Outline: Nuclear radius

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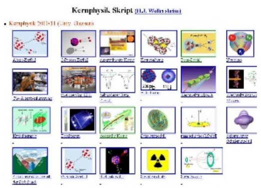
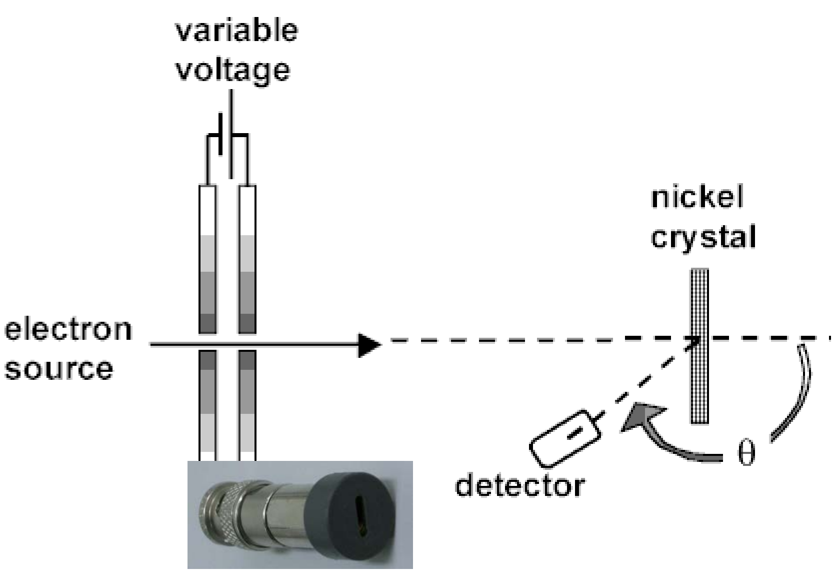
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. de Broglie wavelength
2. double slit experiment
3. electron scattering
4. Mott scattering
5. charge distribution

Nuclear Radii

Clinton Davisson & Lester Germer (1925)



Si-detector with slit cover

Scattered electrons form diffraction pattern characteristic of waves

$$\Psi \approx cos(k \cdot x) = cos\left(\frac{2\pi}{\lambda} \cdot x\right)$$

Wavelength found from Planck's constant and momentum:

$$\lambda = \frac{h}{m \cdot v}$$

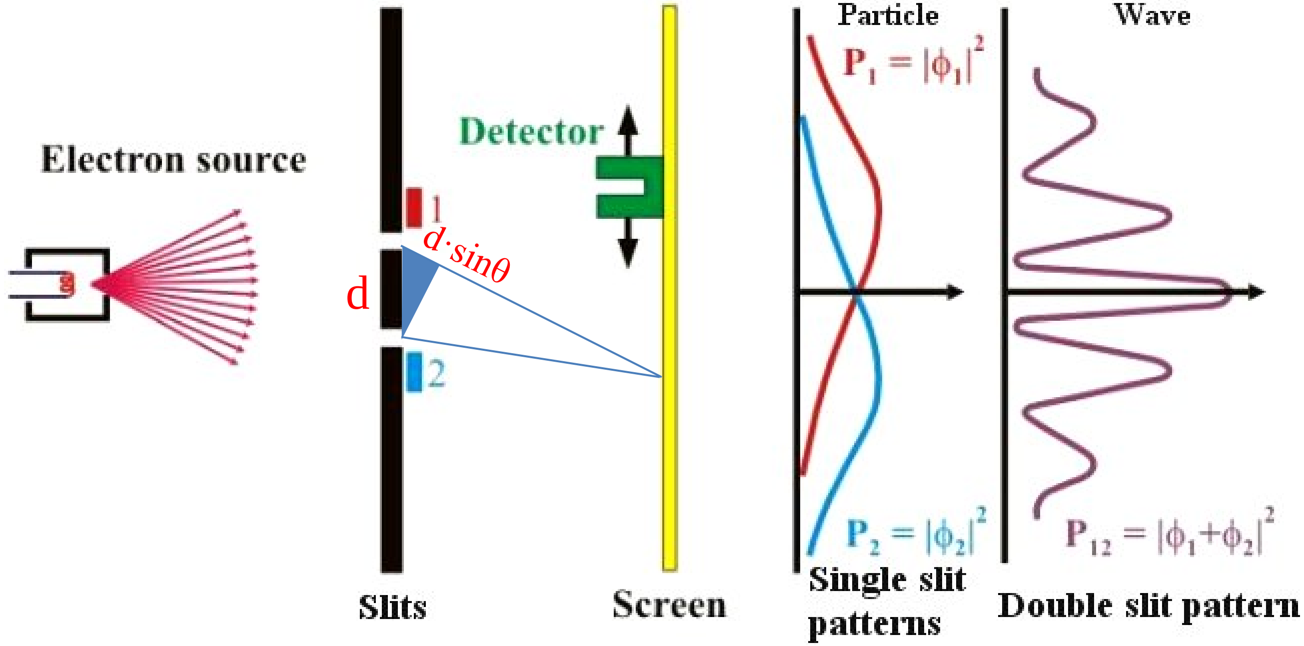
Luis de Broglie (1924): matter particles such as electrons have wave-like properties

