# Outline: Nuclear reactions

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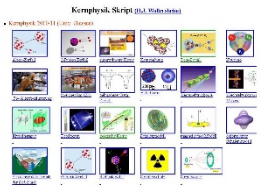
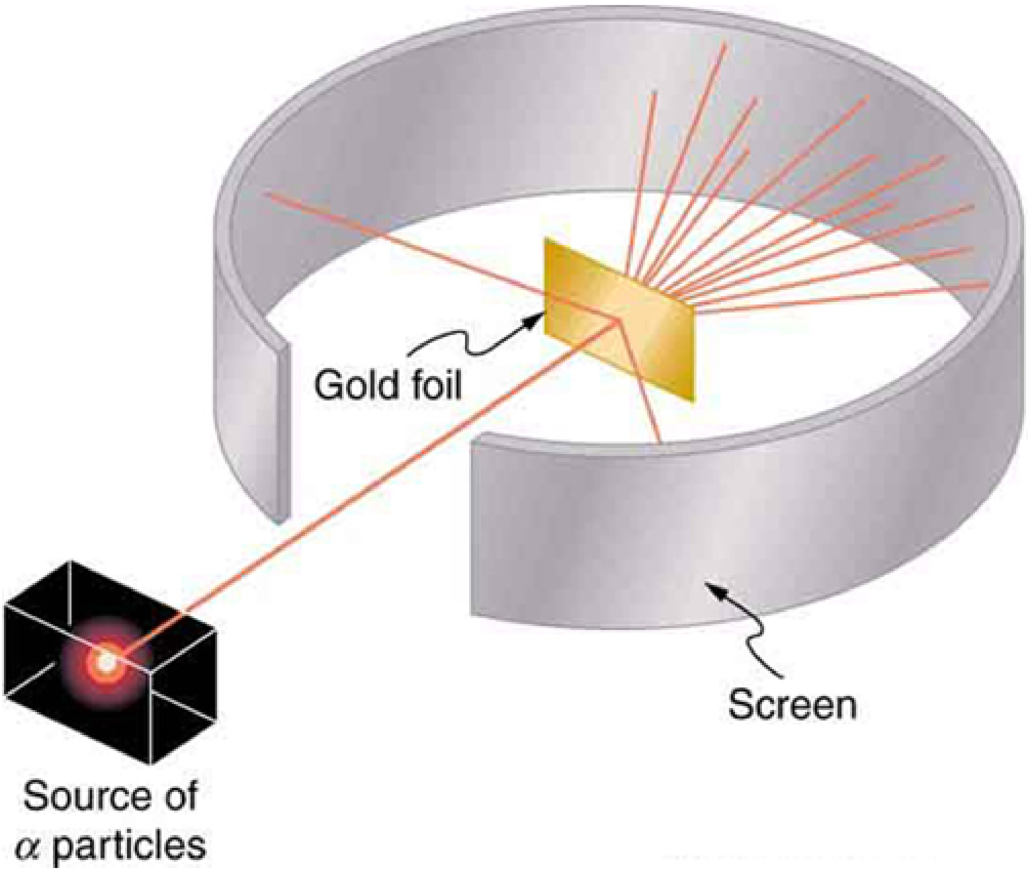
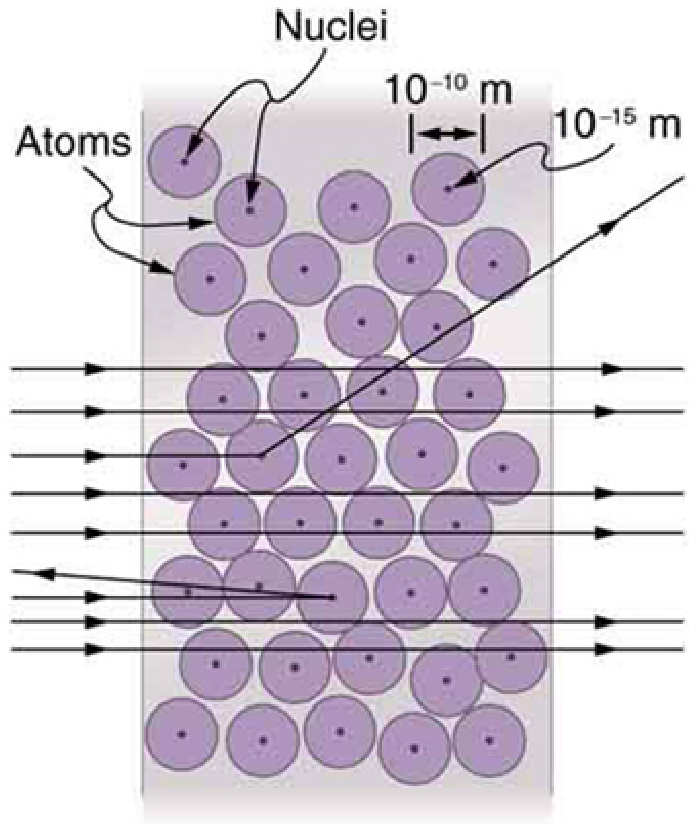
web-page: https://web-docs.gsi.de/~wolle/ and click on



1. luminosity
2. kinematics
3. Fraunhofer and Fresnel diffraction
4. elastic cross section as a function of $\theta$, $\ell$, D
5. elastic scattering and nuclear reactions

