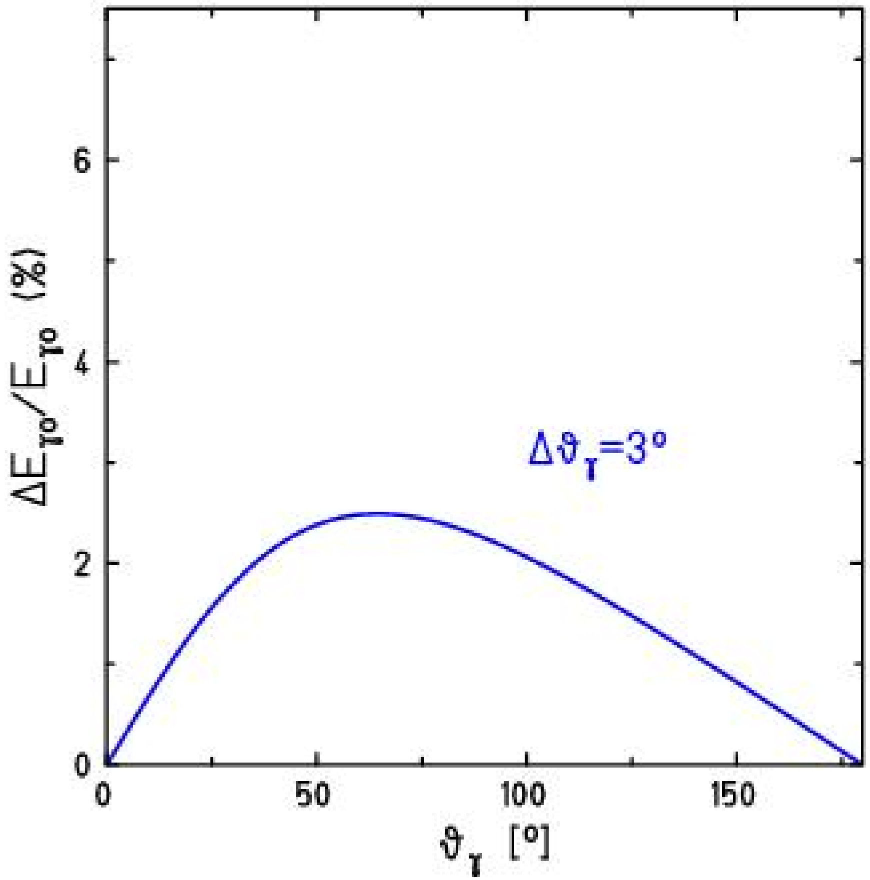
Energy resolution



Doppler broadening (opening angle of detector)

$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta \cdot \sin \vartheta_\gamma}{1 - \beta \cdot \cos \vartheta_\gamma} \cdot \Delta \vartheta_\gamma$$

