Outline: Nuclear fission

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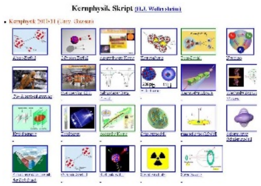
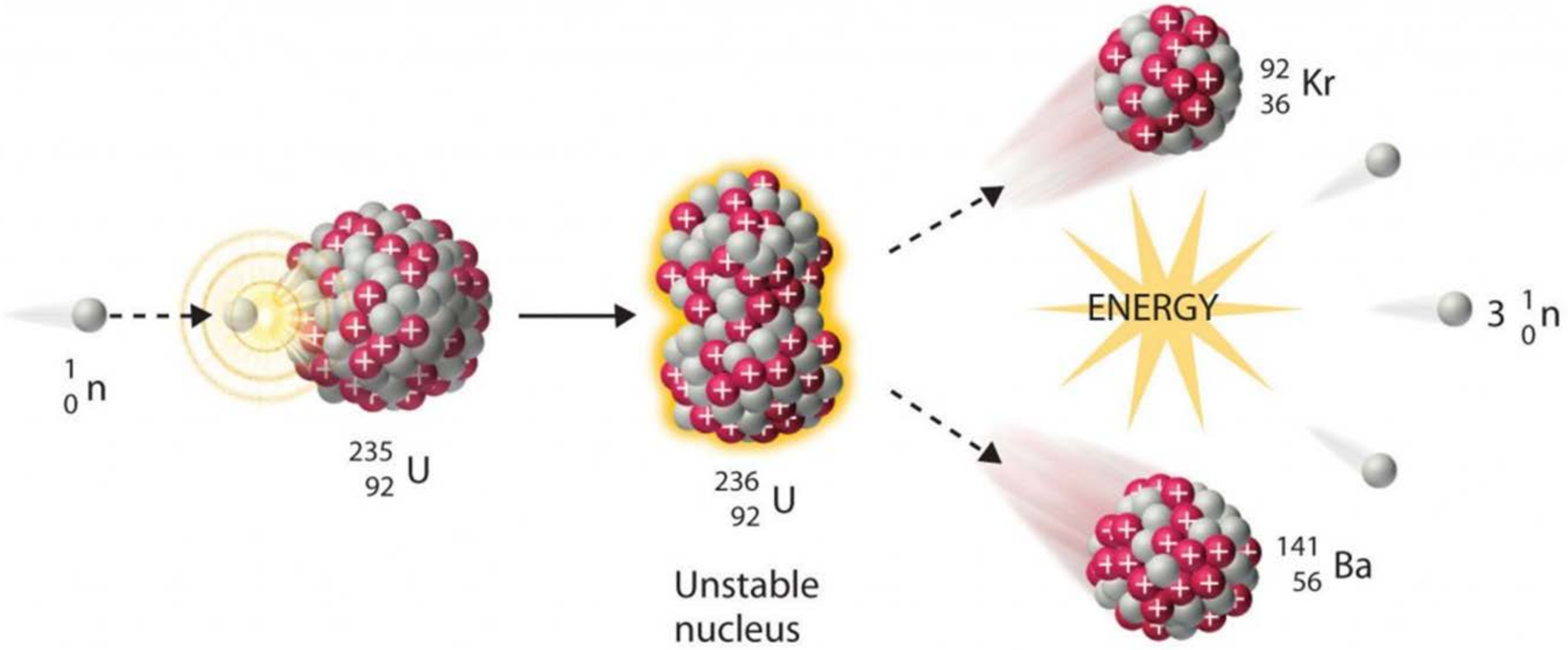


Kernphysik. Skript (H.J. Wollersheim)

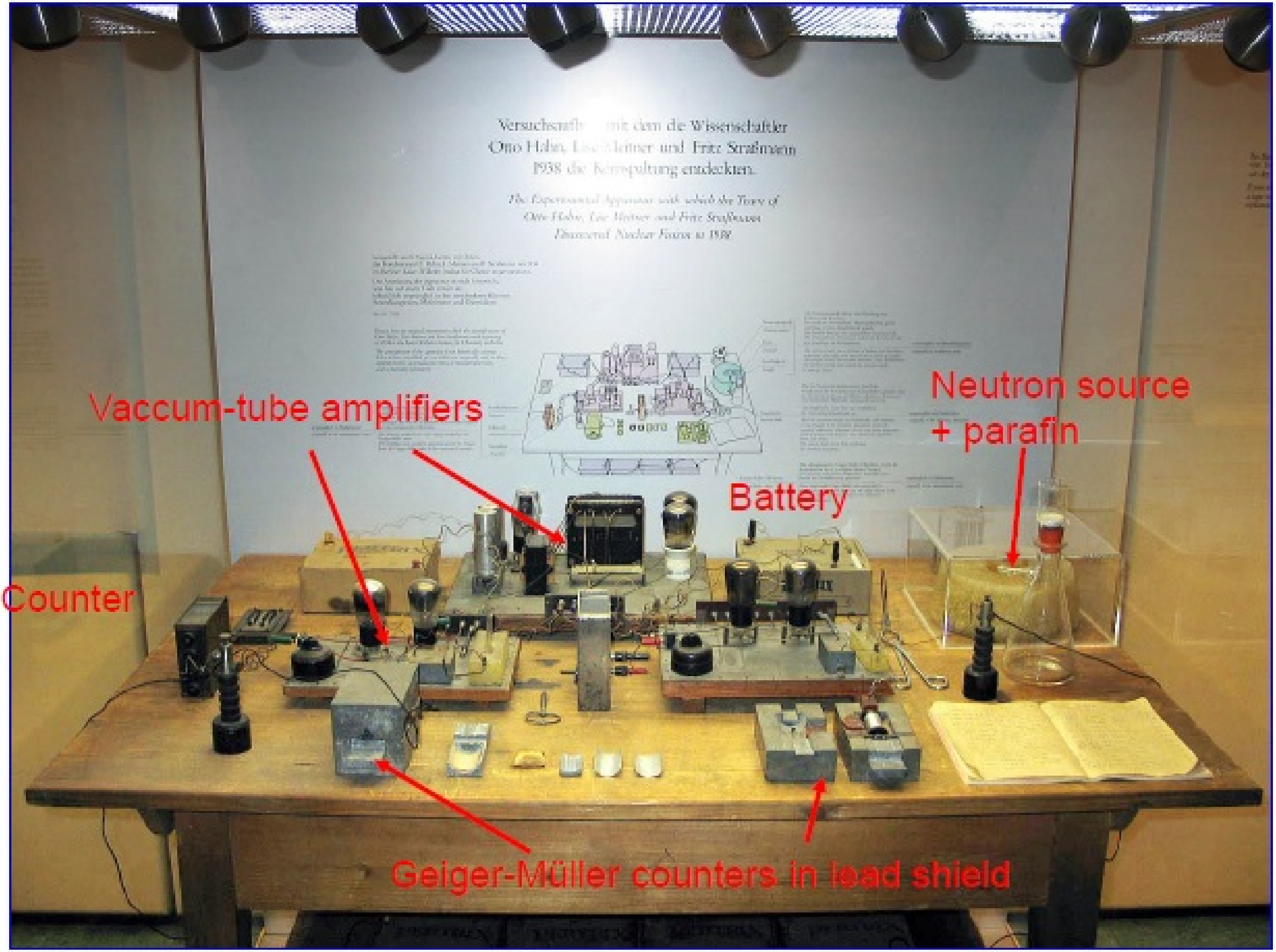
1. history
2. energy gain
3. fission barrier
4. neutron induced nuclear fission
5. nuclear reactor

Nuclear fission



Lise Meitner, Otto Hahn

Hahn-Meitner-Strassmann experimental set-up

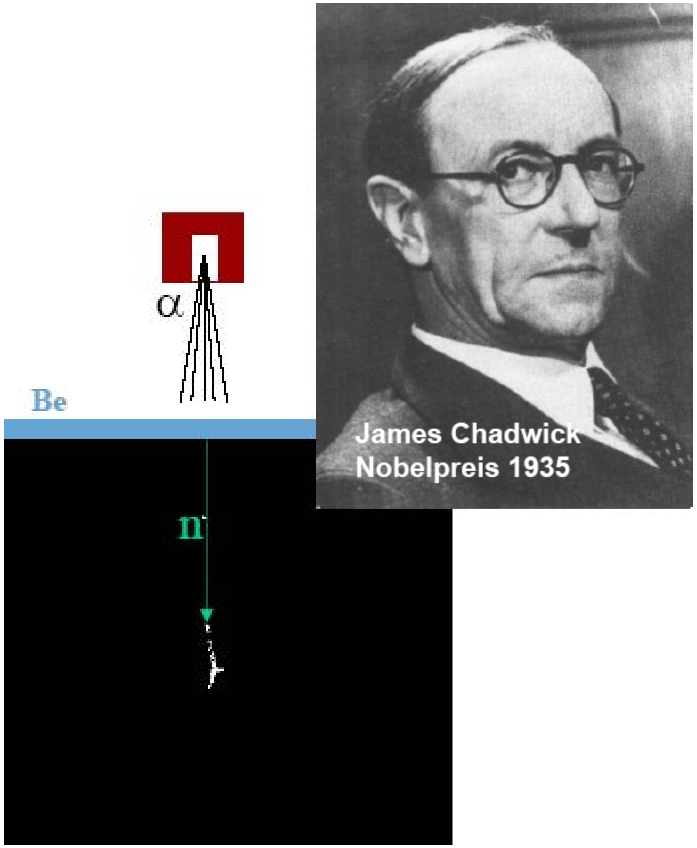


Discovery of fission and the chain reaction

1932 Discovery of the neutron by James Chadwick

($^{4}He + {}^{9}Be \rightarrow {}^{12}C + {}^{1}n + \gamma$)

(Nobel prize 1935)



1933 Fermi bombarded different nuclei with moderated neutrons and discovered the induced radioactivity (Nobel prize 1938 und emigration)

Enrico Fermi (1901-1954)
Theoretician and experimentalist;
Fermi statistics, weak interaction,
first nuclear reactor

Discovery of fission and the chain reaction

1938 Discovery of nuclear fission by Hahn, Meitner, Strassmann using radiochemical methods to verify the fission product Barium (Nobel price 1944 without L.M.)

Otto Hahn (1879-1968)
Lise Meitner (1878-1968)
(Emigration 1938)
Fritz Strassmann (1902-1980)

1939 Detection of fission neutrons, with the potential of a chain reaction (Szilard predicted this in 1933)

1942 Start of the Manhatten projekt, initiator Szilard (1939 letter from Szilard, Einstein, Wigner to Roosevelt)

1942 Fermi built a nuclear reactor and achieved the first controlled nuclear fission reaction

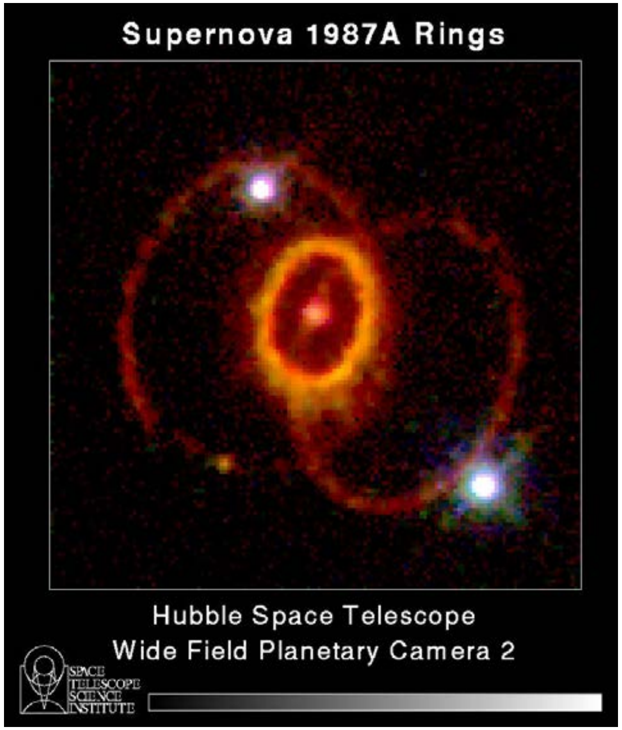
1945 Atomic bomb (fission of U-235 and Pu-239) dropped on Hiroshima und Nagasaki

Uranium decay

❖ In a **Supernova-explosion**, approximately 6 billion years ago, the isotopes Uranium-238 ($T_{1/2}$ =4.5×10^9 years), Uranium-235 ($T_{1/2}$ =0.7×10^9 years) and Plutonium-239 ($T_{1/2}$ =24×10^3 years) have been produced in equal parts.

Today Pu-239 is completely decayed, while from U-235 and U-238 0.3 % and 40 % are left over, respectively. Uranium fuel elements require a U-235 enrichment of at least 3 %.

❖ Good Uranium mining means a content of 0,3 % Uranium in sedimentary rock and it will be mined for $50/kg Uranium, the world reserve amounts to about 10 Mio tons of natural Uranium. In addition, 100 ×10^6 tons are estimated in rocks with mining costs of $300/kg. Certain are 4,2 ×10^9 tons of natural Uranium in sea water which can be extracted for $500/kg.