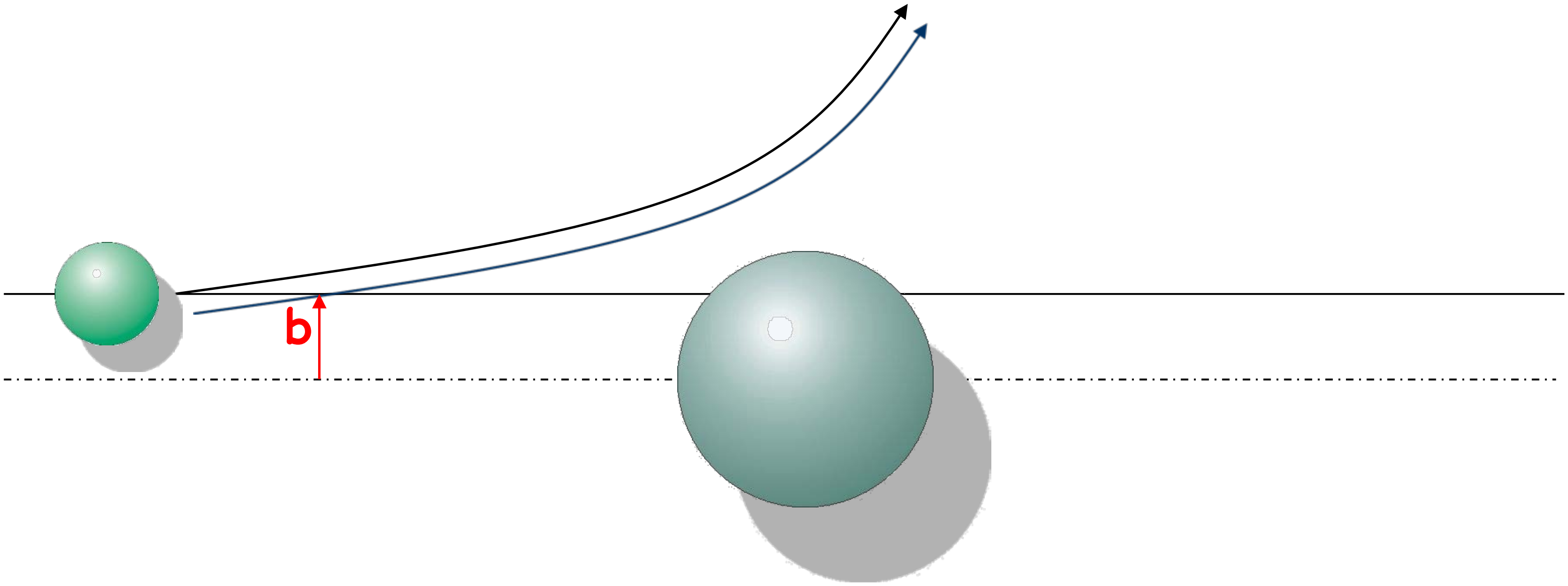
# Semi-classical reaction theory



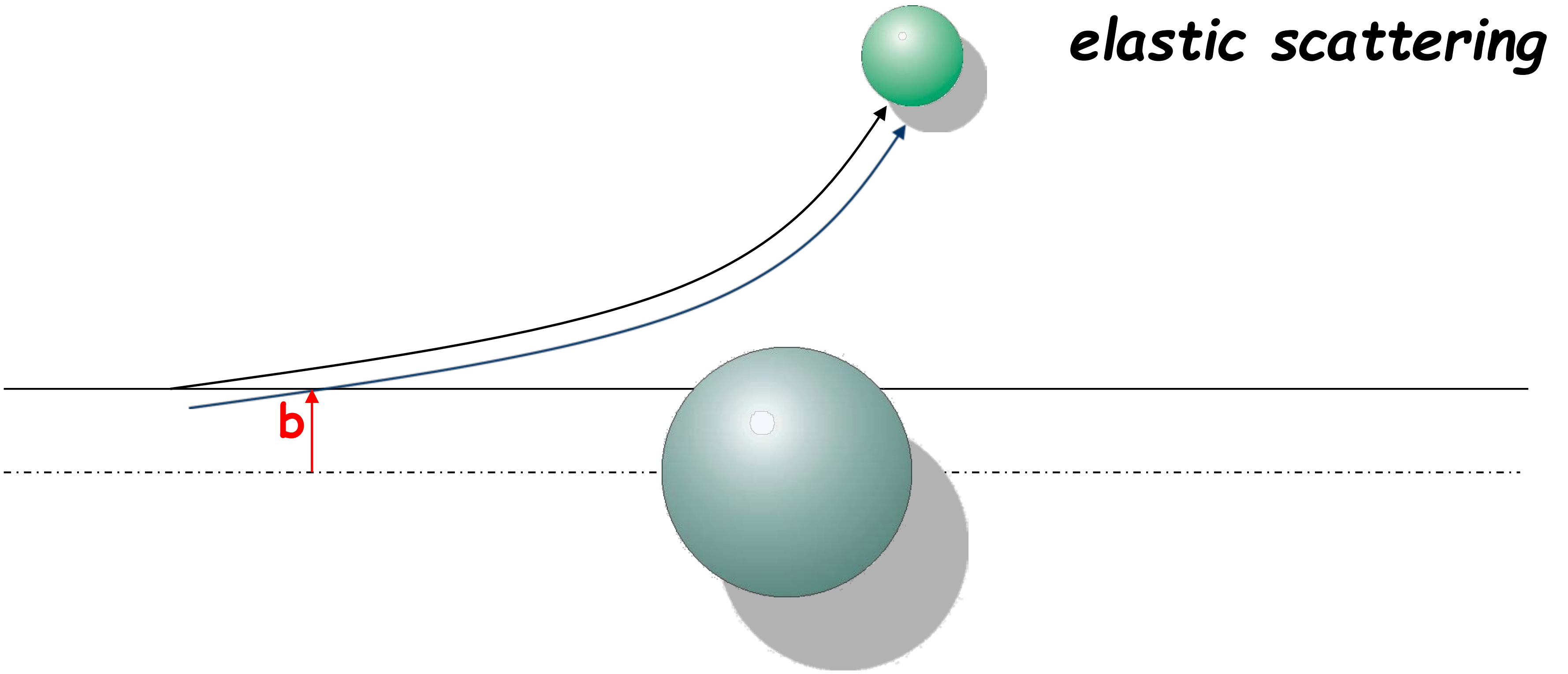



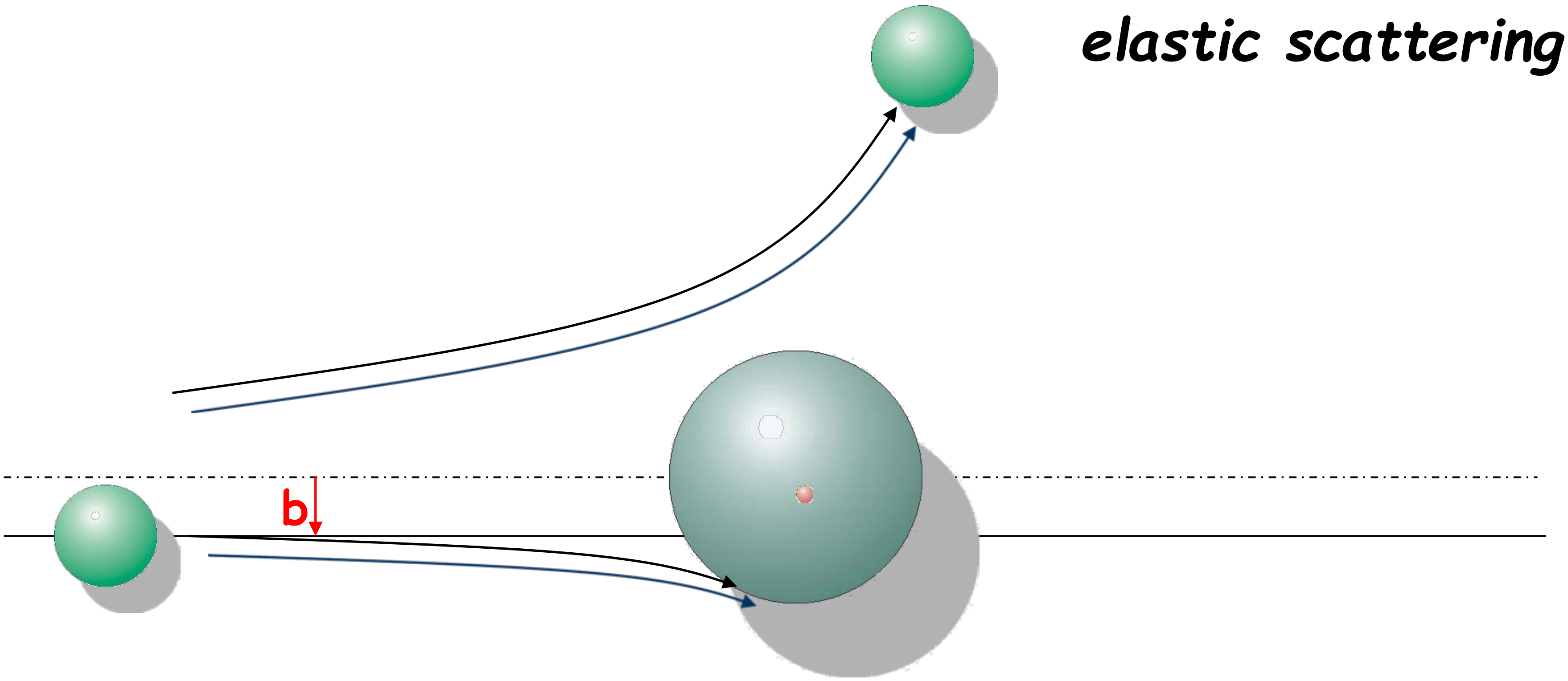