for $\vartheta_p \cong 0^0$



with

$$\Delta \vartheta_\gamma = 0.622 \cdot \arctan \frac{d[mm]}{R[mm] + 30[mm]}$$

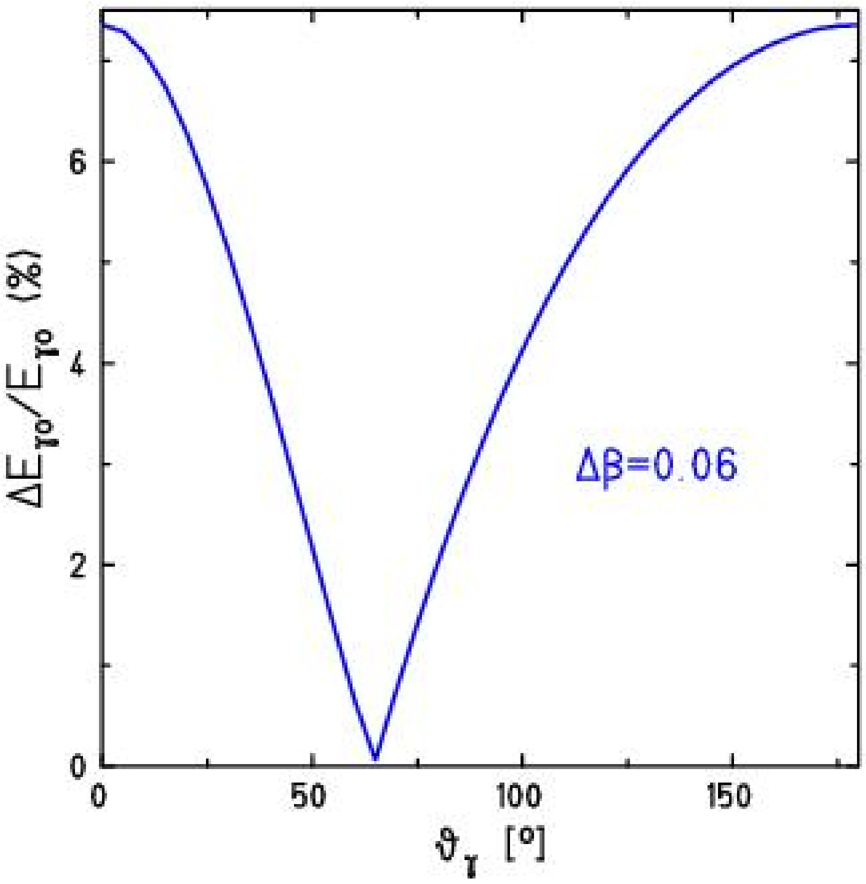
$$R = 700[mm]$$

$$d = 59[mm]$$

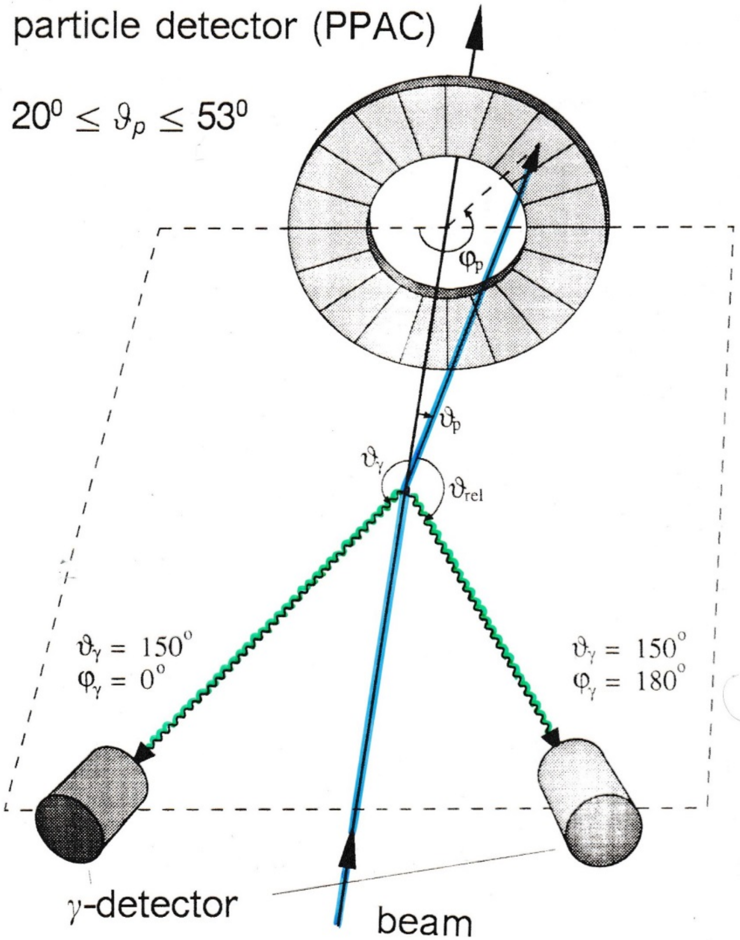
$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta - \cos\vartheta_\gamma}{\left(1-\beta^2\right)\cdot\left(1-\beta\cdot\cos\vartheta_\gamma\right)}\cdot\Delta\beta$$

for $\vartheta_p \cong 0^0$

with $\Delta\beta = 6\%$



Experimental arrangement



experimental problem:

Doppler broadening due to finite size of Ge-detector

$\frac{\Delta E}{E} \sim 1\%$ for $\Delta\vartheta_\gamma = 20^0$ $\beta_1 \cong 10\%$