$$\lambda = \frac{h}{p} = \frac{h \cdot c}{\sqrt{E_{kin} \cdot (E_{kin} + 2m_o c^2)}} = \frac{1239.84[MeV\ fm]}{\sqrt{E_{kin} \cdot (E_{kin} + 2m_0 c^2)}}$$

Electrons at **keV** energies:
"interfere" with Angstrom (~10^{-10} m) scale atomic lattice structure

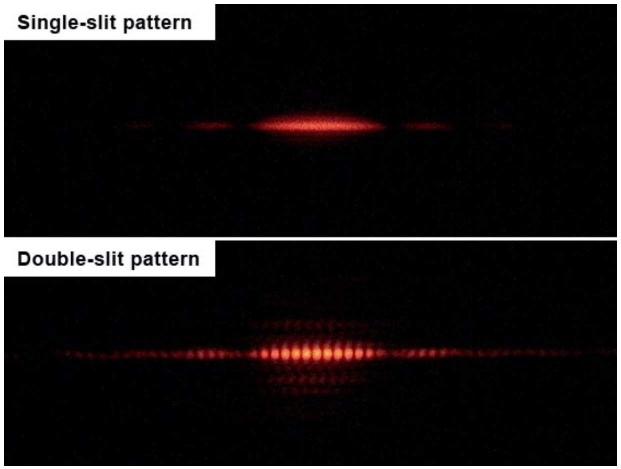
m_0 = 0.511 [MeV] $\hbar = 6.58 \cdot 10^{-22}$ [MeV s]

Double slit electron diffraction



Interference minima when path length from holes differs by half wavelength:

$$d \cdot sin(\theta_{min}) = \lambda/_2$$



Electron scattering on nuclei

How do we measure nuclear radii?

Use electrons as probe ⟶ point like particles, experience only electromagnetic interaction and not strong (nuclear) force, probe the entire nuclear volume.

What energy do we need?

Hint: consider required de Broglie wavelength

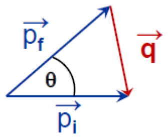
$$\lambda = \frac{h \cdot c}{\sqrt{E_{kin} \cdot (E_{kin} + 2m_o c^2)}} = \frac{1239.84[MeV\ fm]}{\sqrt{E_{kin} \cdot (E_{kin} + 2m_0 c^2)}}$$

λ = 5 [fm] for E_{kin} ~ 250 [MeV]

Study angular distribution of scattered electrons

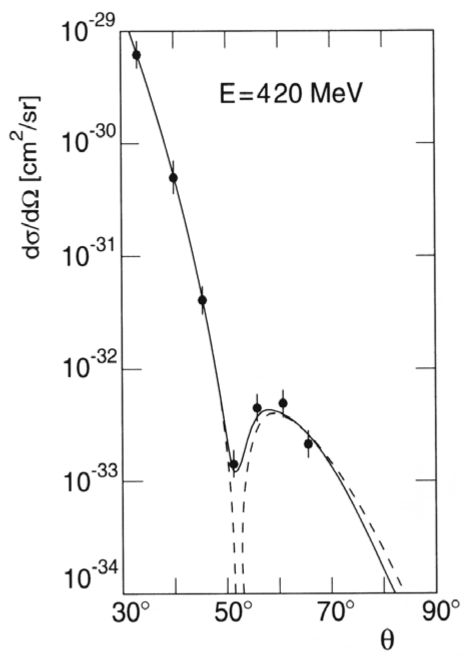
→ observe diffraction effects
→ analog with optics