# Semi-classical reaction theory

***elastic scattering***

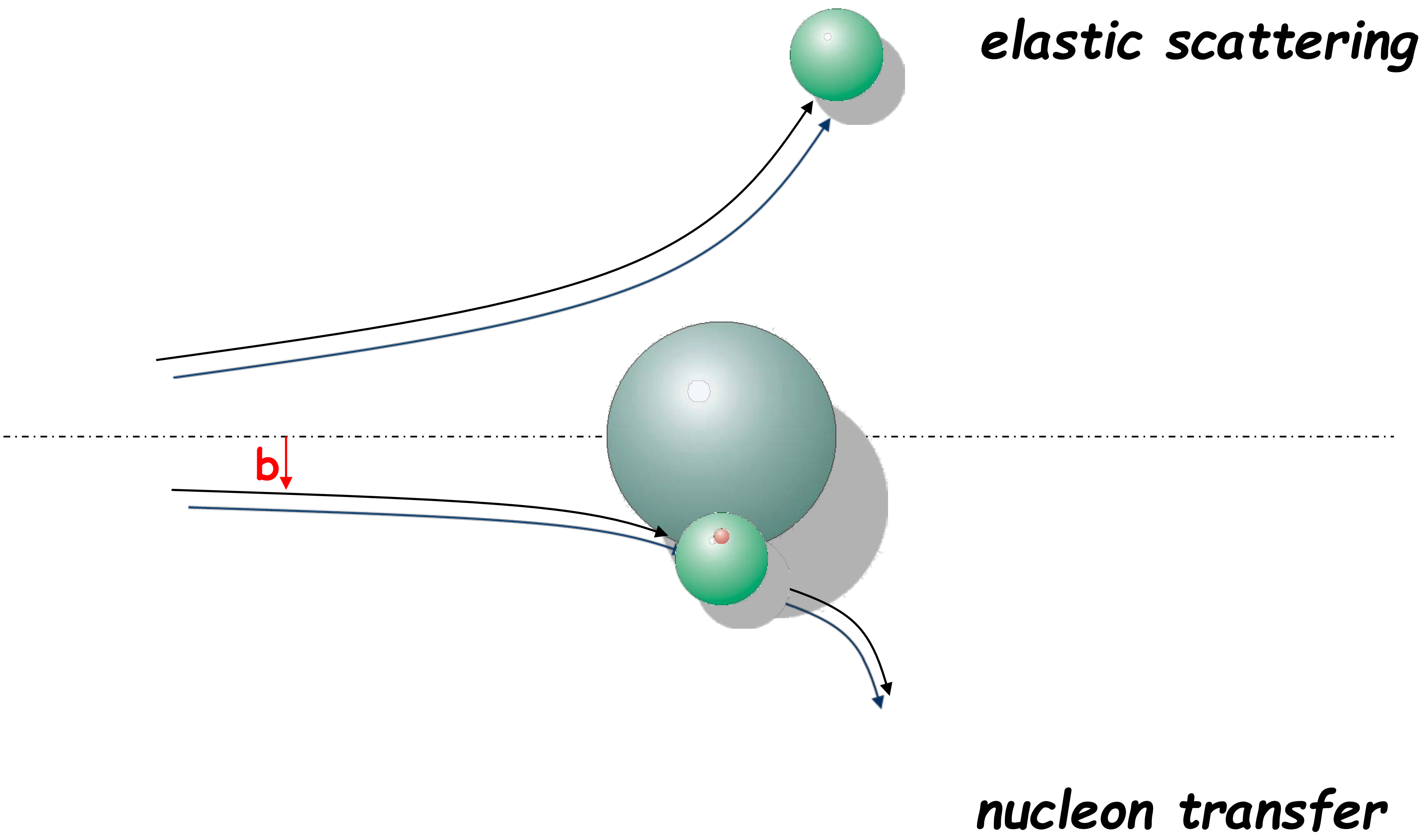


# Semi-classical reaction theory




elastic scattering
b

# Semi-classical reaction theory