For projectile excitation:

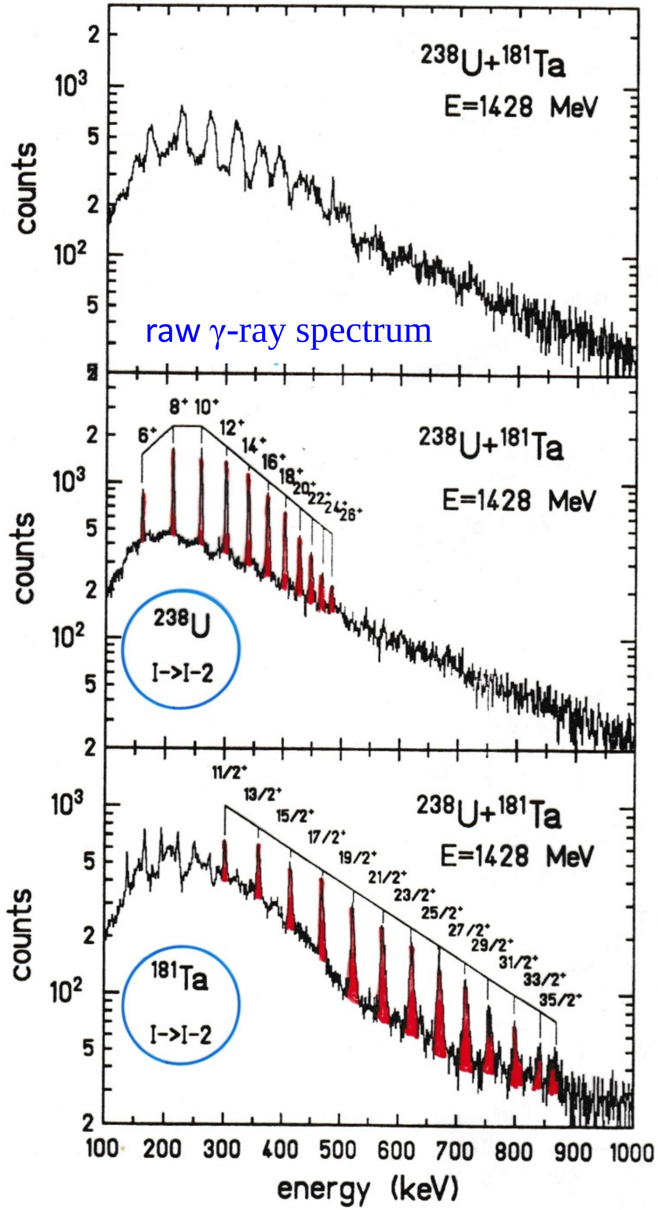
$E^* = \gamma \cdot E \cdot (1 - \beta_1 \cdot cos\theta_{\gamma 1})$ Doppler shift

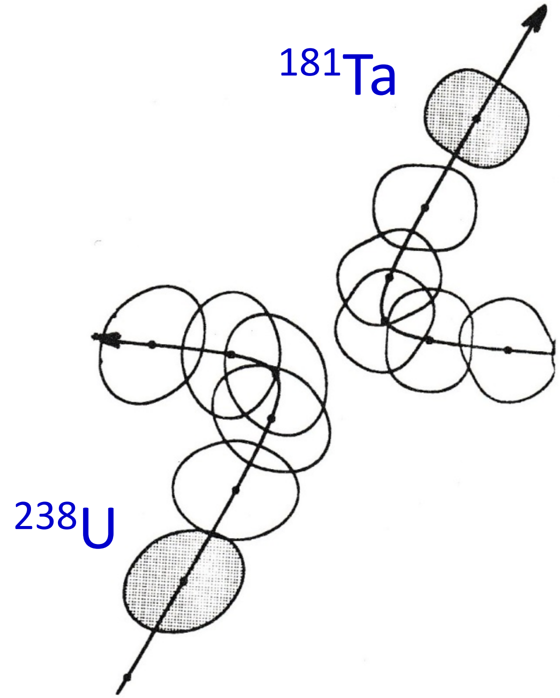
with

$cos\theta_{\gamma 1} = cos\vartheta_1 cos\vartheta_\gamma + sin\vartheta_1 sin\vartheta_\gamma cos(\varphi_\gamma - \varphi_1)$