- Alpha-decay:

$$^{238}_{92}U \rightarrow\ ^{234}_{90}Th + \alpha$$

- Spontaneous fission:

  - decay of natural Uranium isotopes

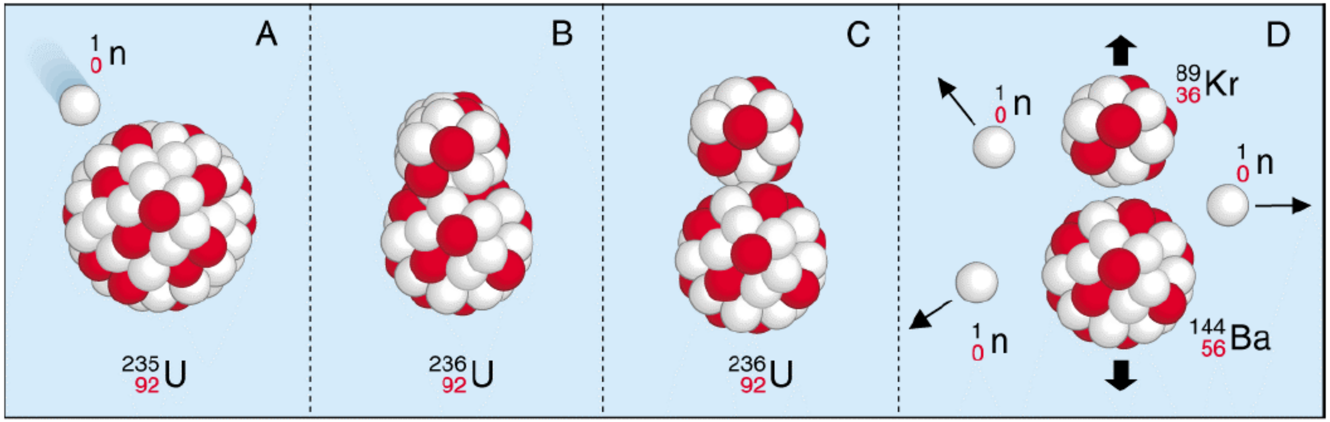
$$^{238}_{92}U \rightarrow\ ^{140}_{54}Xe + \ ^{96}_{38}Sr + 2\ ^{1}_{0}n$$

$$^{235}_{92}U \rightarrow\ ^{142}_{56}Ba + \ ^{90}_{36}Kr + 3\ ^{1}_{0}n$$

Uranium is a silver white shining, soft heavy metal

Energy gain for ^{235}U

Energy balance of $^{235}_{92}U$ - fission



Mass data: www.nndc.bnl.gov/qcalc http://nrv.jinr.ru/nrv/webnrv/qcalc/

Mass (1u=931.478MeV/c^2):

236.045562u → 88.917633u + 143.922940u + 3.025995u

Energy gain: 166.73MeV

Binding energy [M(A,Z) - Z·M(^{1}H) - N·M(^{1}n)] :

-1790.415MeV → -766.908MeV - 1190.239MeV

Energy gain: 166.73MeV

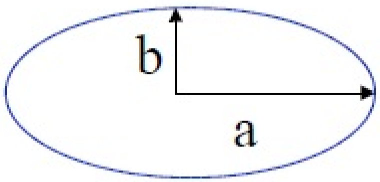
1g Uranium → fission products 68 Million kJ

Mass excess [M(A,Z) - A] :

42.441MeV → -76.725MeV – 71.780MeV + 24.214MeV

Energy gain: 166.73MeV

Nuclear fission – spontaneous fission



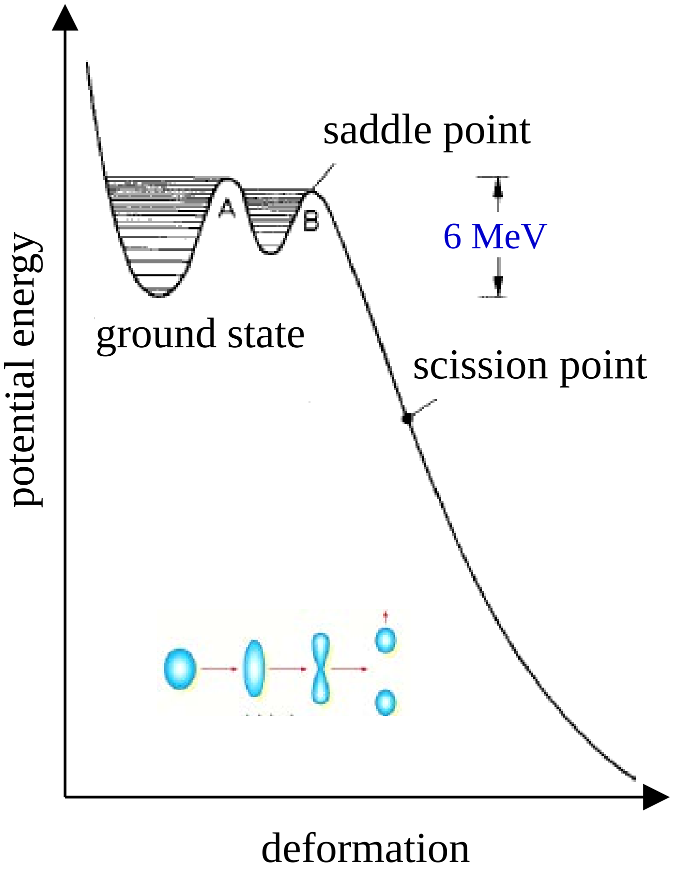
$$a = R \cdot (1 + \varepsilon)$$

$$b = R \cdot (1 + \varepsilon)^{-1/2}$$

$$\boxed{E_S = a_S \cdot A^{2/3} \cdot \left(1 + \frac{2}{5}\varepsilon^2 + \cdots\right)}$$

$$\boxed{E_C = a_C \cdot Z^2 \cdot A^{-1/3} \cdot \left(1 - \frac{1}{5}\varepsilon^2 + \cdots\right)}$$

$$\boxed{\Delta E = \frac{\varepsilon^2}{5} \cdot \left(2 \cdot a_S \cdot A^{2/3} - a_C \cdot Z^2 \cdot A^{-1/3}\right)}$$

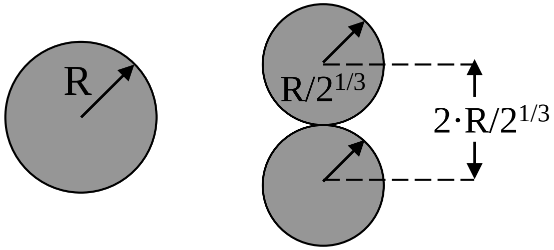


fission barrier ΔE disappears for $\frac{Z^2}{A} \geq \frac{2a_S}{a_C} \approx 48$

This is the case for nuclei with Z>114 and A>270

Spontaneous nuclear fission

Fission is caused in heavy nuclei by the increasing **Coulomb force** between the protons.



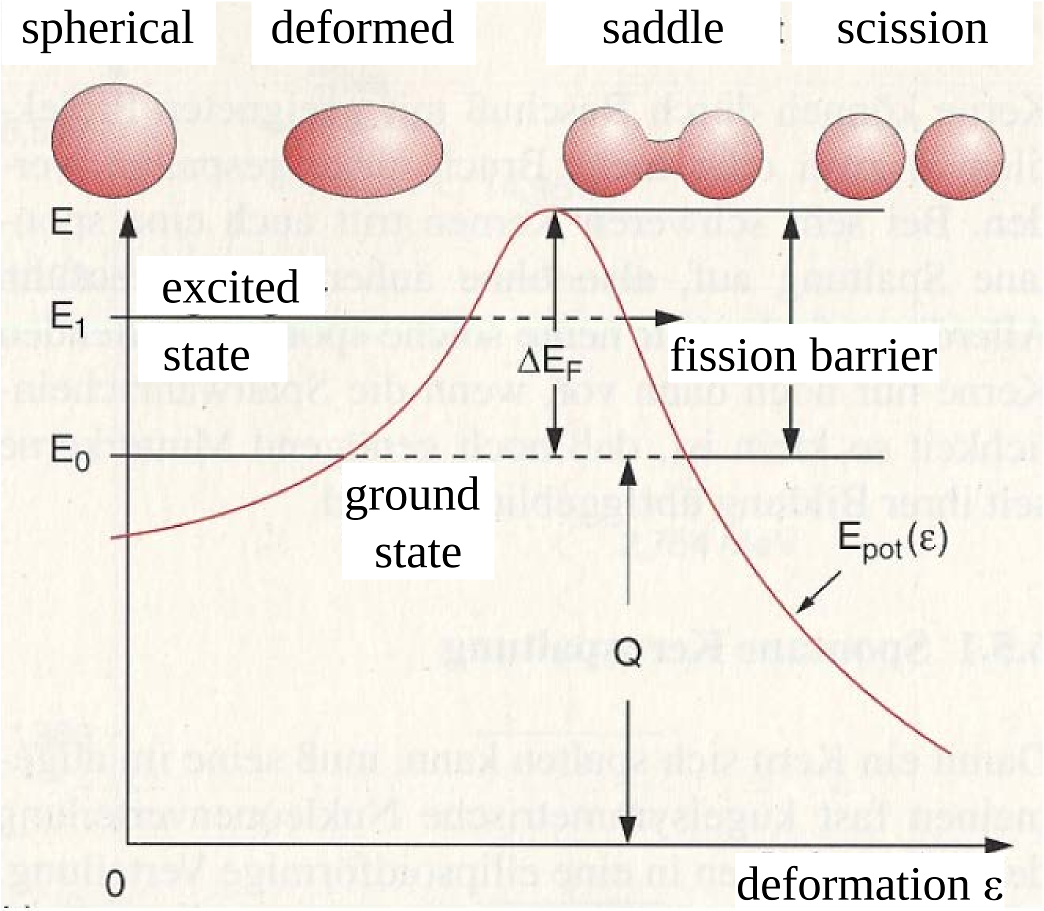
$$V_C = \frac{Z^2\cdot e^2}{8\cdot(1/2)^{1/3}\cdot r_0\cdot A^{1/3}} = 0.189\cdot\frac{Z^2}{A^{1/3}} \quad [MeV]$$

$$\boxed{E_{\exp} = 0.1071\cdot\frac{Z^2}{A^{1/3}} + 22.2 \quad [MeV]}$$

Fission fragments are deformed

The surface energy and the Coulomb energy is changed due to the ellipsoidal deformation during the fission process.

By exanimating both energy terms one realizes that nuclei with **Z²/A ≥ 51** undergo fission spontaneously.



$$X_s = \frac{1}{2}\cdot\frac{E_C}{E_S} = \frac{a_C}{2\cdot a_S}\cdot\frac{Z^2}{A} > 1$$

Spontaneous nuclear fission

Fission barrier:

$$\Delta E_F = E_{Coul} - E_0$$

liquid drop model:

$$\frac{\Delta E_F}{E_S^0} = \begin{cases} 0.38 \cdot (0.75 - X_S) & 1/3 < X_S < 2/3 \\ 0.83 \cdot (1 - X_S)^3 & 2/3 < X_S < 1 \end{cases}$$

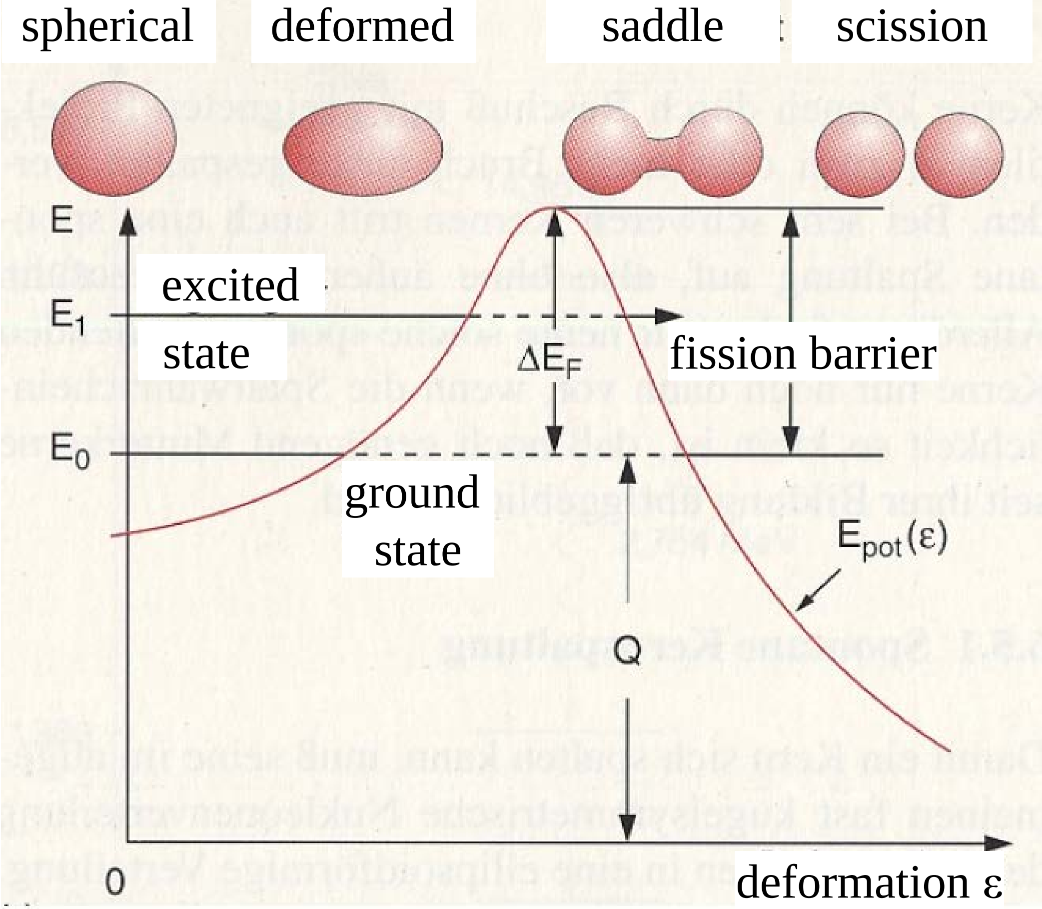
$$E_S^0 = 17.9439 \cdot \left[1 - 1.7826 \cdot \left(\frac{N-Z}{A} \right)^2 \right] \cdot A^{2/3} \quad [MeV]$$