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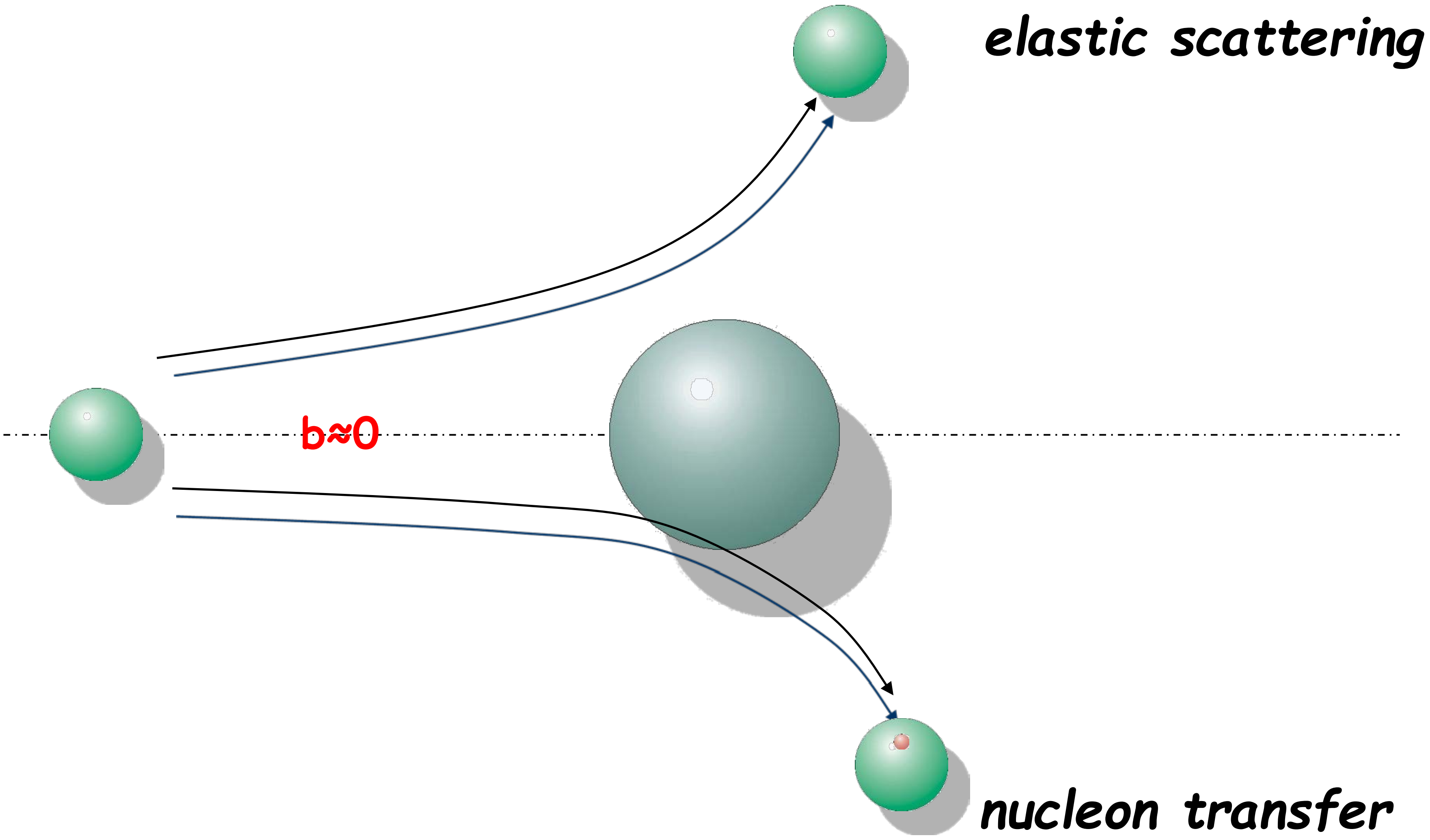
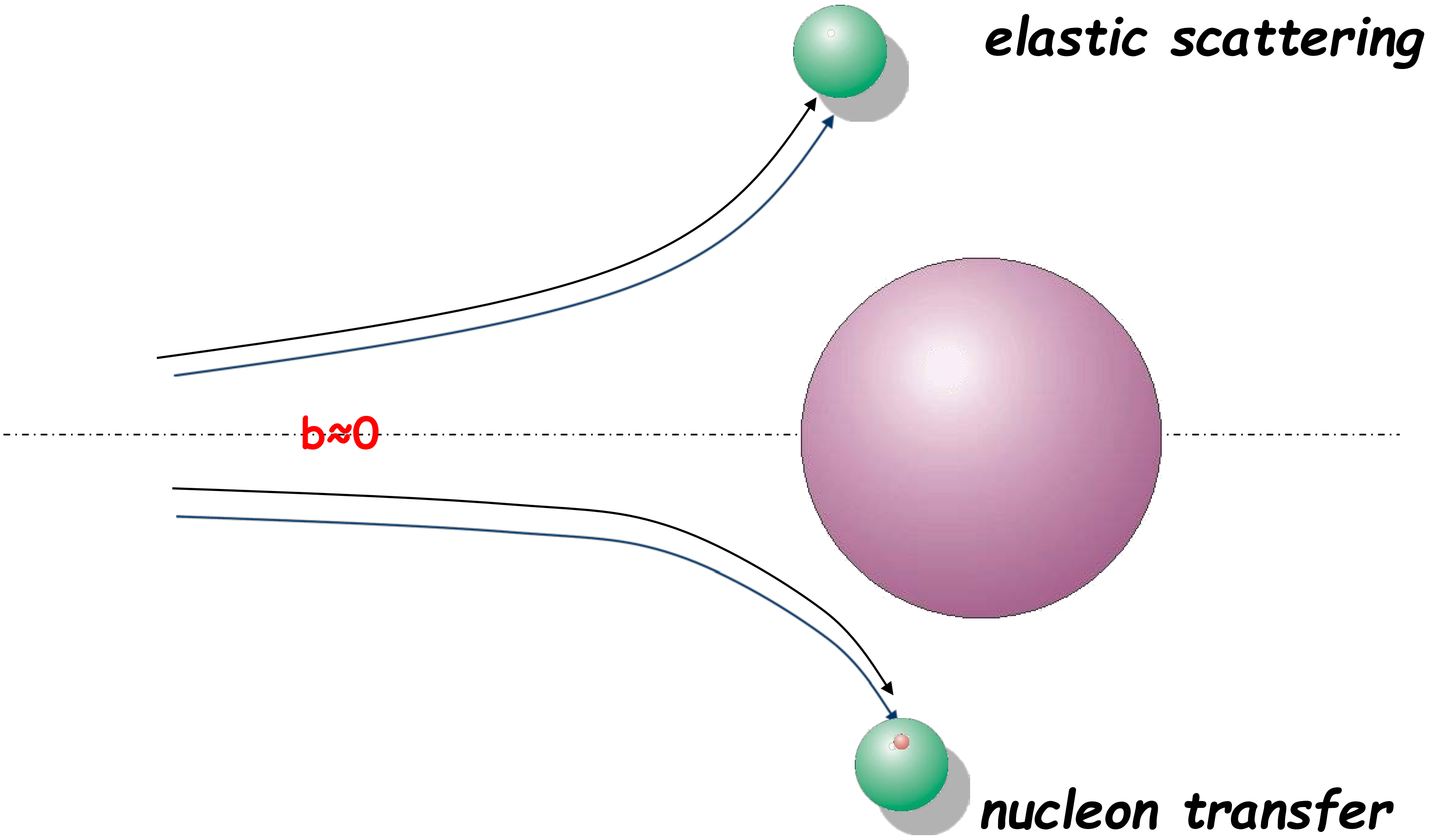
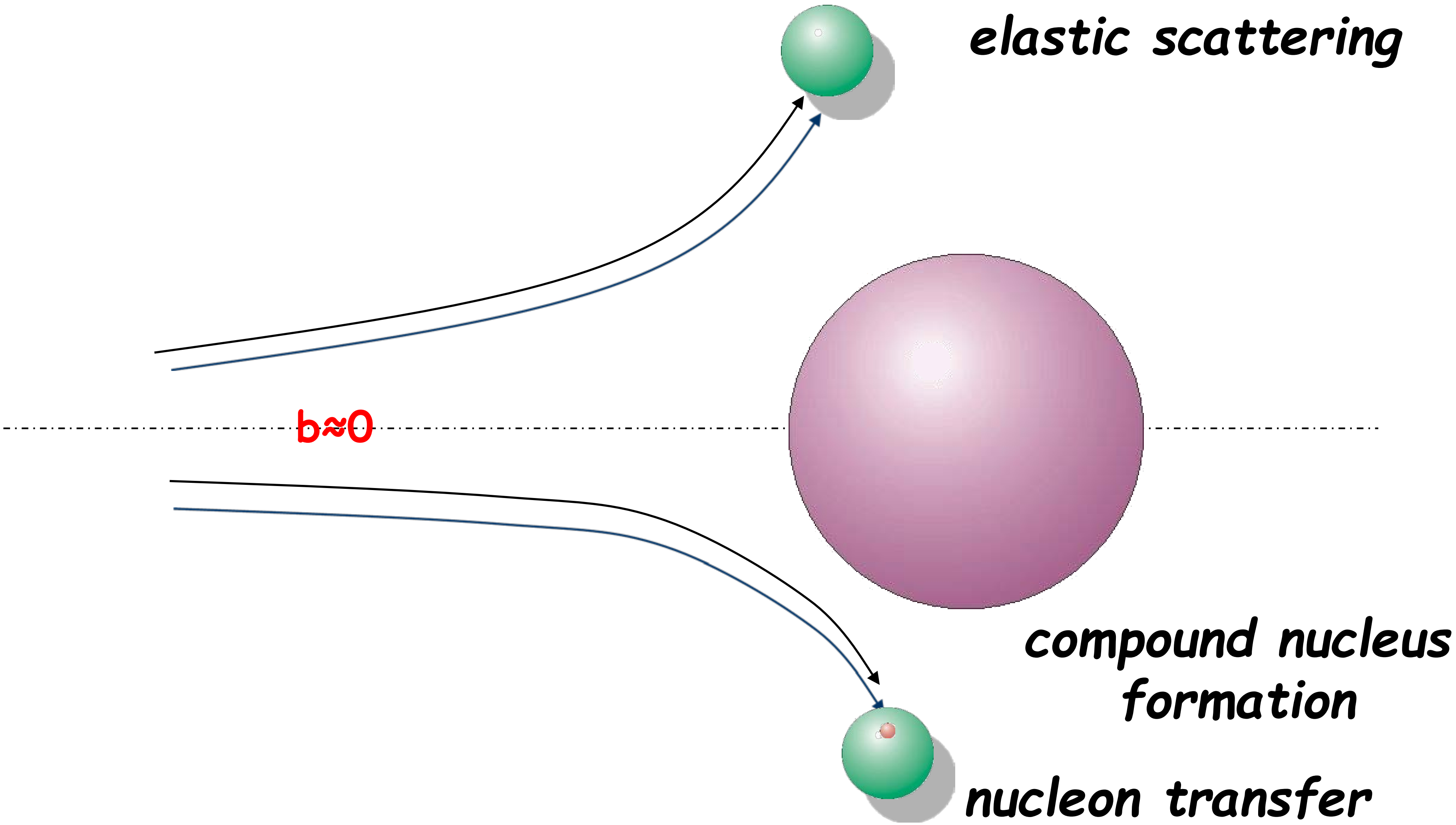