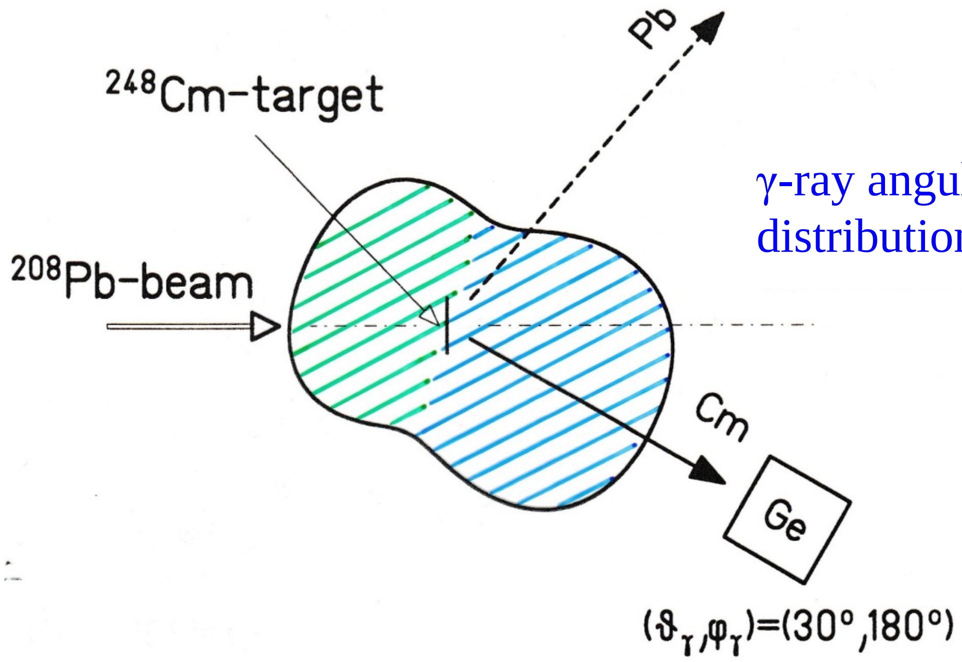
$\Delta E \cong E^* \cdot \beta_1 \cdot sin\theta_{\gamma 1} \cdot \Delta\theta_{\gamma 1}$ Doppler broadening

Example: inelastic heavy-ion scattering





Lorentz transformation



γ-ray angular distribution

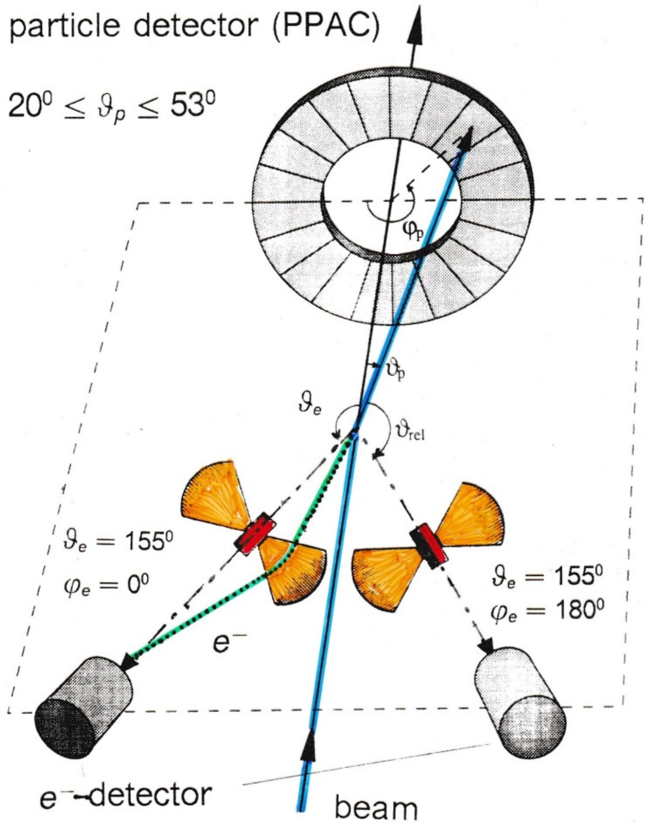
Contraction of the solid angle element in the laboratory system

$$\frac{d\Omega}{d\Omega^*} = \left\{\frac{E^*}{E}\right\}^2$$

with

$$E^* = \gamma \cdot E \cdot (1 - \beta \cdot cos\theta)$$ Doppler formula

Experimental arrangement (electron detection)