$$E_C^0 = 0.7053 \cdot \frac{Z^2}{A^{1/3}} \quad [MeV]$$

$$X_s = \frac{1}{2} \cdot \frac{E_C^0}{E_S^0}$$

this ratio plays an enormous role

Cohen, Plasil, Swiatecki, Ann. of Phys. 82 (1974), 557

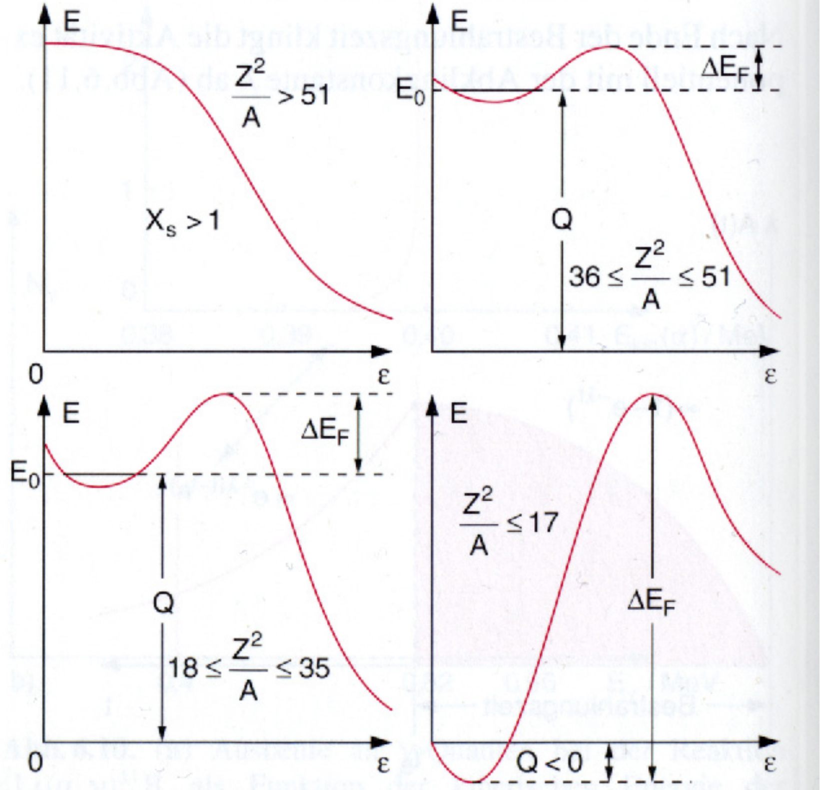


	Z^2/A	E_S^0 [MeV]	E_C^0 [MeV]	X_S	ΔE_F [MeV]
^{235}U	36.02	626.0	967.4	0.773	6.1
^{238}U	35.56	625.9	963.3	0.770	6.4

Energy barrier for spontaneous fission

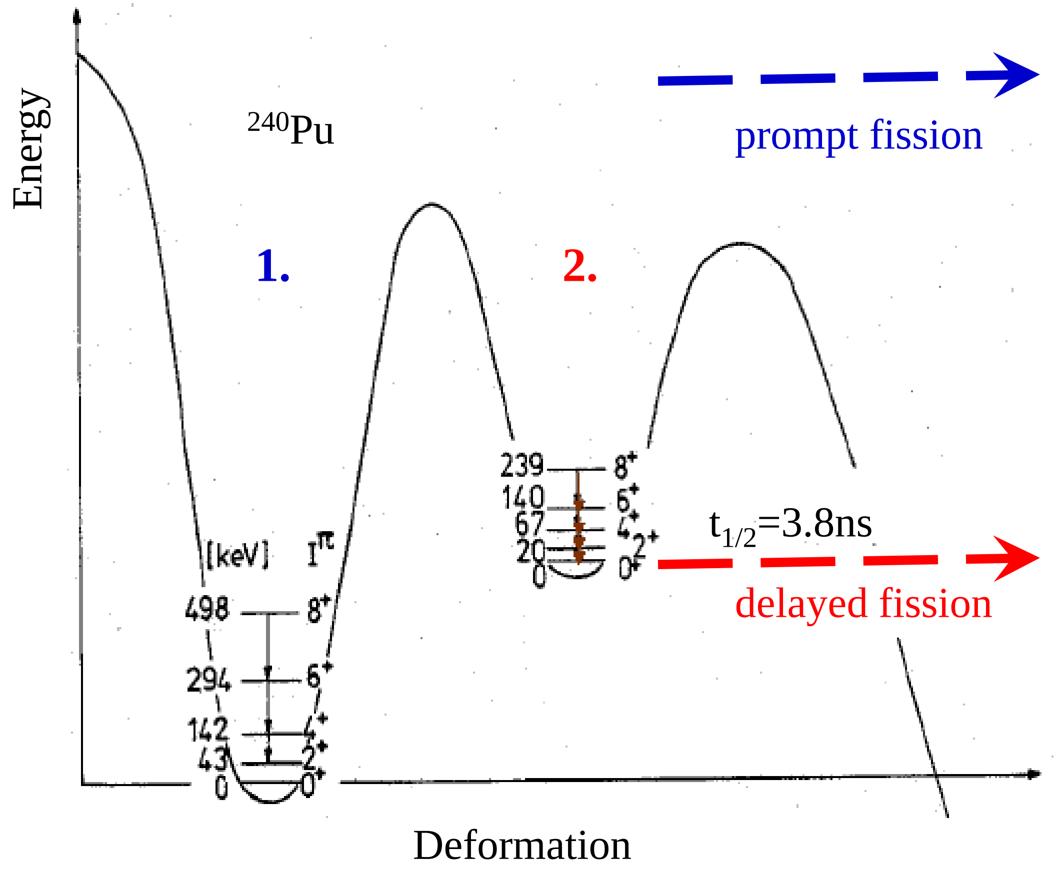
For nuclei with $Z^2/A < 51$ one has to add **energy ΔE_F** in order to observe fission. However, fission is still possible due to the tunneling effect.

The probability for tunneling decrease however very rapidly with decreasing values of Z^2/A, since the fragments have comparatively large masses.



fission barrier ΔE_F for different values of Z^2/A

Double humped fission barrier

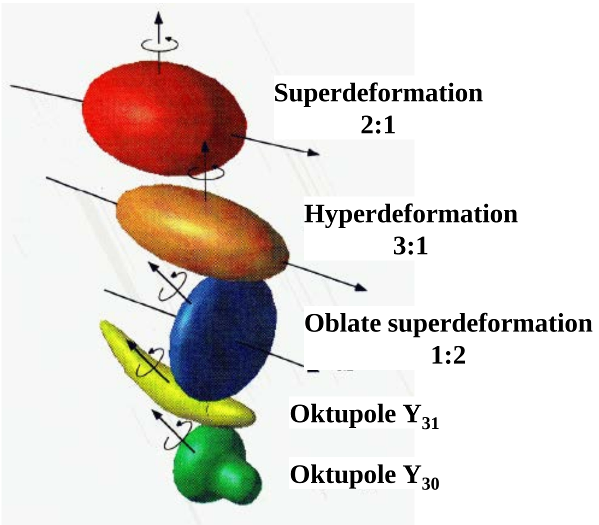


^{238}U(α,2n)^{240f}Pu, E_α=25 MeV

Measurement of conversion electrons

$$\frac{\hbar^2}{2\cdot\Im} = 3.34\ keV$$

(axis ratio 2:1)



Induced nuclear fission

Neutrons have not to overcome a Coulomb barrier. Therefore, slow neutrons can also induce nuclear fission.

$$ {}^{238}_{92}U + n \rightarrow {}^{239}_{92}U^{*} $$

Compound-nucleus: even-odd

$Q_{fission} = [M(^{238}U) + M(^{1}n) - M(^{239}U)]\cdot c^2$ = 4.8 MeV

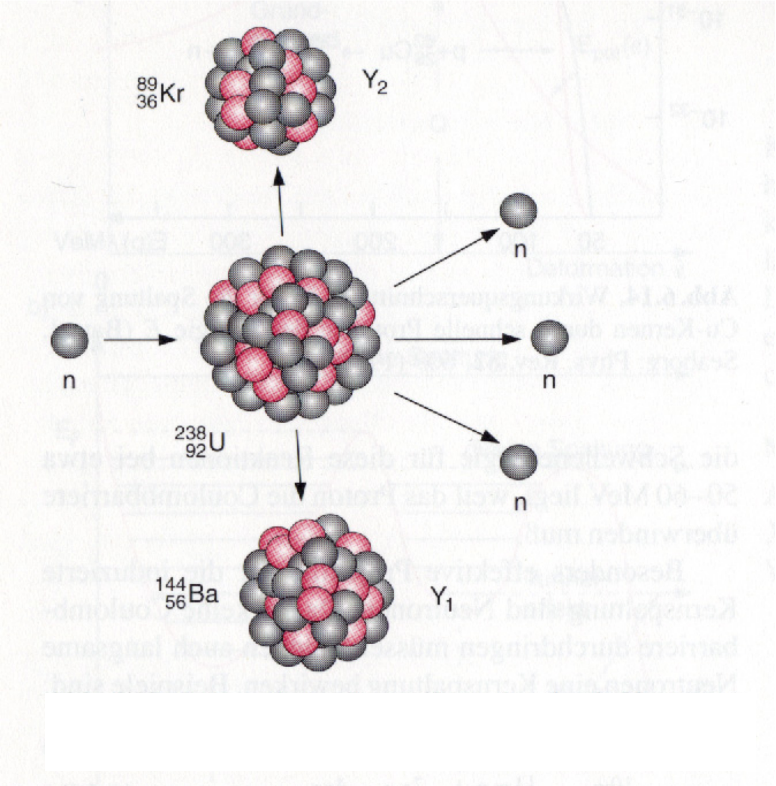
excitation energy relatively small $Q_{fission} < \Delta E_F$ = 6.4 MeV
fission is not easily possible

$$ {}^{235}_{92}U + n \rightarrow {}^{236}_{92}U^{*} $$

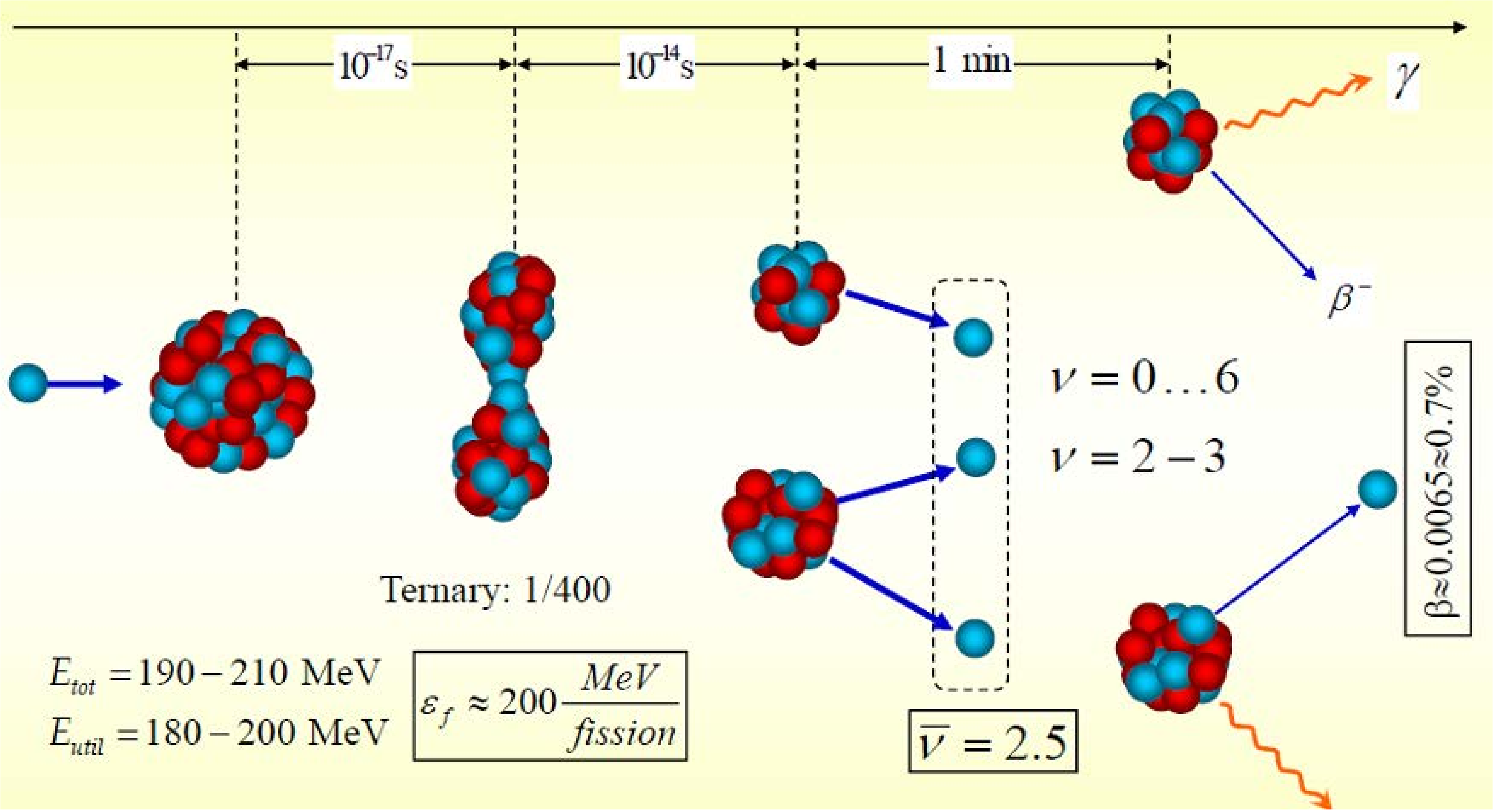
Compound-nucleus: even-even

$Q_{fission} = [M(^{235}U) + M(^{1}n) - M(^{236}U)]\cdot c^2$ = 6.5 MeV

excitation energy relatively large $Q_{fission} > \Delta E_F$ = 6.1 MeV
fission is easily possible



Characteristic properties of nuclear fission



Ternary: 1/400

$\nu = 0 \dots 6$

$\nu = 2 - 3$

$\overline{\nu} = 2.5$

$\beta \approx 0.0065 \approx 0.7\%$

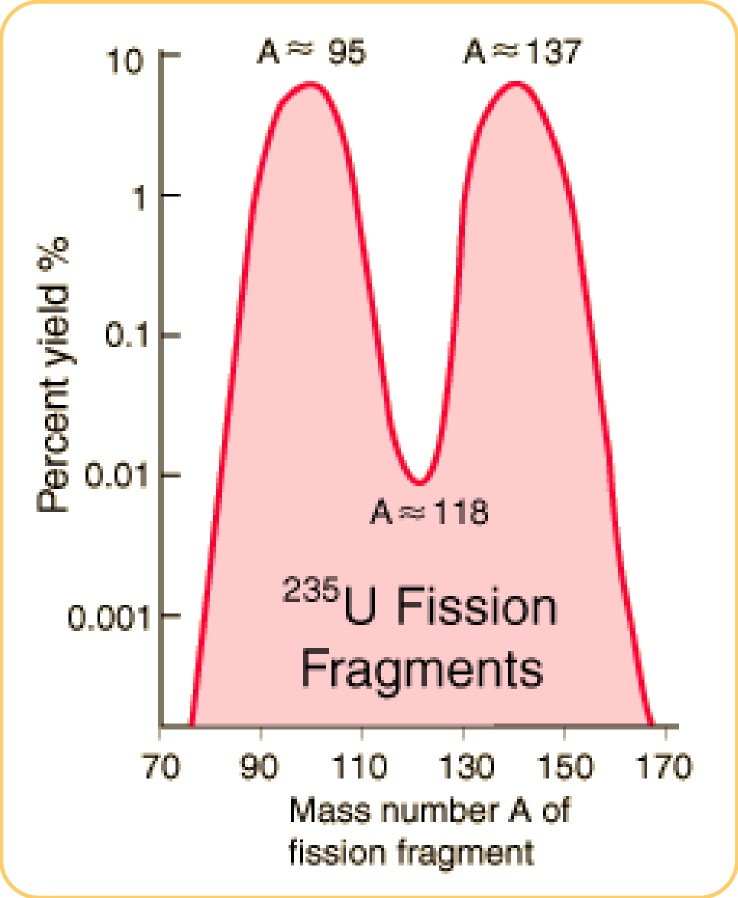
$E_{tot} = 190 - 210$ MeV

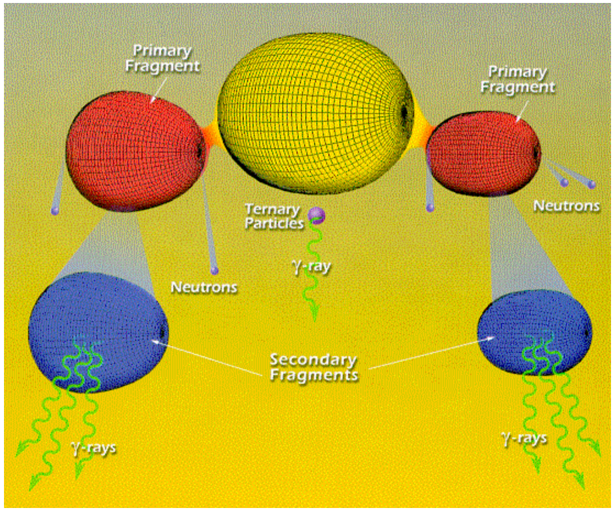
$E_{util} = 180 - 200$ MeV

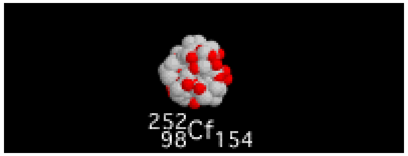
$$\varepsilon_f \approx 200 \frac{MeV}{fission}$$