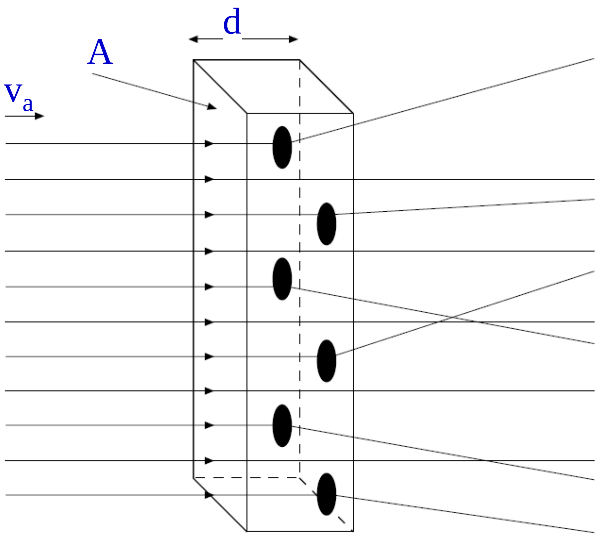
# Semi-classical reaction theory



# Semi-classical reaction theory



# Nuclear reaction cross section




$\Phi_a = n_a \cdot v_a$ $\quad N_b = n_b \cdot A \cdot d$

Consider a beam of projectiles of intensity $\boldsymbol{\Phi_a}$ particles/sec which hits a thin foil of target nuclei with the result that the beam is attenuated by reactions in the foil such that the transmitted intensity is $\boldsymbol{\Phi}$ particles/sec.
The fraction of the incident particles disappear from the beam, i.e. react, in passing through the foil is given by

$$d\Phi = -\Phi \cdot n_b \cdot \sigma \cdot dx$$

The number of reactions that are occurring is the difference between the initial and transmitted flux

$$\Phi_{initial} - \Phi_{trans} = \Phi_{initial}(1 - exp[-n_b \cdot d \cdot \sigma])$$

$$\approx \Phi_{initial} \cdot N_b \cdot \sigma \qquad \text{(for thin target)}$$

## Example:

A particle current of 1 pnA consists of $6 \cdot 10^9$ projectiles/s.

A $^{132}$Sn target (1 mg/cm$^2$) consists of $5 \cdot 10^{18}$ nuclei/cm$^2$

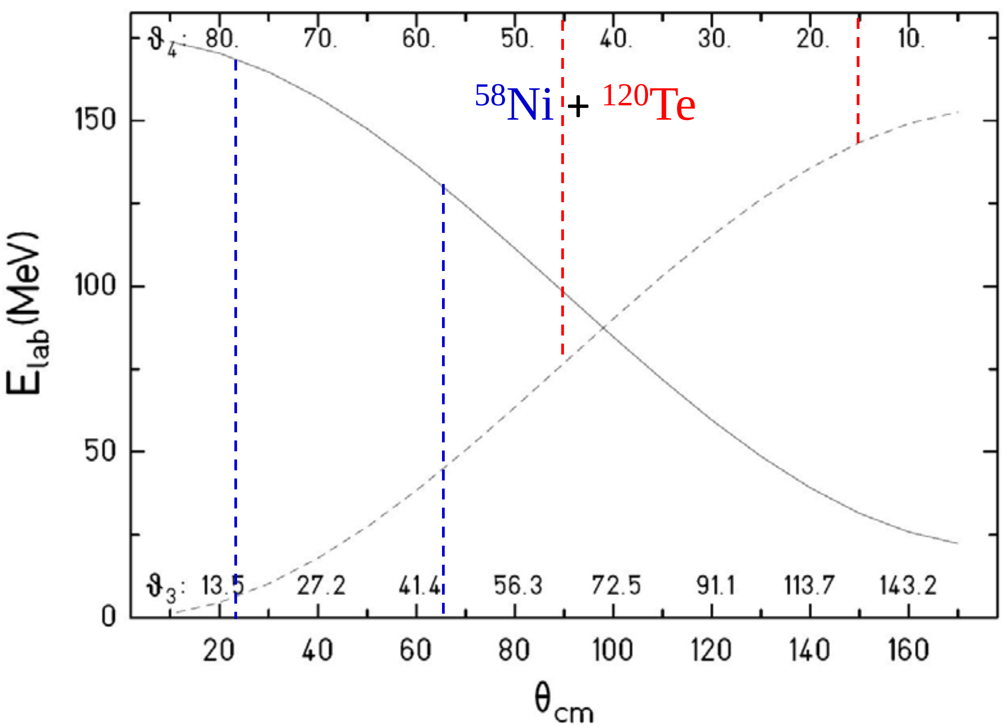
$$\frac{6 \cdot 10^{23} \cdot 10^{-3} \ g/cm^2}{132g} = 4.5 \cdot 10^{18} \quad \left[\frac{target\ nuclei}{cm^2}\right]$$

Luminosity = projectiles [s$^{-1}$] · target nuclei [cm$^{-2}$]

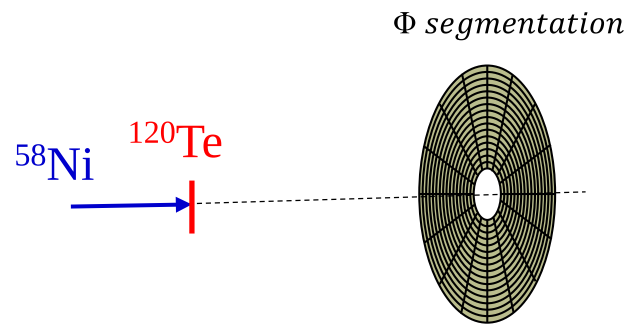
Luminosity (projectile → $^{132}$Sn) = $3 \cdot 10^{28}$ [s$^{-1}$cm$^{-2}$]

Reaction rate [s$^{-1}$] = luminosity · cross section [cm$^2$]
= projectiles [s$^{-1}$] · target nuclei [cm$^{-2}$] · cross section [cm$^2$]

# Kinematics




$$= \frac{1}{2}(180^0 - \theta_{cm})$$




projectile and target nucleus are measured in PPAC

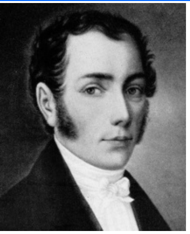
electrode is made of concentric rings (proportional to polar angle $\vartheta$)