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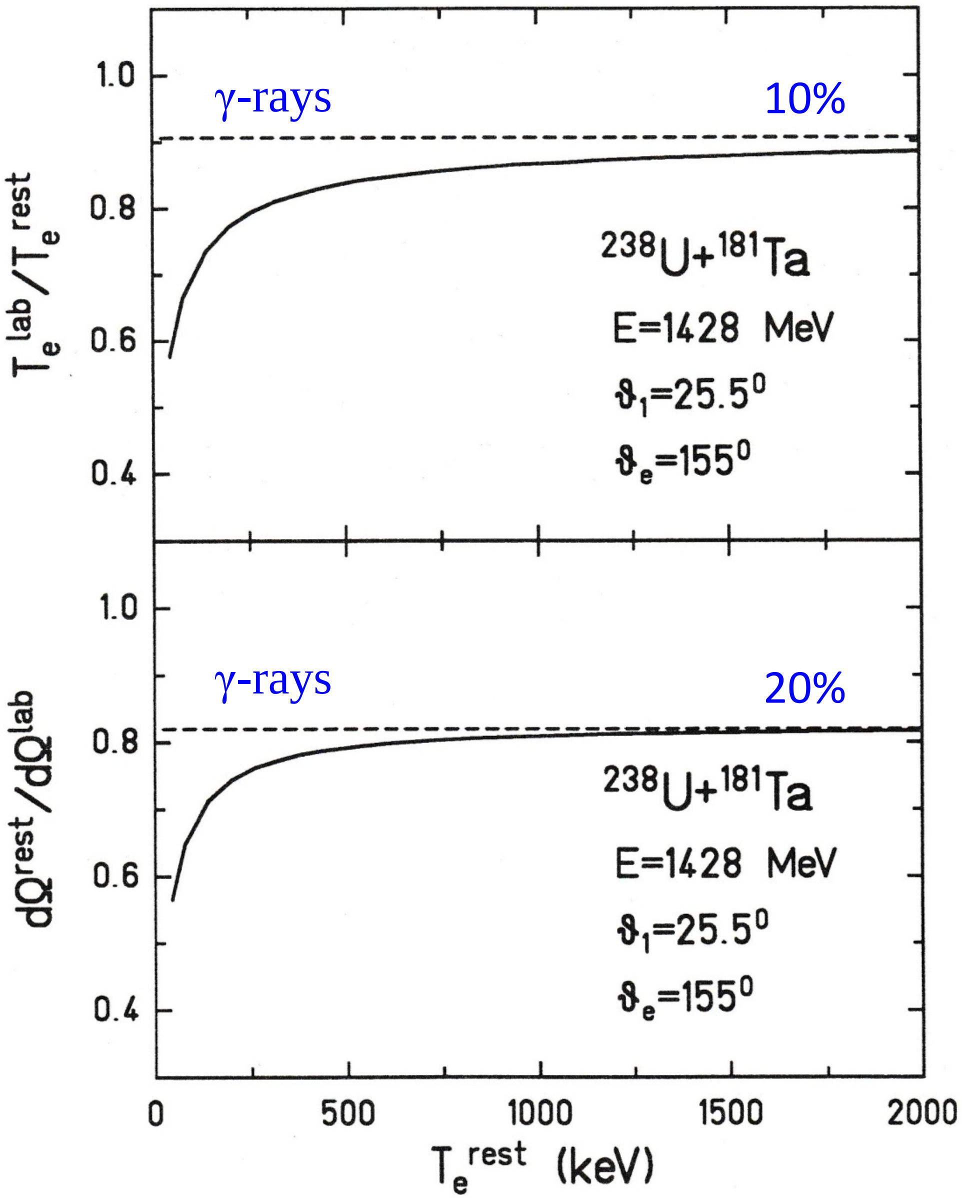
with