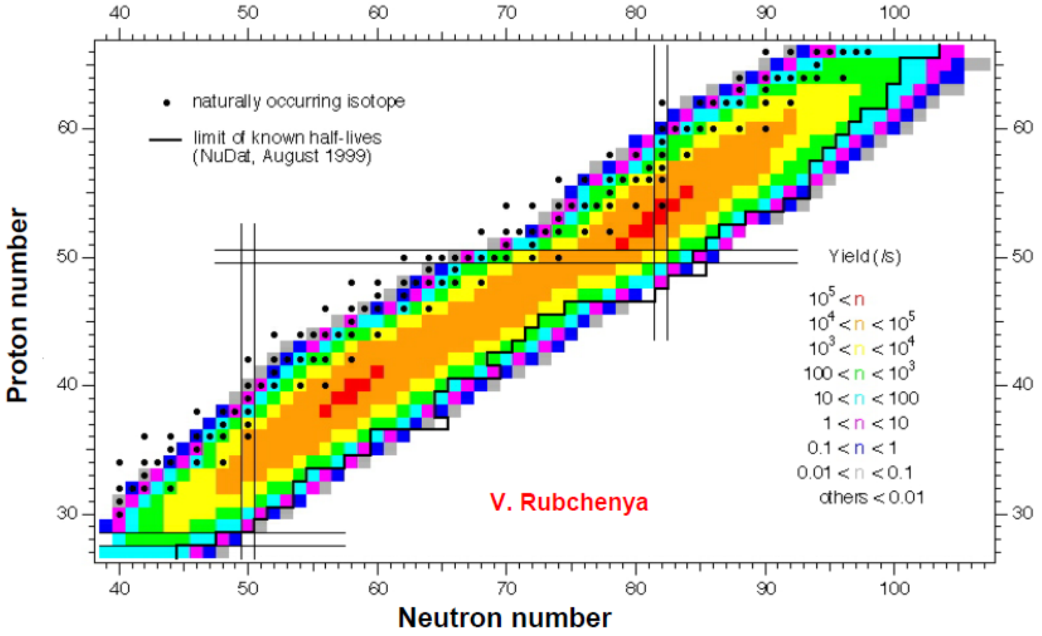
Characteristic properties of nuclear fission

a) fission **asymmetric** ⇒ multiple **highly excited** daughter nuclei



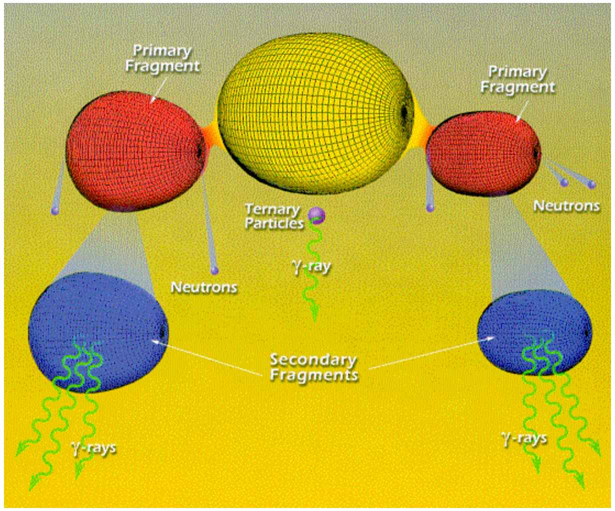






Characteristic properties of nuclear fission

a) fission **asymmetric** ⇒ multiple **highly excited** daughter nuclei



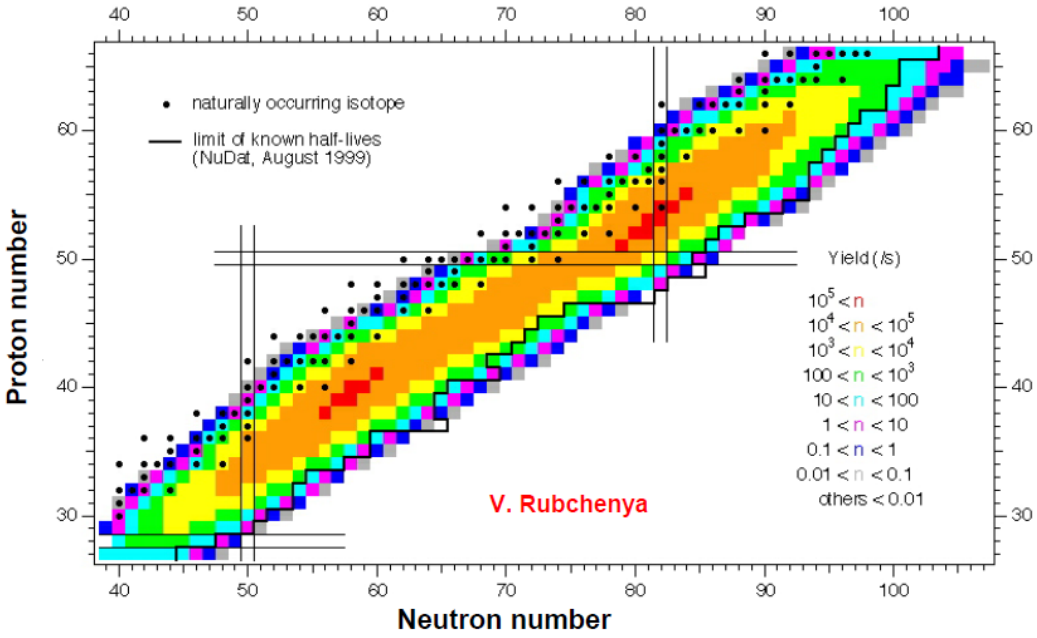
b) Neutron excess in the daughters: $\left.\frac{Z}{A}\right|_{U} < \left.\frac{Z}{A}\right|_{A<100}$

⇒ many **β⁻-instable** daughter nuclei (often long living)

examples:

$$^{140}_{54}\text{Xe} \xrightarrow{16\text{ s}} {}^{140}_{55}\text{Cs} \xrightarrow{66\text{ s}} {}^{140}_{56}\text{Ba}$$

$$^{99}_{41}\text{Nb} \xrightarrow{2{,}4\text{ min}} {}^{99}_{42}\text{Mo} \xrightarrow{67\text{ a}} {}^{99}_{43}\text{Tc}$$



- ➢ **≈1000 different β⁻-instable** nuclei afte
- ➢ **long lived β⁻**-emitter together with ^{239}

Characteristic properties of nuclear fission

c) fragments highly excited & neutron excess

$\Rightarrow$ **prompt** ($\Delta t < 10^{-16}$s) **neutron emission**

examples:

$$n_{\text{thermal}} + {}^{235}_{92}\text{U} \rightarrow Y_1 + Y_2 + \nu_n \cdot n \qquad \bar{\nu}_n = 2{,}42$$

$$n_{\text{thermal}} + {}^{239}_{94}\text{Pu} \rightarrow Y_1 + Y_2 + \nu_n \cdot n \qquad \bar{\nu}_n = 2{,}87$$

neutron-energy spectrum ↔ evaporation from a moving source

Maxwell-Boltzmann distribution

$k = 8.617 \cdot 10^{-5}$ eV/^{0}K

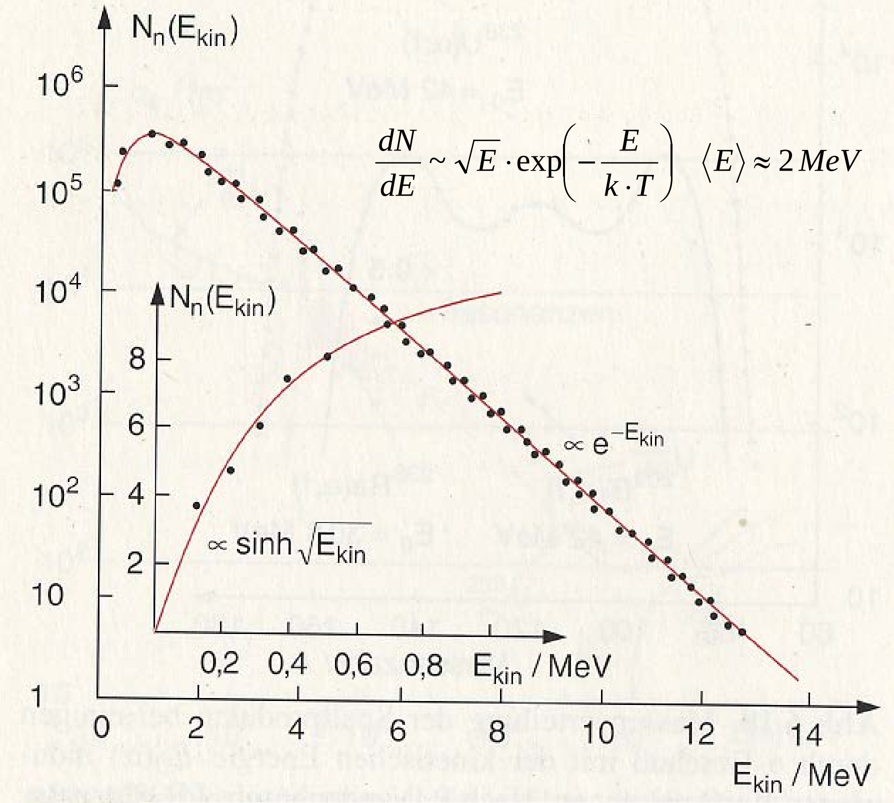
$k \cdot T_0 = 0.0253$ eV for $T_0 = 293.61\ ^0$K

Important:

Approximately 99% of all neutrons are immediately released

Only 1% are delayed emitted within a time window of 0.05s < t < 60s

→ Controlling a nuclear power plant



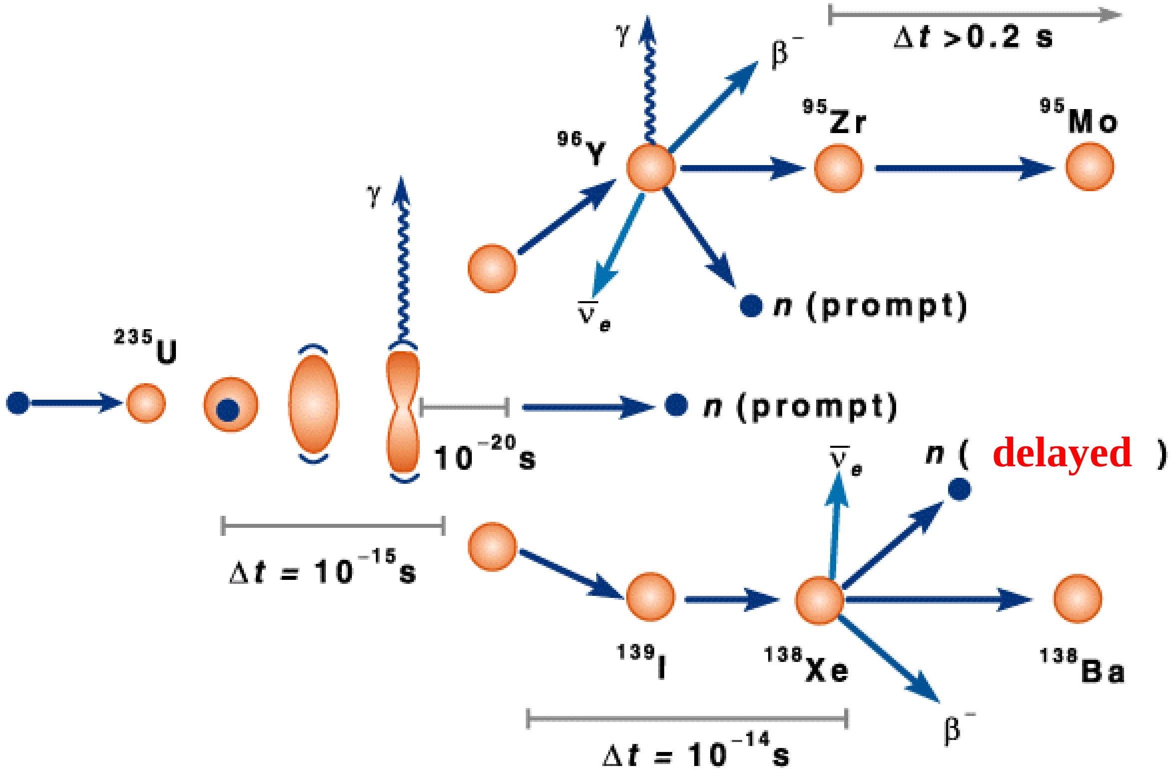
energy distribution of the fission neutrons

Characteristic properties of nuclear fission

d) **Delayed** neutrons (Δt = 0,2 s … 60 s)

$$nucleus_1 \xrightarrow[t_{1/2}\,=\,\text{time delay}]{\beta^-\text{-decay}} \text{nucleus}_2^* \rightarrow nucleus_3 + n$$

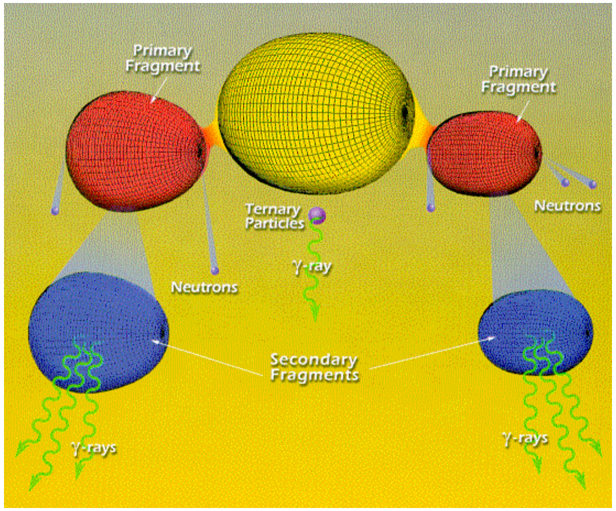
→ ~1% of the neutrons are delayed



Characteristic properties of nuclear fission

e) **Energy balance** of the $^{235}_{92}U$-fission

Y_{small}	100 MeV	β^- (fission nuclei)	8 MeV
Y_{large}	70 MeV	γ (fission nuclei)	7 MeV
$\overline{\nu}_n \cdot n$	5 MeV	neutrinos ($\overline{\nu}_e$)	12 MeV
γ (prompt)	7 MeV	total:	210 MeV

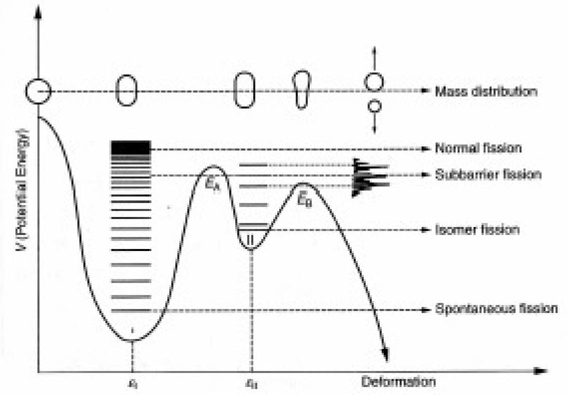


Absorption- and fission cross sections for neutrons

The neutrons, created during fission, can participate in different reactions and are hence lost for future fission processes.