$$15^0 \leq \vartheta \leq 45^0$$

Solid angle: $\Omega = \iint sin\vartheta \; d\vartheta \; d\varphi$

$$\Omega = 2\pi \cdot (1 - cos\vartheta)$$

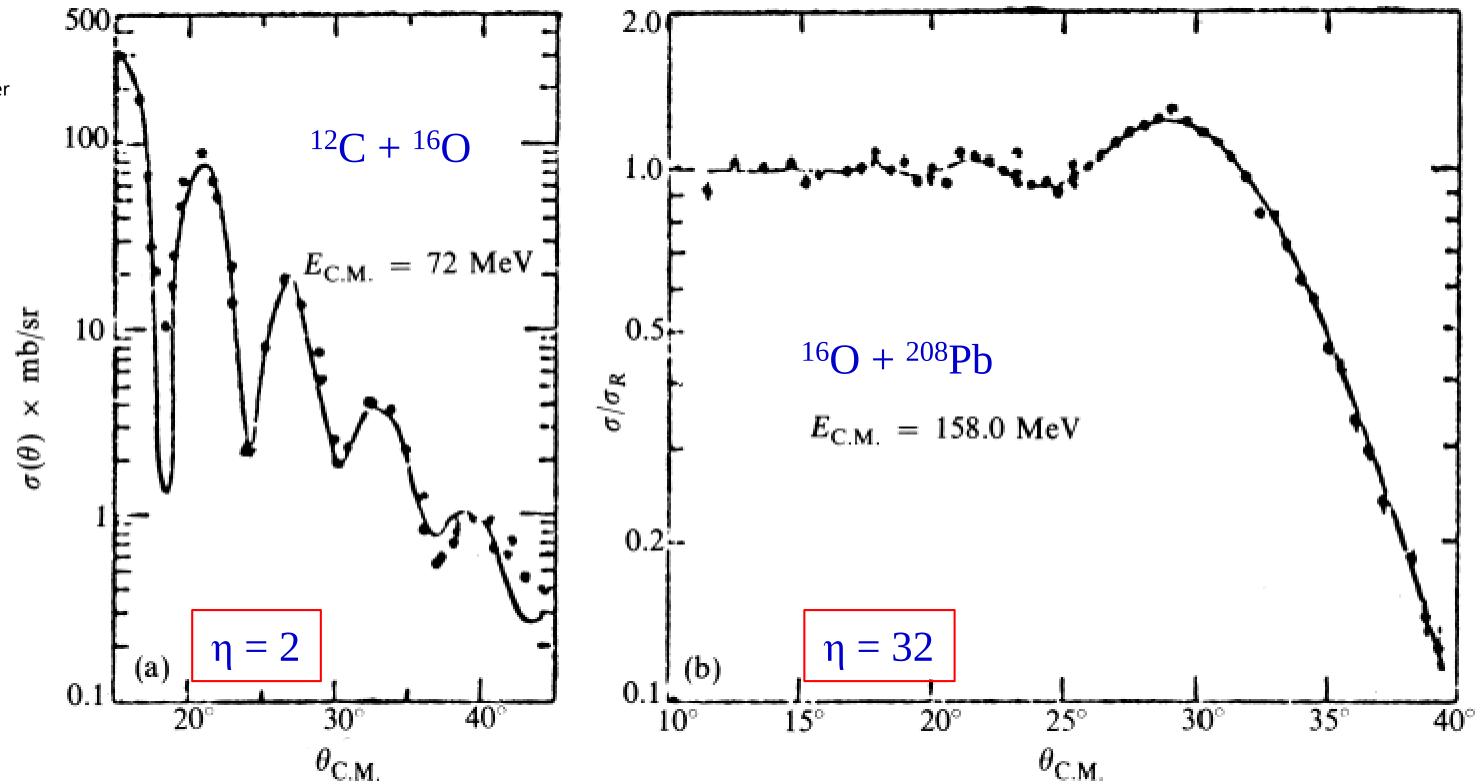
PPAC: 1.626 sr



Joseph von Fraunhofer
1787 – 1826

**Fraunhofer** (left) **and Fresnel** (right) **diffraction**

Augustin Jean Fresnel
1788 -1827



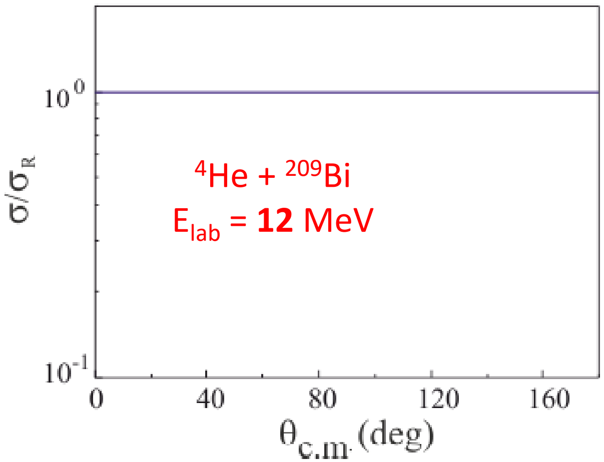

***Born approximation*** (quantum description) or ***classical description***: $\eta = \frac{a}{ƛ}$

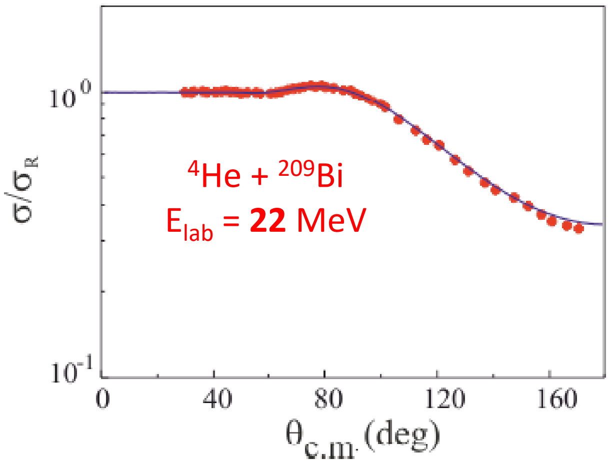
half distance of closest approach for head-on collision $a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$

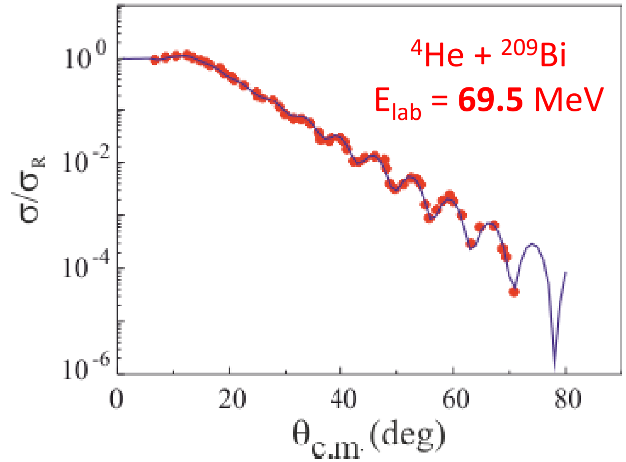
wave length of projectile ƛ = $(k_\infty)^{-1}$ $k_\infty = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$