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

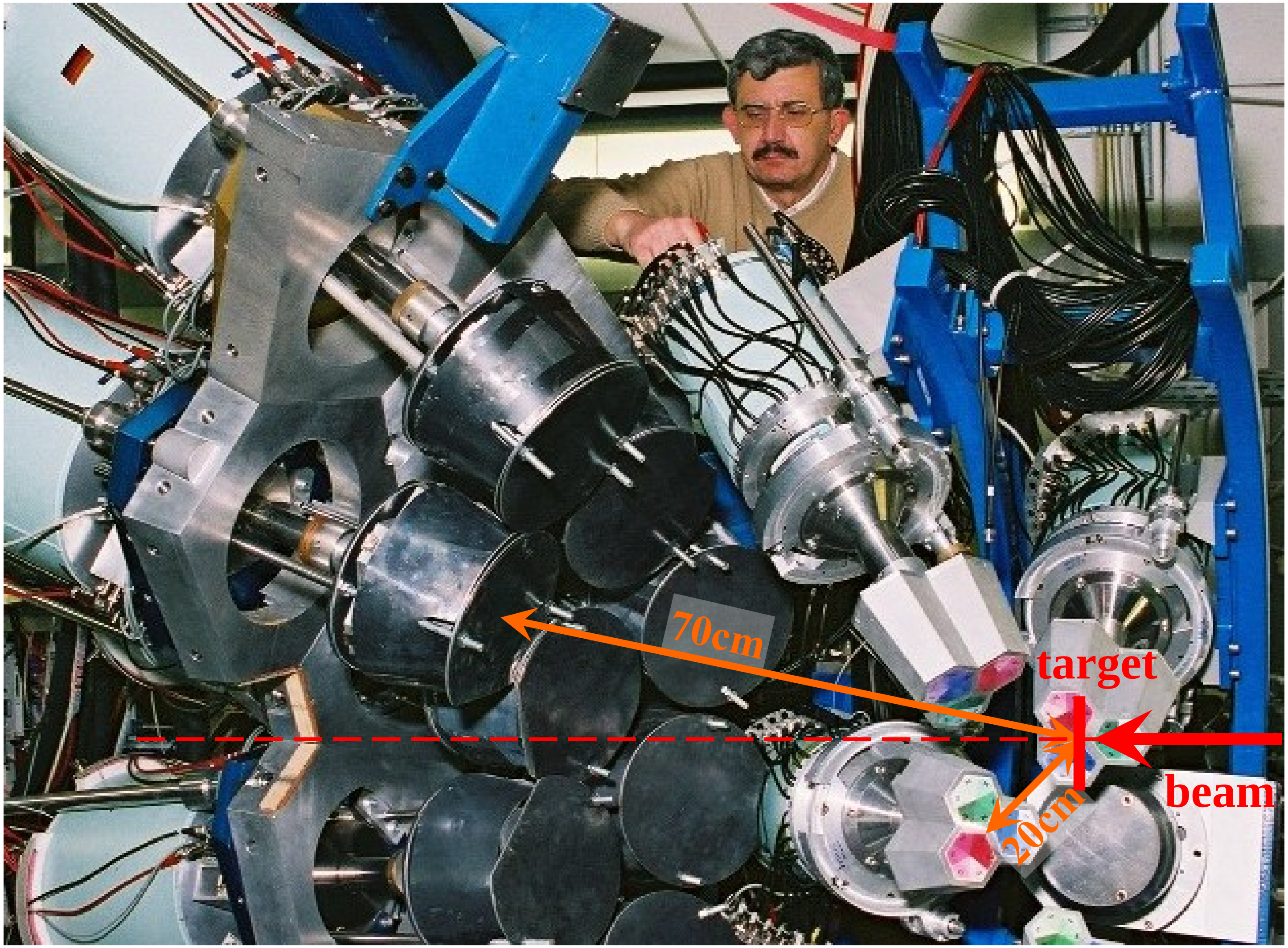
Lorentz transformation



energy shift

solid angle contraction

Segmented detectors

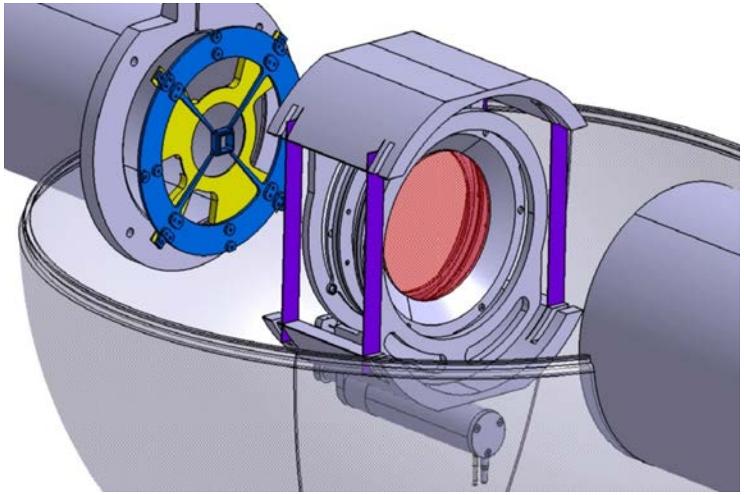