example: **(n,γ) absorption reaction**

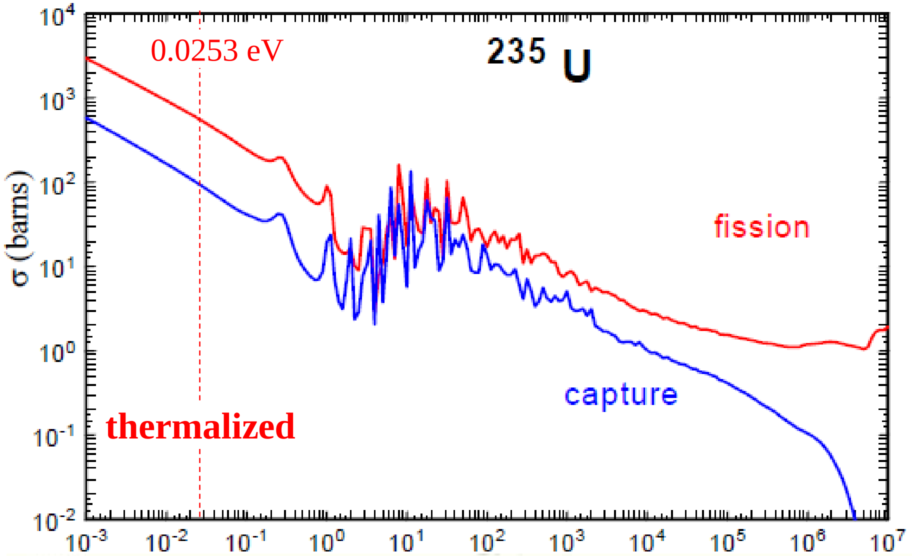
$n + U \rightarrow U^{*} \rightarrow U + \gamma$

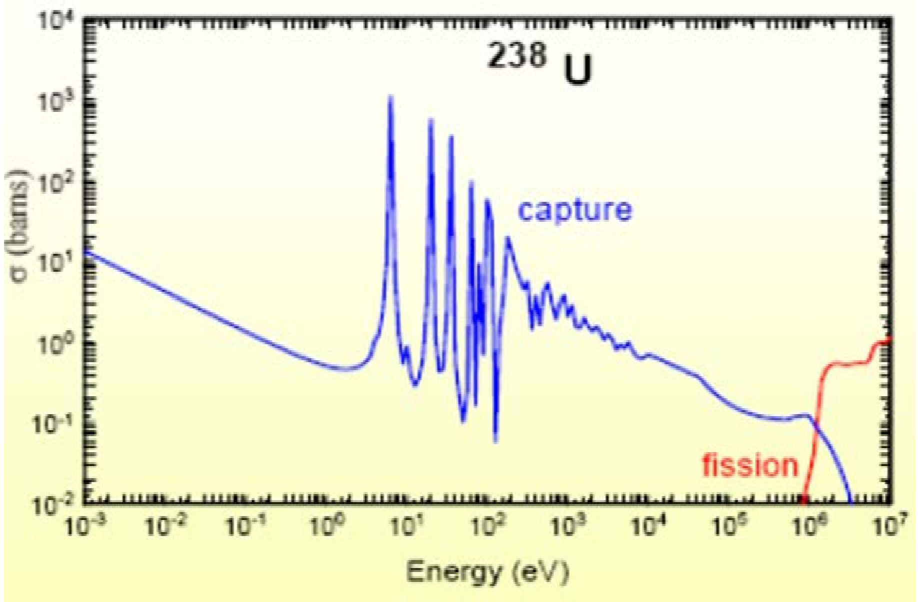


For ^{238}**U** the cross section for inelastic collisions σ(n,n‘γ) is larger than the fission cross section σ(n,f). In ^{238}U a chain reaction can not occur.

A chain reaction can occur only with thermalized neutrons and fission of ^{235}U:

⟶ slowing down (moderation) of neutrons.





Absorption- and fission cross sections for neutrons

We start with thermalized neutrons,

η is the average number of fission neutrons per thermalized neutron.

$$\eta = \frac{\sigma_{fission}}{\sigma_{fission} + \sigma_{abs}} \cdot \langle \nu \rangle$$

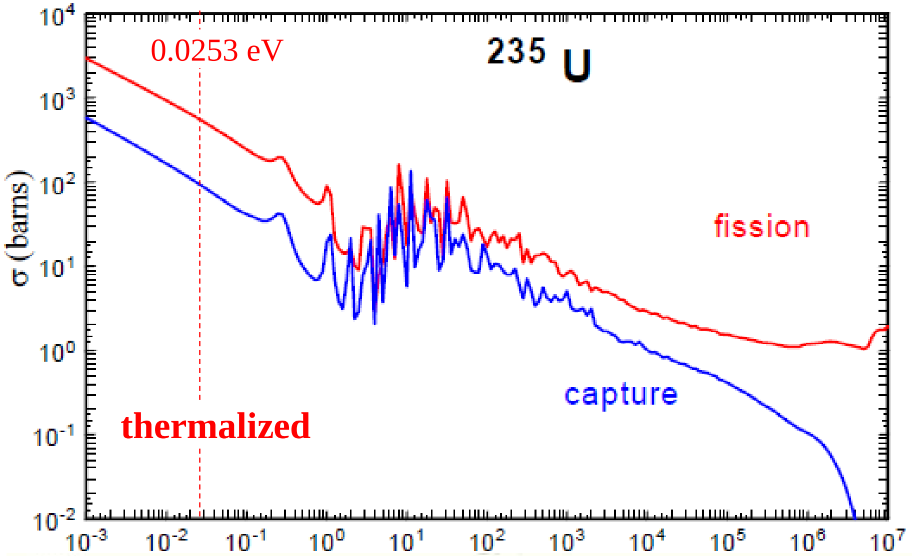
$$\sigma_{fission} = \frac{0.72}{100} \cdot \sigma_f\left({}^{235}U\right) + \frac{99.28}{100} \cdot \sigma_f\left({}^{238}U\right) = 4.20\,b$$

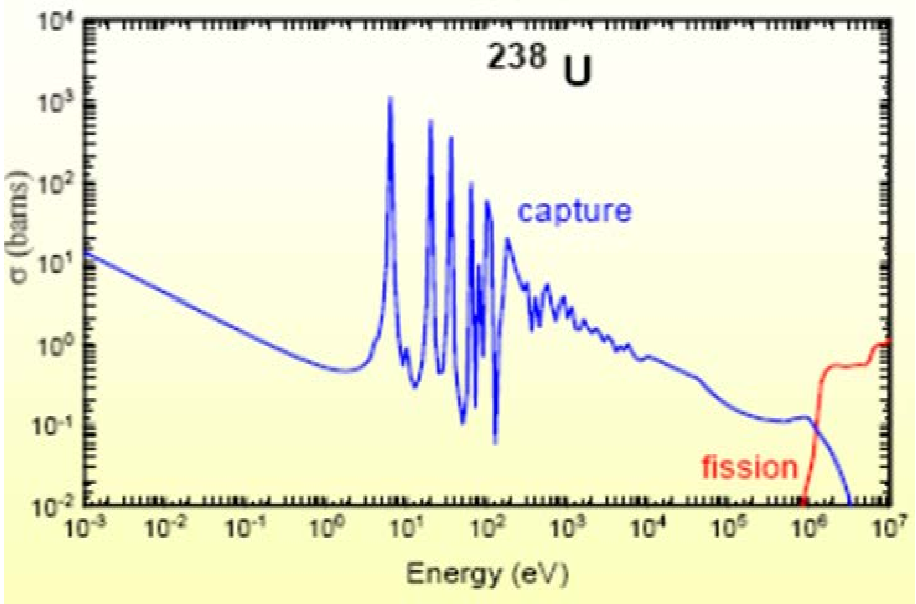
$$\sigma_{capture} = \frac{0.72}{100} \cdot \sigma_a\left({}^{235}U\right) + \frac{99.28}{100} \cdot \sigma_a\left({}^{238}U\right) = 3.43\,b$$

For ^{235}**U**: $\sigma_f = 584$ b and $\sigma_a = 97$ b, $\langle \nu \rangle = 2.4$
For ^{238}**U**: $\sigma_f = 0$ b and $\sigma_a = 2.1$ b

Effective value of **η = 1.3** for natural Uranium is too small for chain reaction.

⟶ ^{235}U has to be enriched to 3% (η=1.8)

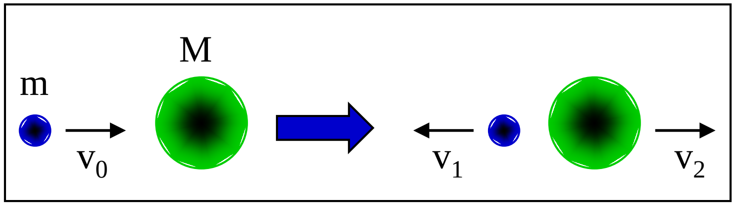




Interactions of neutrons with matter

Slowing down of neutrons by elastic nuclear collisions:

$$n(E_n) + {}^{A}K \rightarrow n(E'_n) + {}^{A}K$$



Kinematic of the reaction ⇒ $\boxed{\left(\frac{A-1}{A+1}\right)^2 E_n \le E'_n \le E_n}$

no excitation, no capture,
no fission

example:

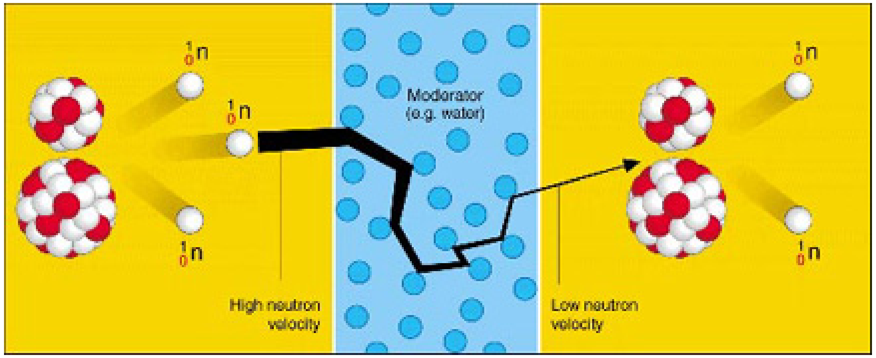
$$A = 1 \qquad 0 \le E'_n \le E_n$$

$$A = 238 \qquad 0{,}992\, E_n \le E'_n \le E_n$$

Average energy loss of the neutrons per collision:

$$\left\langle \frac{\Delta E_n}{E_n} \right\rangle = 1 - \left\langle \frac{E'_n}{E_n} \right\rangle = \frac{1}{2}\left(1 - \left(\frac{A-1}{A+1}\right)^2\right) \Rightarrow \boxed{\left\langle \frac{\Delta E_n}{E_n} \right\rangle = \frac{2A}{(A+1)^2}}$$

Thermalization of neutrons



Example: water (H_2O) as moderator → scattering on protons, $A = 1$

$$\left.\left\langle \frac{\Delta E_n}{E_n} \right\rangle\right|_{A=1} = 50\%$$

Rough estimate of the number of collisions k until thermalisation:

$$0{,}5^k \cdot E_n \approx k_B T \approx \tfrac{1}{40}\,\mathrm{eV} \quad \Rightarrow \quad k \approx \frac{1}{\ln 2} \ln \frac{E_n}{k_B T} \qquad E_n \approx 1\,\mathrm{MeV} \quad \Rightarrow \quad k \approx 25$$

Moderator	Average collision number for a slowing down from 1,75 MeV to 0.025 eV	Tendency to capture thermalized neutrons in relative units
Hydrogen	18	650
Deuterium	25	1
Beryllium	86	7
Carbon	114	10

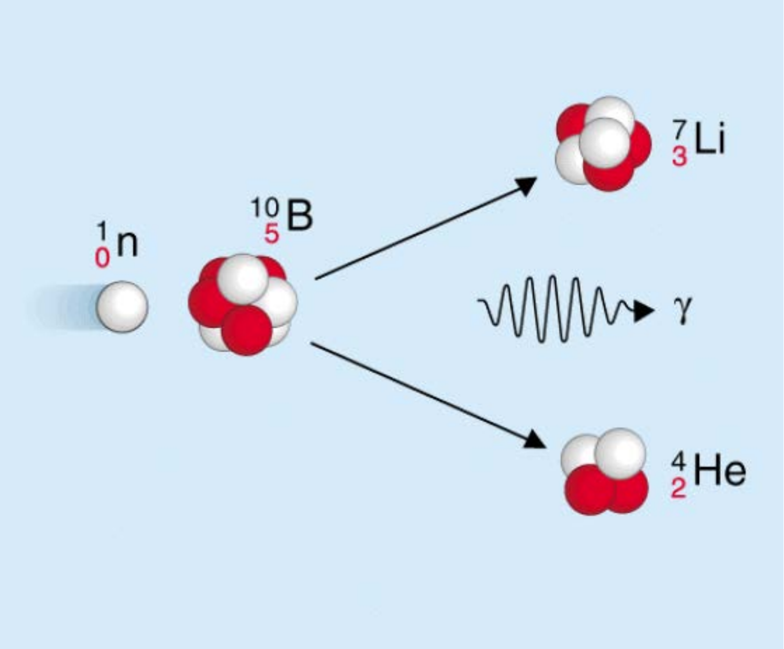
Further neutron losses

- ^{238}U-absorption
- Reactor poison, e.g. the fission product ^{135}Xe: $\sigma_f(\,^{235}U\,) \approx 500$ b

  $\sigma_{abs} \approx 3\,000\,000$ b
- Control rod material (Cd, B) ⇒ controlled neutron-absorption
- Reactor fuel: $\sigma_{tot}(\,^{235}U\,) > \sigma_f(\,^{235}U\,)$

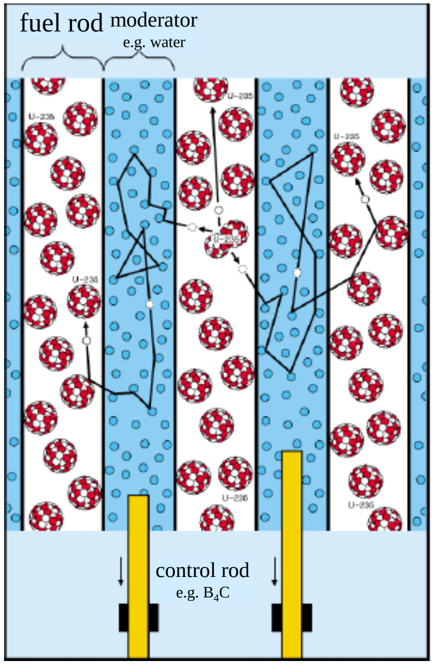
Steering of the chain reaction

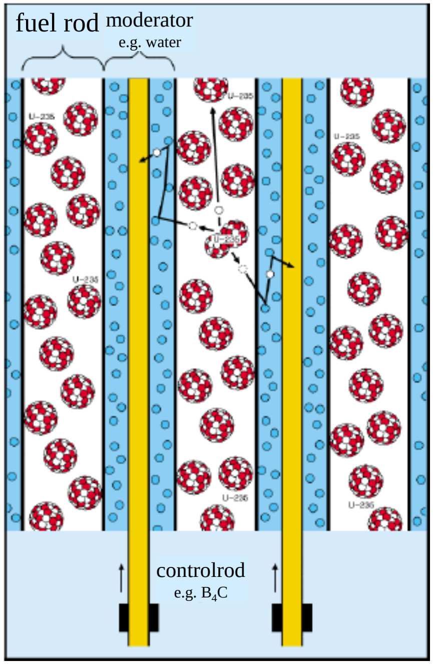
- ❖ Control rods:
  Material with large neutron-absorption: B, Cd, In, Ag



$$n + {}^{10}_{5}B \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He + \gamma$$

$$n + {}^{113}_{48}Cd \rightarrow {}^{114}_{48}Cd + \gamma$$



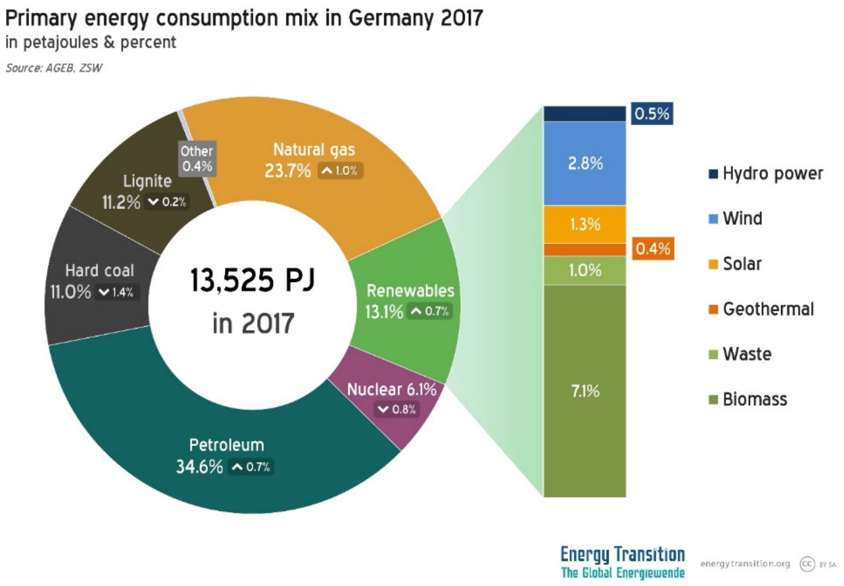


Energy consumption of humans in Germany

Information in power (Watt) = energy / time (Joule/sec) per head, annual mean

turnover of the body (food → heat)	100 Watt

total consumption on **primary energy**	5000 Watt
(civilization increases consumption by a factor of 50 !!!)	