The momentum transfer is given by:



$$q = 2\frac{p}{\hbar} \cdot sin(\theta/2) = 2\frac{E}{\hbar c} \cdot sin(\theta/2)$$

$$q = \frac{840MeV}{197MeV\ fm} \cdot sin(52/2) = 1.8\ [fm^{-1}]$$



e^- scattering on ^{12}C

The cross section describes the scattering on a point-like particle:

$$\frac{d\sigma}{d\Omega} = \frac{0.13 \cdot Z}{E^2} \cdot \frac{1}{sin^4(\theta/2)} \cdot \{1 - \beta^2 \cdot sin^2(\theta/2)\}$$

Mott scattering

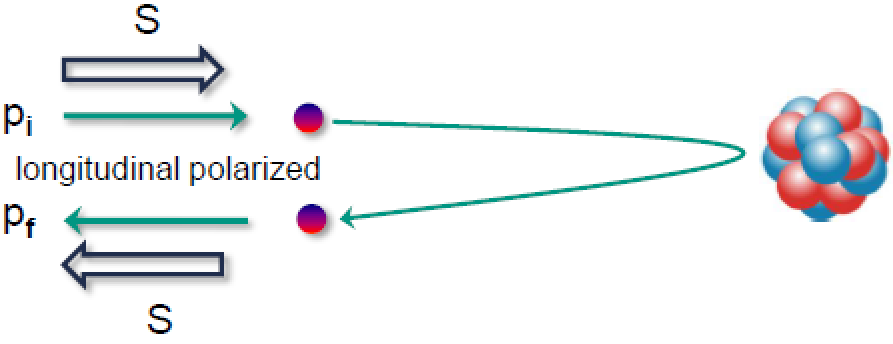
Mott scattering for relativistic projectiles with spin (no recoil effect)

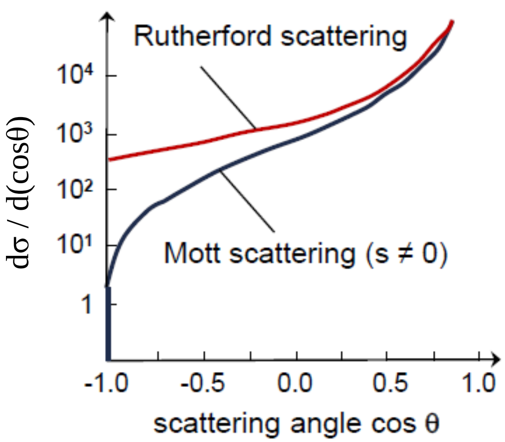
$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \{1-\beta^2 \sin^2(\theta/2)\} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2\left(\frac{\theta}{2}\right)$$

for $\beta = v/c \rightarrow 1$

Nevill F. Mott
1905-1996

Central collision ($\theta = 180^0$, $\ell = 0$) of an electron ($s = ½$)





Electron spin has to perform a spin-flip

→ backward scattering heavily suppressed

Electron scattering on nuclei

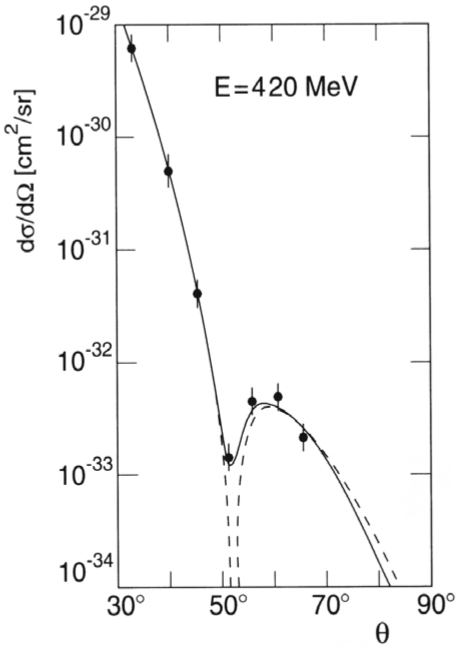
Experimental cross section:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot |F(q^2)|^2$$

$F(q^2)$ is the form factor, which is the Fourier transform of the charge distribution

The form factor of a homogenously charged sphere:

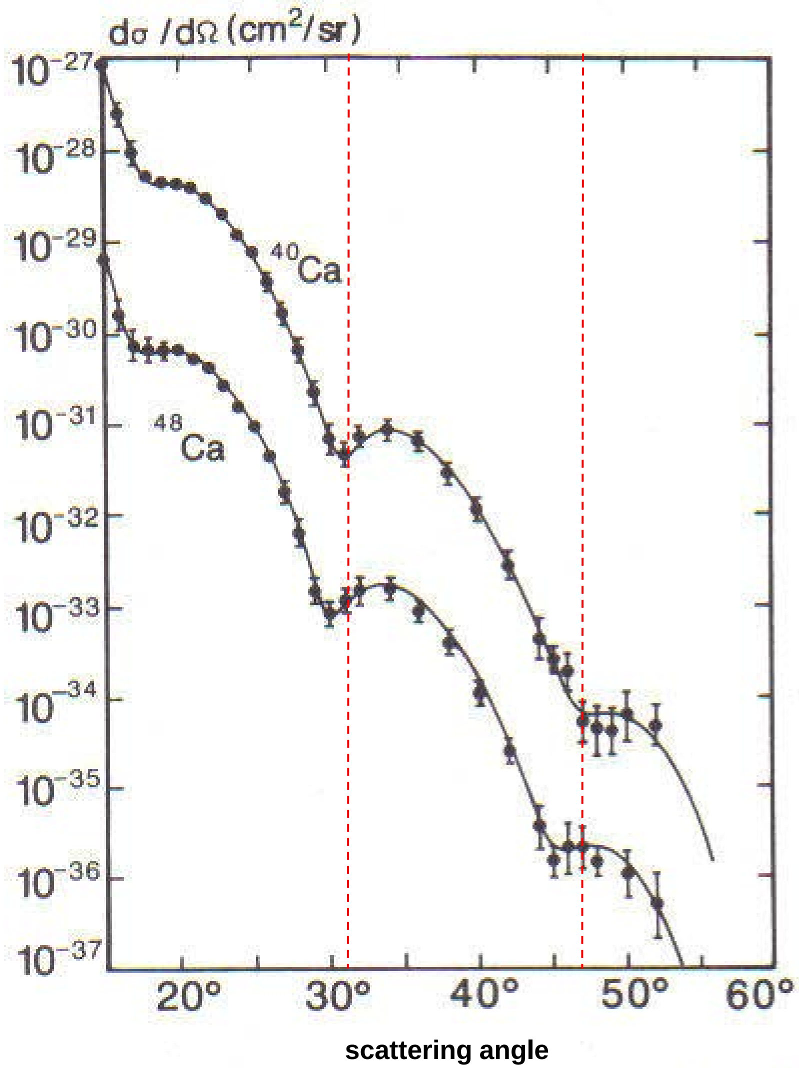
$$\boxed{F(q^2) = \frac{3}{(qR)^3} \cdot \{sin(qR) - qR \cdot cos(qR)\}}$$



❖ Comparison with experimental cross section on ^{12}C

$q \cdot R = 4.5 \longrightarrow R = 2.5$ [fm] for $q = 1.8$ [fm^{-1}]