$$\eta = k_\infty \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

# Elastic scattering



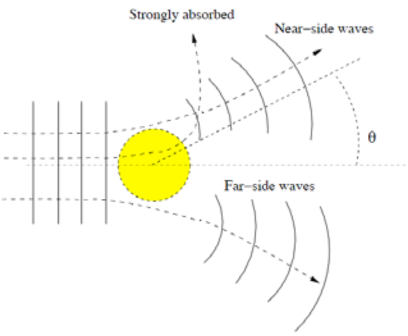



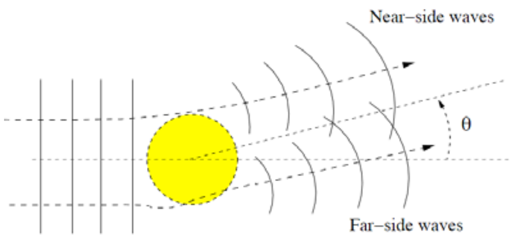

Rutherford scattering
$\eta = 15$

Fresnel scattering
$\eta = 11$

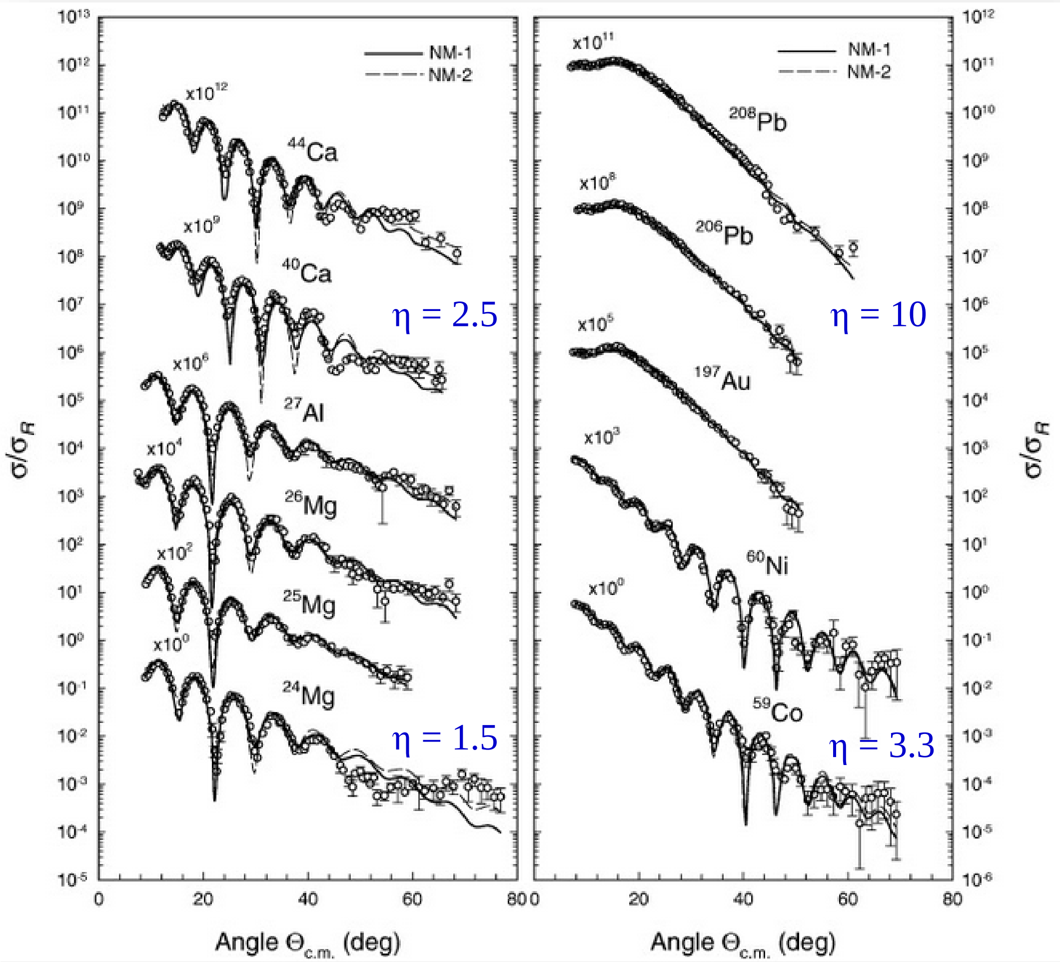
Fraunhofer scattering
$\eta = 6$

Transition from classical (optical) picture to quantum picture



$^6$Li elastic scattering @ 88 MeV




particle

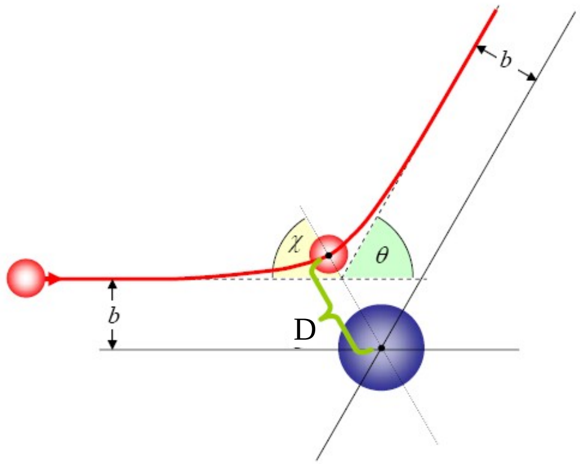
Fresnel scattering (η≥10)

wave

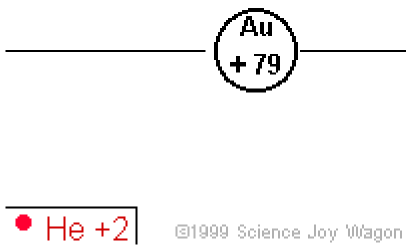
Fraunhofer scattering (η<10)

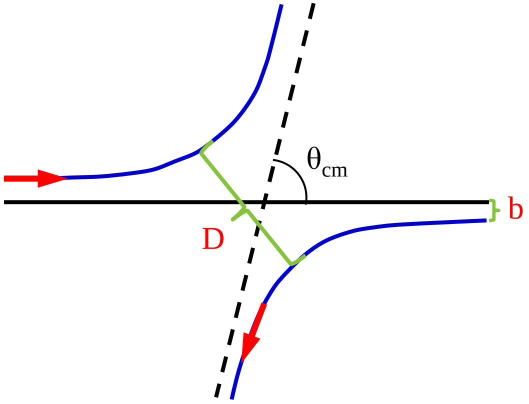
Oscillation in angular distribution → good angular resolution required

S. Hossain et al. Phys. Scr. 87 (2013) 015201




$$\theta = \pi - 2\chi$$




center of mass system

impact parameter:

$$b = a \cdot cot \frac{\theta_{cm}}{2}$$

distance of closest approach:

$$D = a \cdot \left[ sin^{-1} \frac{\theta_{cm}}{2} + 1 \right]$$

orbital angular momentum:

$$\ell = k_{\infty} \cdot b = \eta \cdot cot \frac{\theta_{cm}}{2}$$

half distance of closest approach in a head-on collision ($\theta_{cm}=180^0$):

$$a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$$

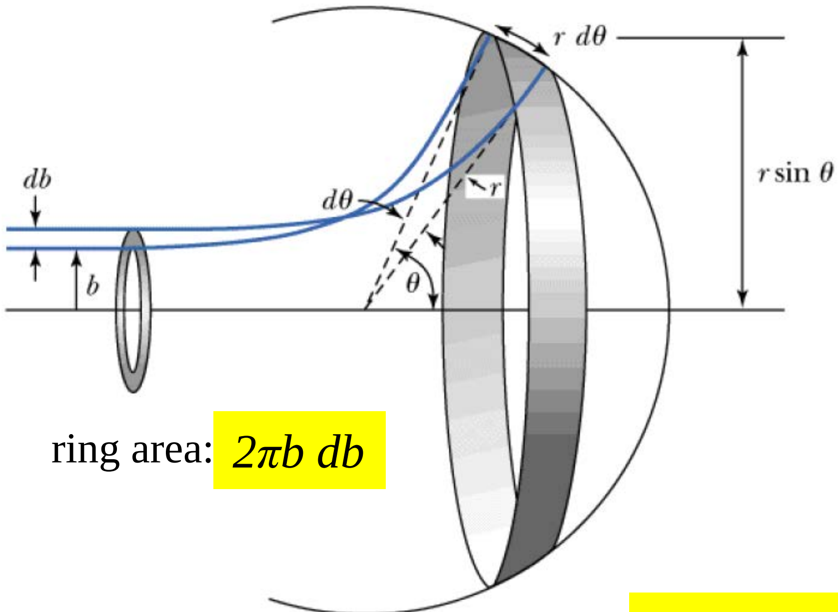
asymptotic wave number:

$$k_{\infty} = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$$

Sommerfeld parameter:

$$\eta = k_{\infty} \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

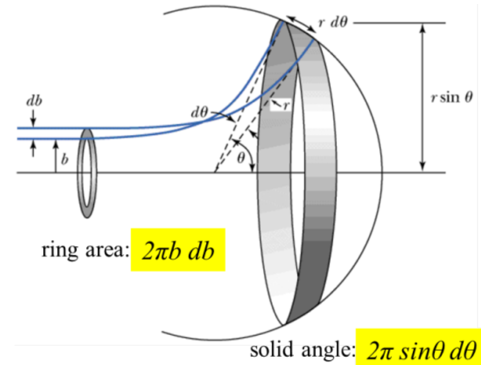
# Scattering theory

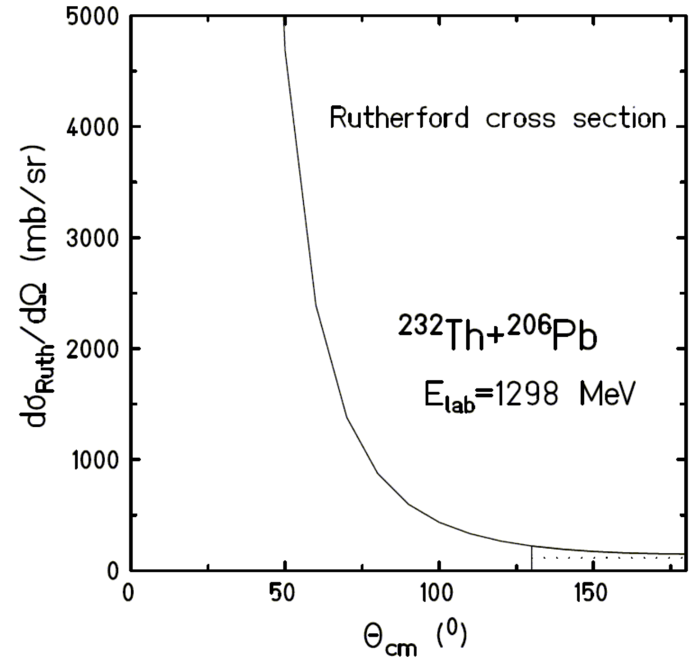



ring area: $2\pi b\, db$

solid angle: $2\pi \sin\theta\, d\theta$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$