Primary energy consumption mix in Germany 2017
in petajoules & percent
Source: AGEB, ZSW

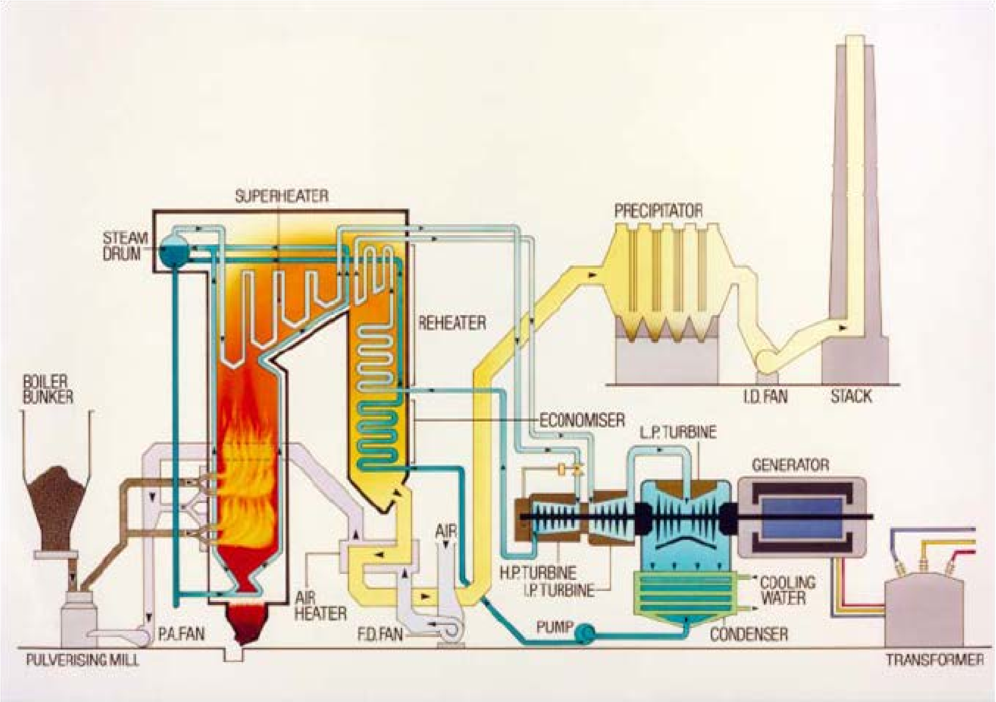
total consumption on **final energy** (2005)	3700 Watt
electrical power consumption (with industry)	750 Watt
Private households heating and hot water	1000 Watt

Chemical energy

Chemical energy is **atomic energy** in the truest sense of the word.

Origin of the chemical energy: Change of the **covalent bonds between atoms** which are the molecular building blocks.

12g Carbon-burning with 32g Oxygen (O_2)
Thermal energy: 393 kJ ~ 30 kJ/gC
reaction $C + O_2 \rightarrow CO_2 + 4.1\ \mathrm{eV}$



Thermal energy ($Q = m \cdot c \cdot \Delta T$, $c = 4180\ JK^{-1}kg^{-1}$) **:**
30 kJ can heat up 1 liter of water by 7^0 Celcius .

Electrical energy (W = power times time) **:**
30 kJ can keep a 100 Watt lamp 5 minutes long switched on.

Mechanical energy (W = mass · earth acceleration · height) **:**
30 kJ lift a mass of 70kg 43 meter high.

Comparison with coal burning

If one burns 1 kg of hard coal one obtains an available energy of 8.14 kWh.

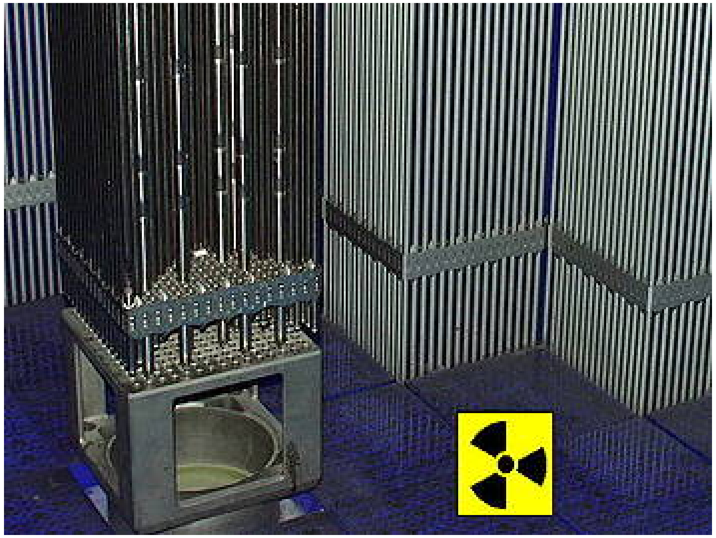
In nuclear fission of 1 kg Uranium one obtains an available energy of 22 700 000 kWh.

Nuclear bond 200 MeV is significant stronger than **molecular bond 4.1 eV.**

Uranium is as „fuel“ three million times more effective than hard coal.

In fission of 1 kg Uranium we obtain the same energy as if we would burn 2800000 kg of Carbon to 10.2 millions kg of Carbon dioxyd!!!

Nuclear reactor (functional principle)



Reactor core contains

→ **fuel** enriched Uranium with ~3% U-235
(comparison: enrichment of bomb: 80% U-235)

→ **moderator** water under high pressure (150 bar), to **slow down** the neutrons (increase of the fission probability) and for **cooling** (fission energy is turns into kinetic energy of fission products, which heats up the fuel)

→ **absorber** movable control rods (B, Cd, Gd) to adjust the absorption of the neutrona, so that **k=1** (critical) to keep up the chain reaction.

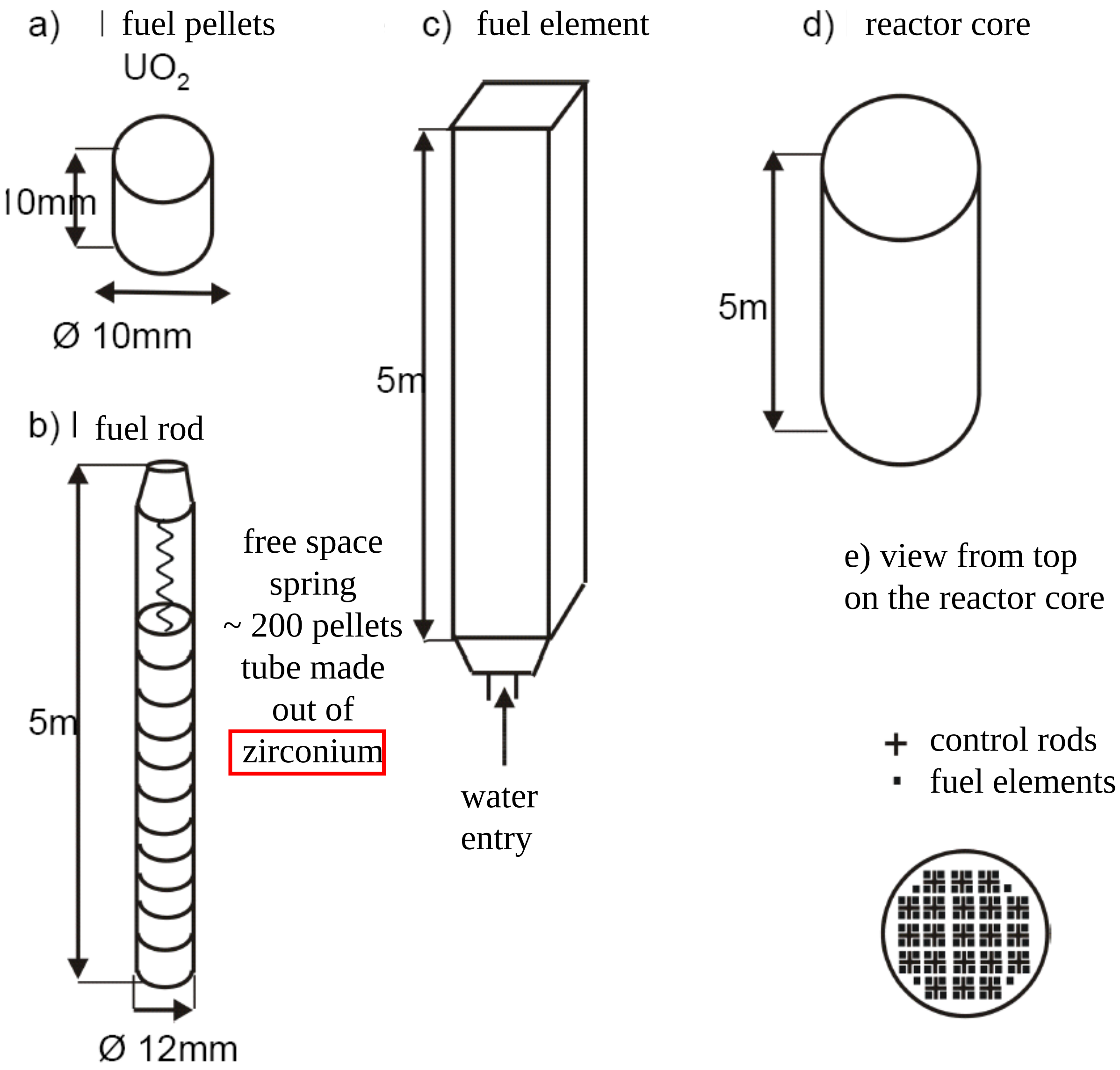
A fuel rod and Uranium-oxide pellets, the fuel of most power reactors.

From the fuel pellet to the reactor core

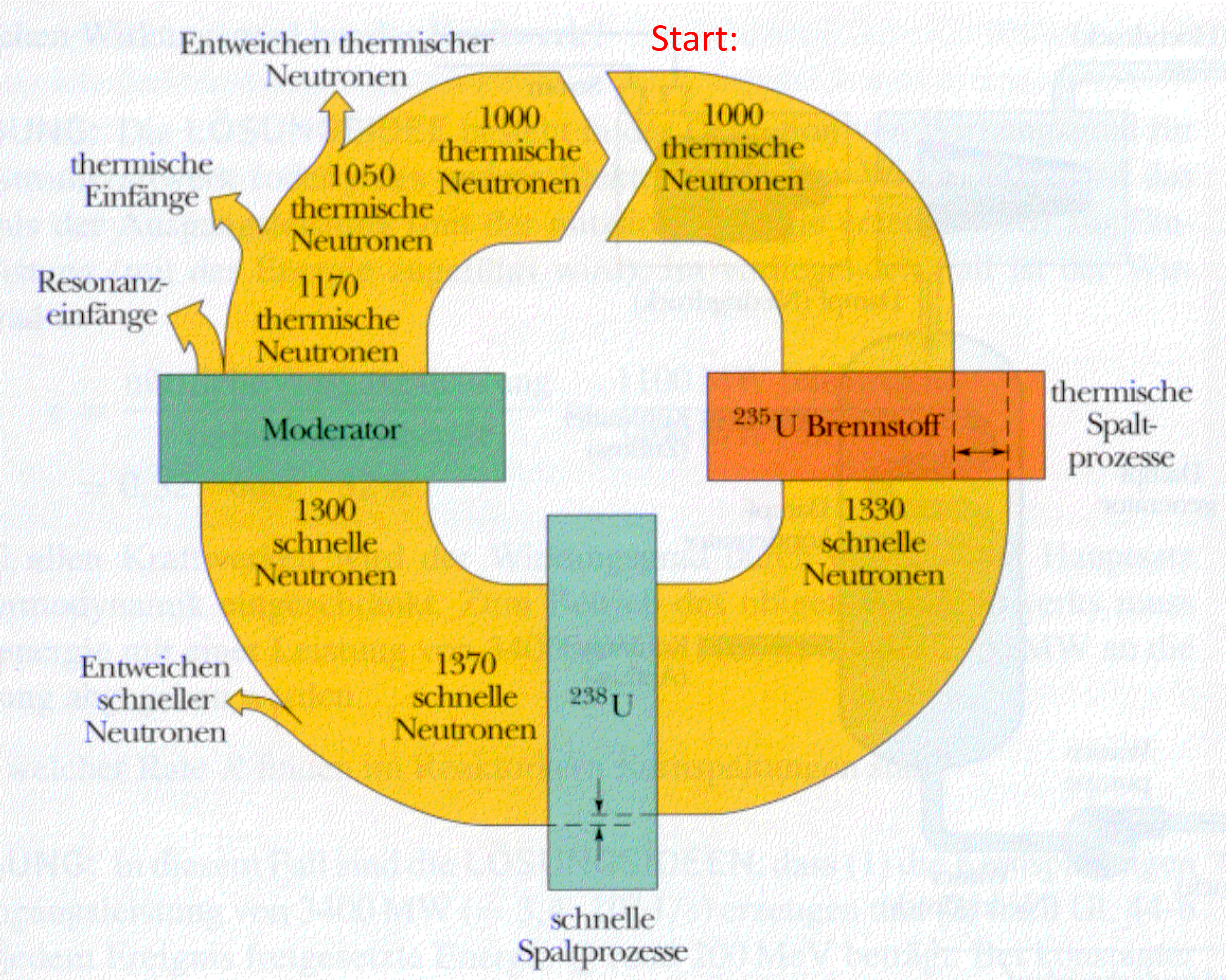
Fuel in form of **fuel elements**, ~200 of these, each individual water- moderated and cooled. Each element consists of 20-30 **fuel rods**, each fuel rod consists of 200 **Uranium-oxide pellets**

size of a pellet:

1 cm in height, 1 cm diameter.