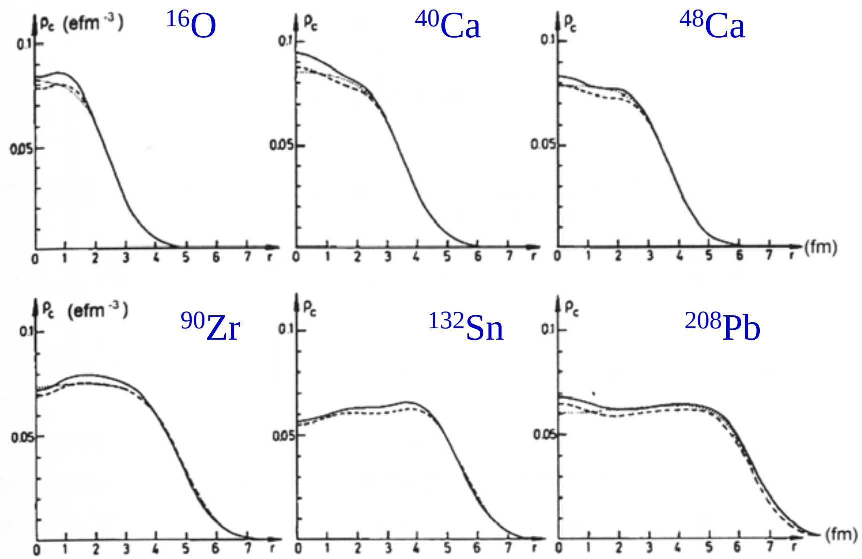
Isotope effect of the nuclear radius

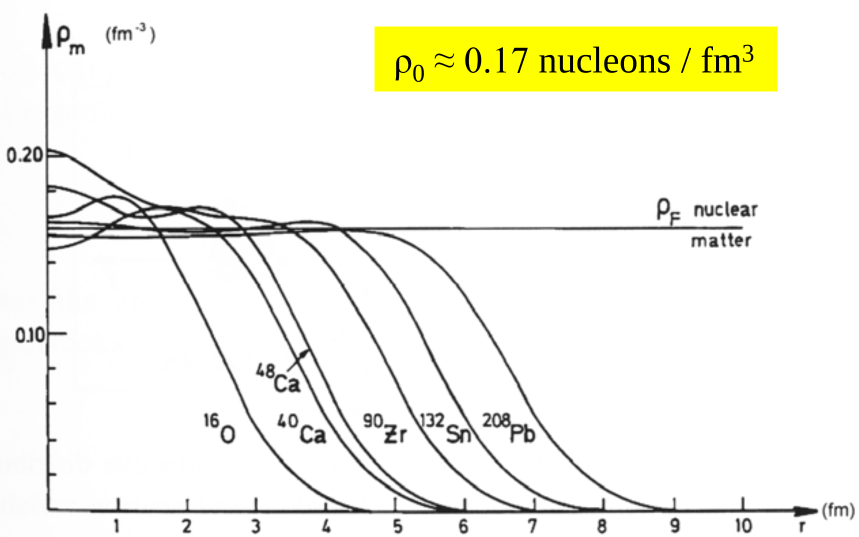


From the position of the cross section minima for ^{48}Ca and ^{40}Ca it is obvious that the nuclear radius **R** increases with mass number **A.**

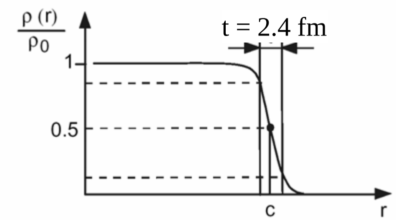
Charge distribution

experimental charge distributions





$\rho_0 \approx 0.17$ nucleons / fm^3



❖ Fermi distribution:

$$\rho(r) = \frac{\rho_0}{1 + exp\left(\frac{r-c}{a}\right)}$$

with $c \approx 1.07 \cdot A^{1/3}$ fm, $a \approx 0.54$ fm

❖ Root mean square radius:

$$r_{rms} = \sqrt{\langle r^2 \rangle} = r_0 \cdot A^{1/3} \quad with\ r_0 = 0.94\ fm$$

❖ Equivalent radius of a sphere:

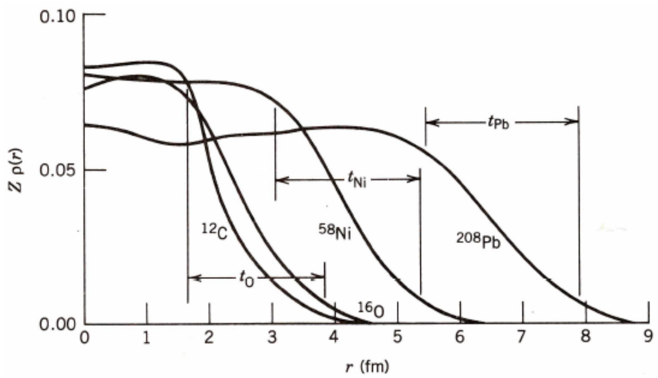
$$R^2 = 5/3 \cdot \langle r^2 \rangle \quad \rightarrow \quad R \approx 1.21 \cdot A^{1/3}$$

Conclusion of nuclear radius measurements

1. The central density, is (roughly) constant, almost independent of atomic number, and has a value about 0.17 fm^{-3}. This is very close to the density of nuclear matter in the infinite radius approximation,

$$\rho_0 = {}^{3}\!/_{4\pi r_0^3}$$

2. The "skin depth", is (roughly) constant as well, almost independent of atomic number, with a value of about $t = 2.4$ fm typically. The skin depth is usually defined as the difference in radii of the nuclear densities at 90% and 10% of maximum value.



3. Scattering measurements suggest a best fit to the radius of nuclei:

$R_N = r_0 \cdot A^{1/3}$ $\qquad r_0 \approx 1.22$ [fm] $\qquad$ 1.2 → 1.25 is also common

4. A convenient parametric form of the nuclear density was proposed by Woods and Saxon

$$\rho_N(r) = \frac{\rho_0}{1 + exp\left(\frac{r - c_N}{a}\right)}$$

with $t = a \cdot 4 \cdot \ln 3 \Rightarrow a = 0.54$ fm