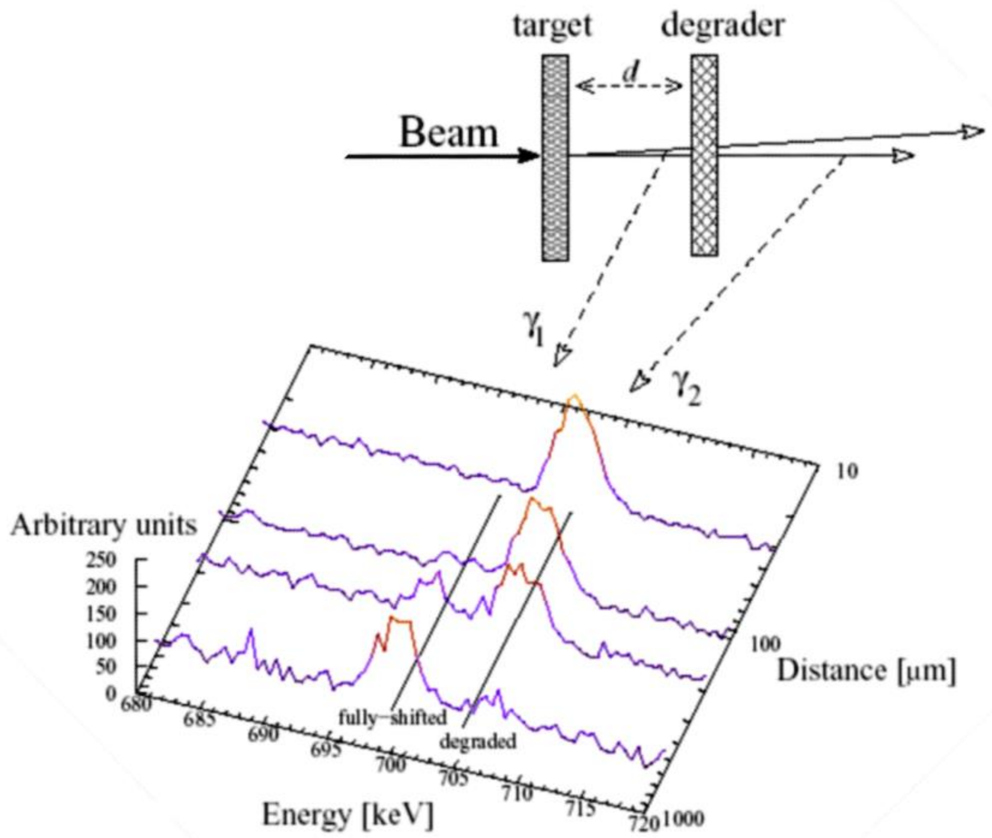
Recoil distance method

$$I_{degraded} = I \cdot e^{-d/v\tau}$$

$$I_{shifted} = \left(1 - e^{-d/v\tau}\right)$$

$$\frac{I_{degraded}}{I_{degraded} + I_{shifted}} = e^{-d/v\tau}$$





Doppler Shift Attenuation Method