Neutron balance in a reactor



Reactor dynamics

neutron lifetime:

- ❖ thermal reactor: ~ 20 μs
- ❖ fast reactor: ~ 0.5 μs

multiplication factor (reactivity):

$$k_{eff} = \frac{neutron\ production\ rate}{neutron\ loss\ rate}$$

- ❖ k<1 reactor runs down (will be stopped)
- ❖ k=1 constant power
- ❖ k>1 reactor runs up (getting started)

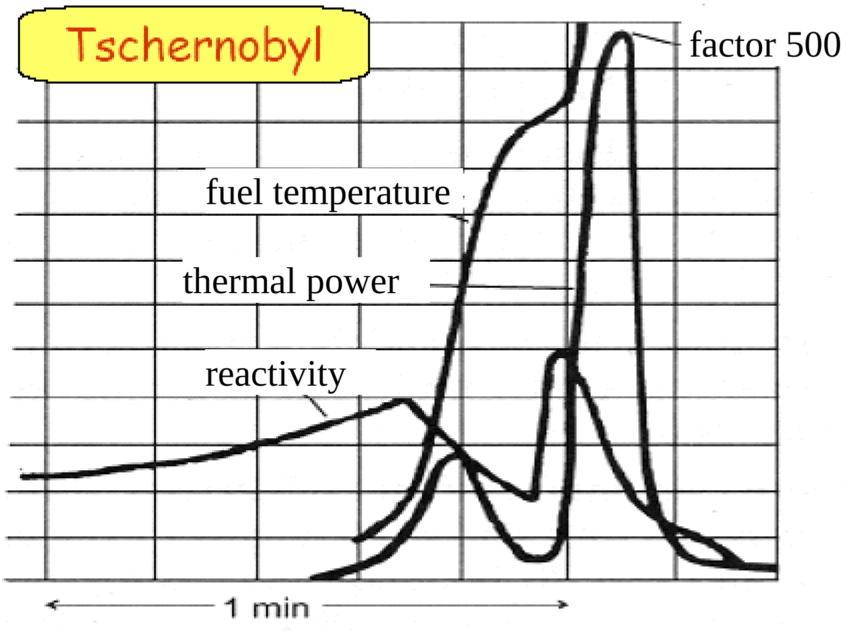
How is a reactor running a stable mode?

assumption: k=1.001

- ❖ after 1 s: 50000 generations
- ❖ power multiplication per second:

$$(1.001)^{50000} = 5 \cdot 10^{21}$$

Why does a reactor not explode?

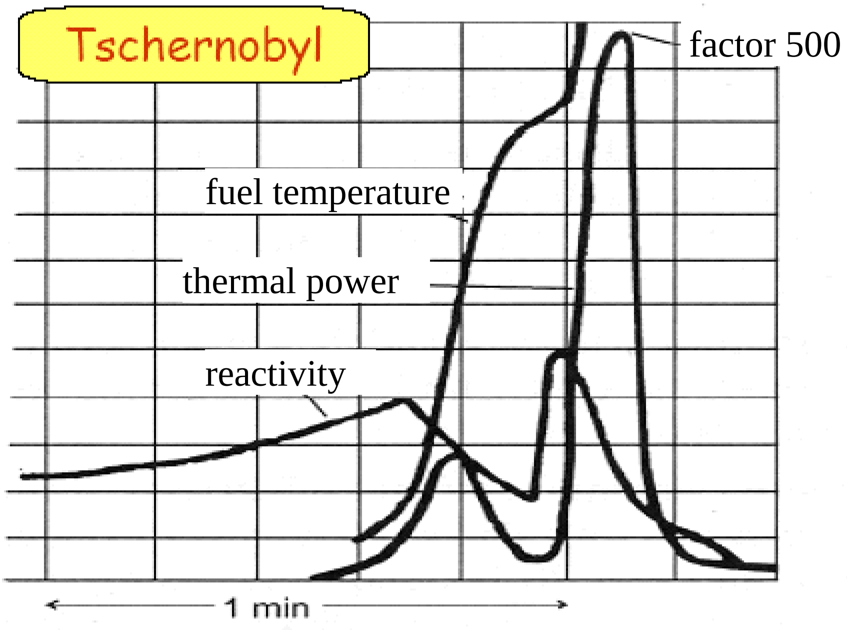


Reactor dynamics

How is a reactor running in a stable mode?

0.5% of the neutrons from 235-U are emitted delayed by about 10 s from the fission fragments

- k<1: under critical reactor
- 1<k<1+β: delayed critical reactor
- k>1+β: prompt critical reactor



Examples for the change of reactivity during operation:

- burn-off of nuclear fuel
- conversion (“breeding”) due to neutron bombardment

$$^{238}U + n \rightarrow {}^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$$

$$^{232}Th + n \rightarrow {}^{233}Th \rightarrow {}^{233}Pa \rightarrow {}^{233}U$$

- decay of fission material
- adding neutron poisons (e.g. Bor in water, control rods)
- change in the moderator-fuel ratio
  - density change of water (temperature)
  - bubble formation (pos./ neg. reactivity coefficient)
  - water losses
- reactivity losses due to Doppler broadening of the reaction cross sections at high temperatures

Reactor dynamics

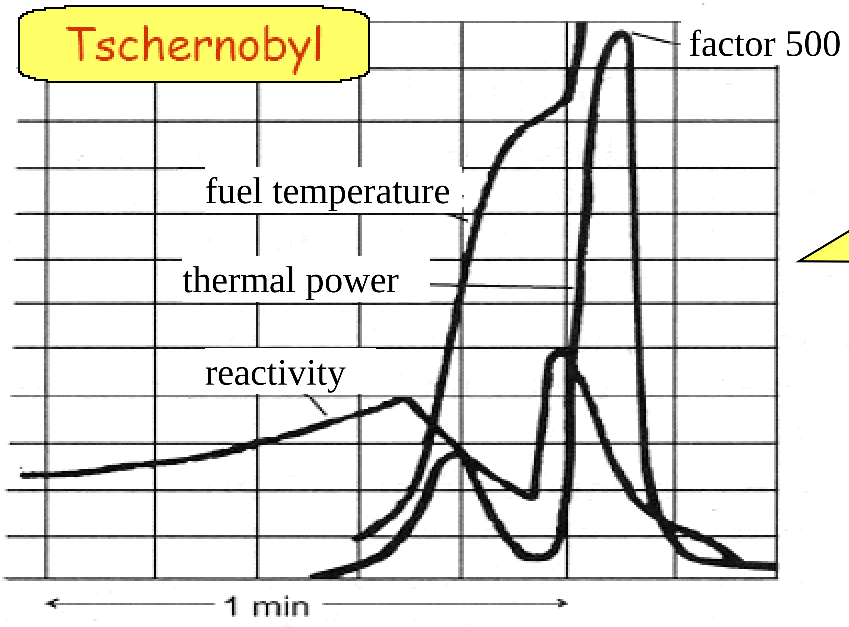
Examples for the change of reactivity during operation:

- Xenon peak: ^{135}Xe is a neutron poison

$$^{135}J \rightarrow {}^{135}Xe \begin{cases} \longrightarrow {}^{135} \\ \xrightarrow{n} {}^{136}Xe \end{cases}$$

If one reduces the reactor power there will be an excess of ^{135}Xe (neutron poison), which will reduce the reactivity even more and only after many hours it will vanish.

During operation ^{135}Xe will be continuously reduced du to neutron bombardment

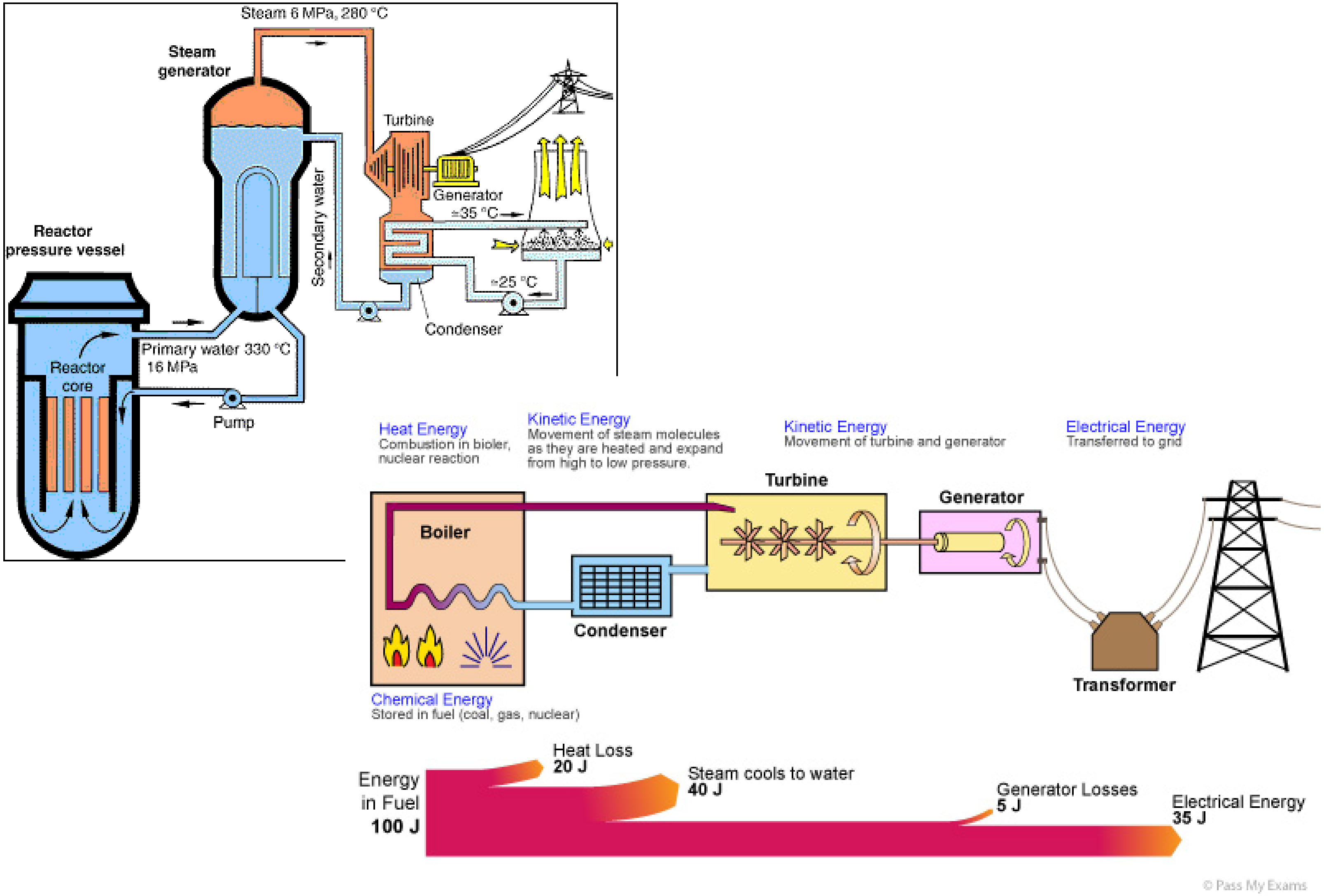


In Tschnobyl all control rods were removed in order to prevent a complete shut-down of the reactor due to the Xe-peak. Then the power increased so fast that the control rods could not been brought back fast enough. The positive bubble coefficient increased the reactivity in addition. First the explosive ejection of fuel made the reactor under-critical.

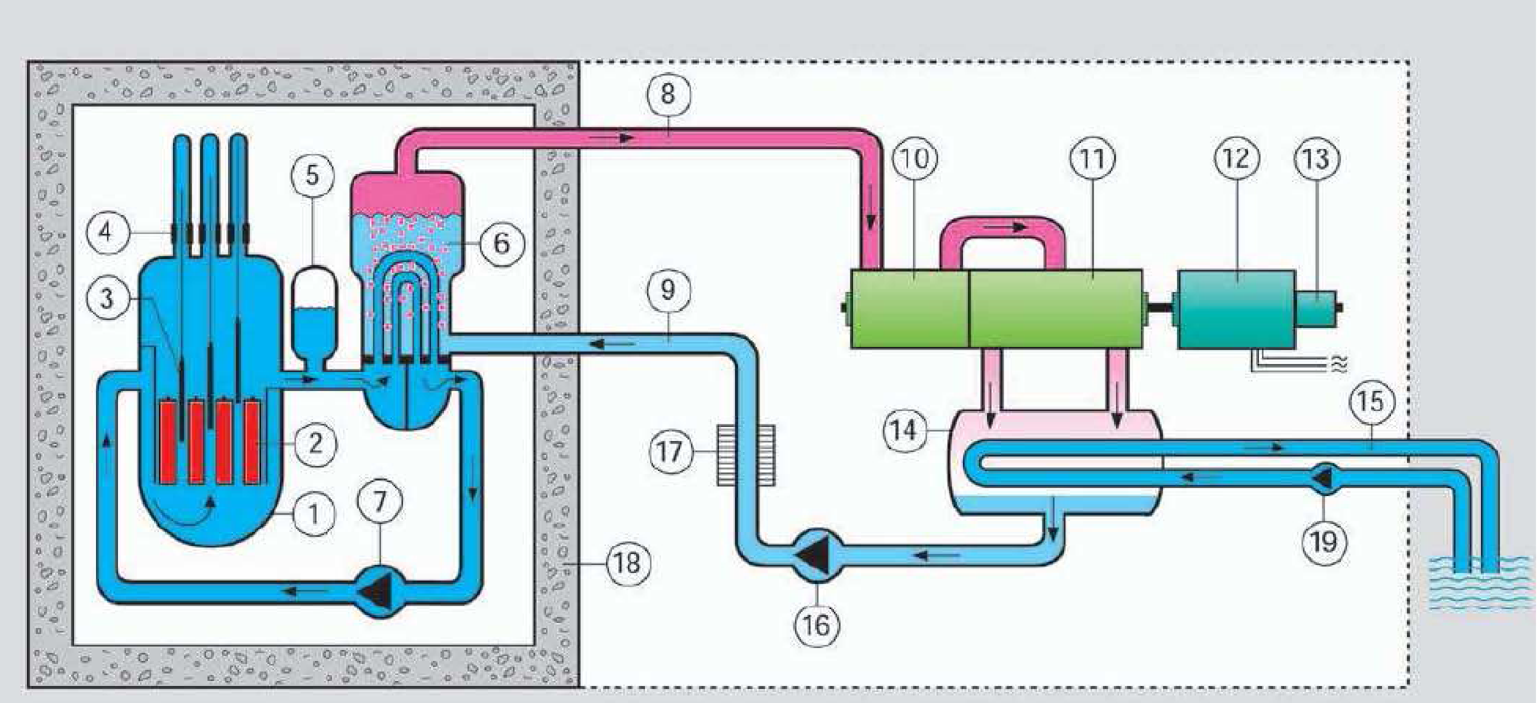
Energy transfer in a nuclear reactor

- release of **nuclear binding energy** during fission
- Transfer into kinetic energy of the fission products
- **Thermal energy due to slowing down** of the fragments (neutrons) in the solid fuel
- Use of the thermal energy to heat and **evaporate the cooling medium** (water)
- Water steam will be guided to a **turbine**
- Transformation of the rotational energy of the turbine into **electrical energy via a generator**
- Supply the electricity into the grid
- The waste heat will be given either directly (e.g. into a river) or indirectly (e.g. via a cooling tower to air) to the environment.

Energy transfer in a hot water reactor



Energy transfer in a pressure water reactor

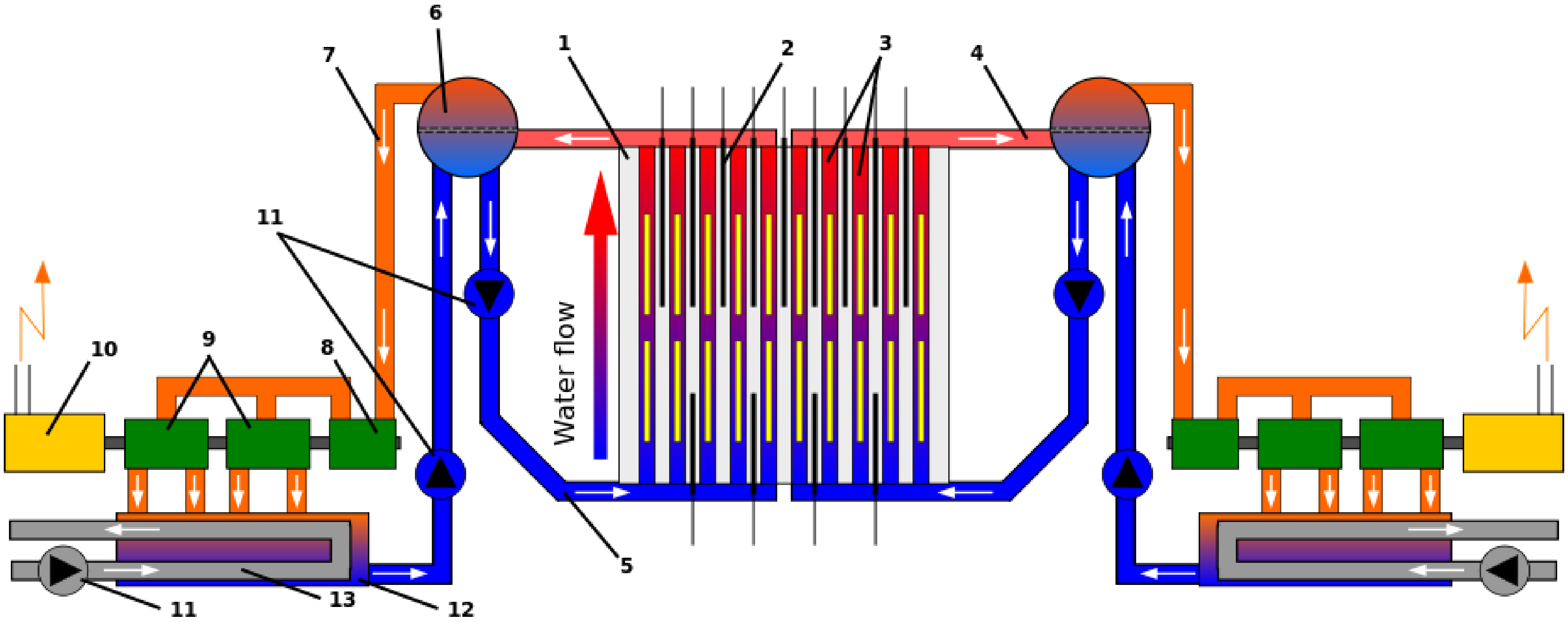


1 Reaktordruckbehälter
2 Uranbrennelemente
3 Steuerstäbe
4 Steuerstabsantriebe
5 Druckhalter
6 Dampferzeuger
7 Kühlmittelpumpe
8 Frischdampf
9 Speisewasser

10 Hochdruckteil der Turbine
11 Niederdruckteil der Turbine
12 Generator
13 Erregermaschine
14 Kondensator
15 Flußwasser
16 Speisewasserpumpe
17 Vorwärmanlage
18 Betonabschirmung

19 Kühlwasserpumpe

Light water reactor in Chernobyl

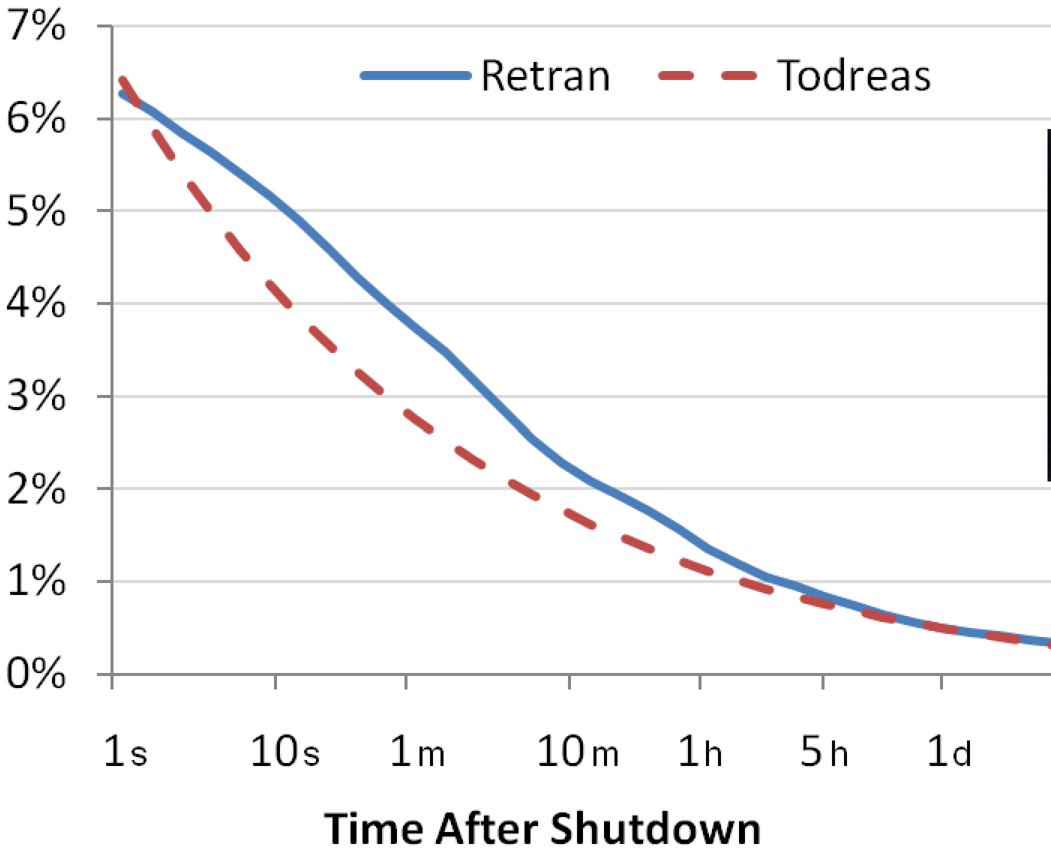


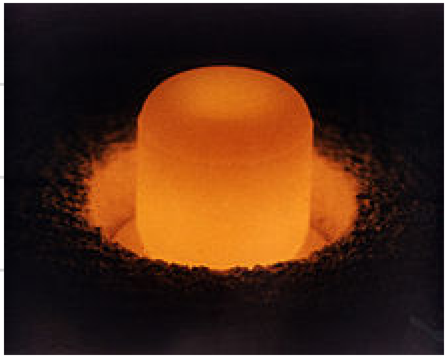
Legend :

1. Graphite moderated reactor core
2. Control rods
3. Pressure channels with fuel rods
4. Water/steam mixture
5. Water
6. Water/steam separator
7. Steam inlet
8. High-pressure steam turbine
9. Low-pressure steam turbine
10. Generator
11. Pump
12. Steam condenser
13. Cooling water (from river, sea, etc.)

The graphite-moderated nuclear power reactor in Chernobyl could use natural Uranium. There are 1661 fuel channels and 211 control rod channels in the graphite blocks.

Decay heat





The Plutonium-dioxide pellet is glowing red because of the heat generated by the radioactive decay.

When a nuclear reactor has been shut down and nuclear fission is not occurring at a large scale, the major source of heat production will be due to the delayed beta decay of these fission products (which originated as fission fragments). For this reason, at the moment of reactor shutdown, decay heat will be about 6.5% of the previous core power if the reactor has had a long and steady power history. About 1 hour after shutdown, the decay heat will be about 1.5% of the previous core power. After a day, the decay heat falls to 0.4%, and after a week it will be only 0.2%. (WIKIPEDIA)

Decay heat after 11 month of operation

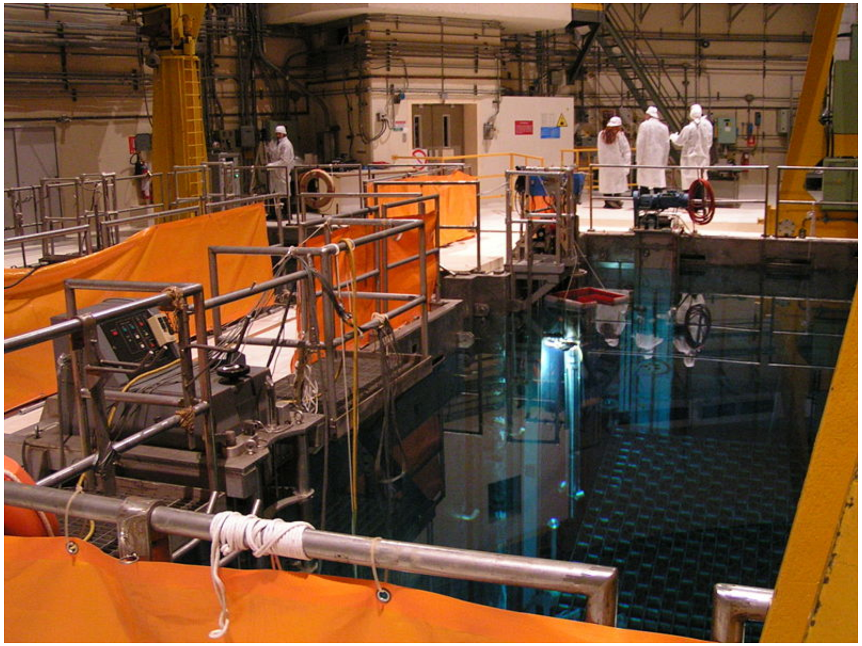
If a reactor is operated for the time T_0 [s] with a power of P_0, then the decay heat power P at the time t [s] after the shut-down of the reactor $P(t) = P_0 \cdot 6.22 \cdot 10^{-2} \cdot \left[t^{-0.2} - (T_0 + t)^{-0.2} \right]$

Time after shut-down	**Decay heat in percent**	**Thermal power for 4000 MW before shut-down**	**Time to heat-up 2500 m³ water from 15 °C to 100 °C**
10 Sekunden	3,72 %	149 MW	100 min
1 Minute	2,54 %	102 MW	146 min
1 Stunde	1,01 %	40 MW	6 h
1 Tag	0,44 %	18 MW	14 h
3 Tage	0,31 %	13 MW	20 h
1 Woche	0,23 %	9 MW	26 h
1 Monat	0,13 %	5 MW	49 h
3 Monate	0,07 %	3 MW	89 h

Storage pool

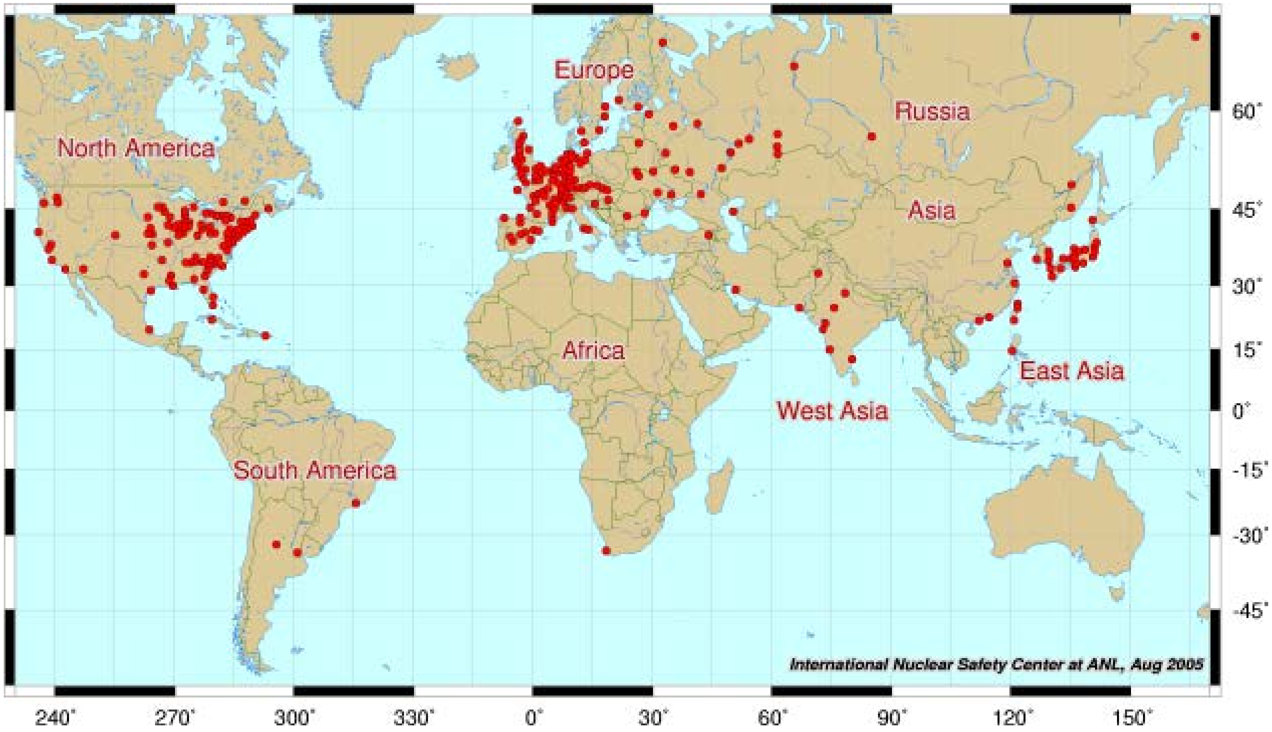
In case of a leakage or failure of the cooling the water can leak out or evaporate. In this case the stored fuel elements can be excessively heated.

Is water still present in the pool, the Zircaloy of the fuel rods can react with the water steam at ~800^0 C. In an exotherm redox reaction Zirconium-oxide and Hydrogen is produced which will create an explosive Knallgas mixture in a short time.
In case of no cooling at all the fuel rods can start burning which will destroy the fuel elements.



World nuclear energy

- ❖ 440 nuclear power reactors in 50 countries (March 2022)
- ❖ ~10% of the world energy production (2022)
- ❖ 55 more reactors in construction



International Nuclear Safety Center at ANL, Aug 2005