angular momentum and scattering angle:

$$\ell = b \cdot p = b \cdot \sqrt{2 \cdot m \cdot T} \qquad k_\infty = \frac{\sqrt{2 \cdot m \cdot T}}{\hbar}$$

$$\ell = \eta \cdot \cot\frac{\theta}{2} \qquad \eta = k_\infty \cdot a$$

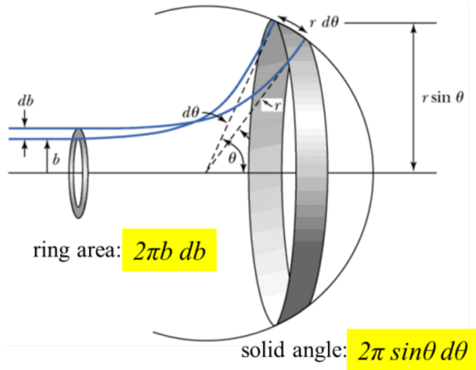
$$\frac{d\sigma}{d\ell} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{d\ell} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{d\Omega}{d\ell}$$

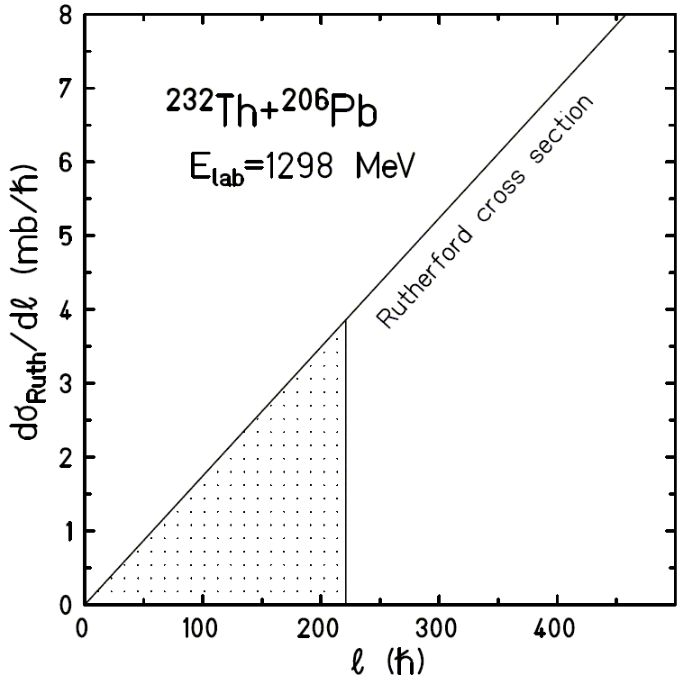
$$\frac{d\Omega}{d\ell} = 2\pi \cdot \sin\theta \cdot \frac{d\theta}{d\ell} \qquad \frac{d\ell}{d\theta} = \frac{\eta}{2} \cdot \sin^{-2}\frac{\theta}{2}$$

$$\frac{d\Omega}{d\ell} = 2\pi \cdot 2 \cdot \cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2} \cdot \frac{2 \cdot \sin^2\frac{\theta}{2}}{\eta} \qquad \cos\frac{\theta}{2} = \frac{\ell}{\eta} \cdot \sin\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{8\pi \cdot \ell}{\eta^2} \cdot \sin^4\frac{\theta}{2}$$

$$\boxed{\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell}$$

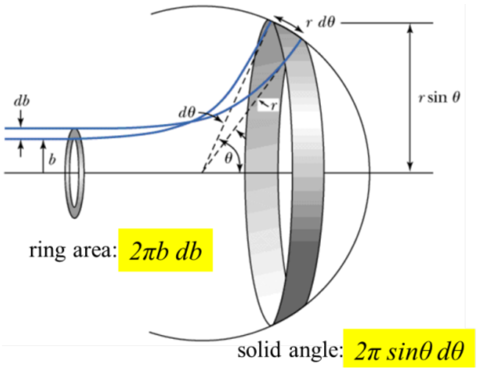


# Scattering theory

distance of closest approach and scattering angle:

$$D = a \cdot \left[ \sin^{-1}\frac{\theta}{2} + 1 \right] \qquad \sin\frac{\theta}{2} = \frac{a}{D-a}$$




$$\frac{d\sigma}{dD} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{dD} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{d\Omega}{dD}$$

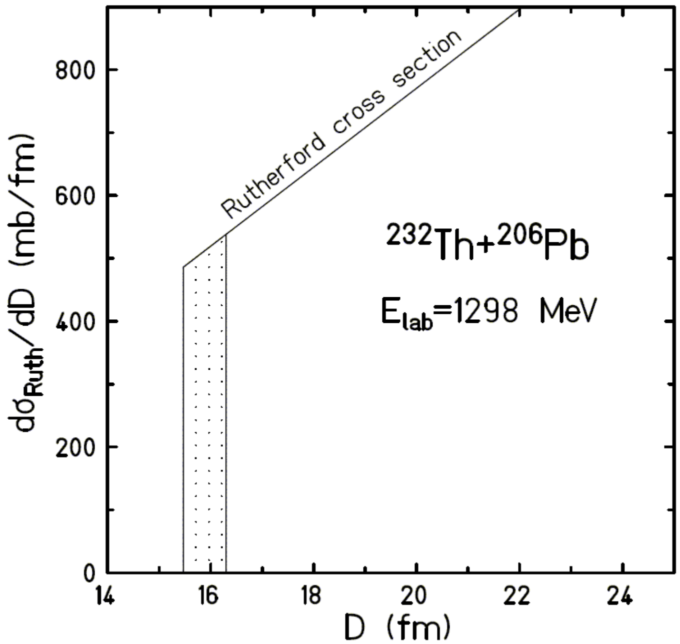
$$\frac{d\Omega}{dD} = 2\pi \cdot \sin\theta \cdot \frac{d\theta}{dD} \qquad \frac{dD}{d\theta} = \frac{a}{2} \cdot \frac{-\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}$$

$$\frac{d\Omega}{dD} = 2\pi \cdot 2 \cdot \cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2} \cdot \frac{2}{a} \cdot \frac{\sin^2\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\frac{d\Omega}{dD} = \frac{8\pi}{a} \cdot \sin^3\frac{\theta}{2}$$

$$\frac{d\sigma}{dD} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2} \cdot \frac{8\pi}{a} \cdot \sin^3\frac{\theta}{2} = \frac{2\pi \cdot a}{\sin\frac{\theta}{2}}$$

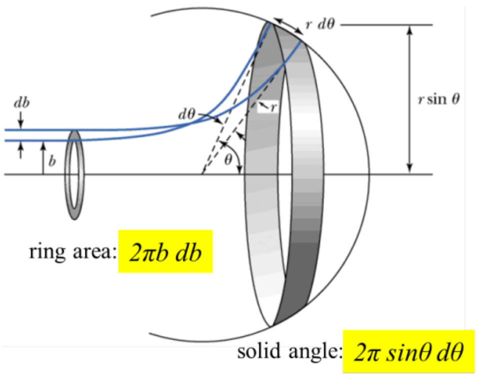
$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$



❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2} \qquad a = \frac{Z_p \cdot Z_t \cdot e^2}{2 \cdot E_{cm}}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$




❖ angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2} \qquad \eta = k_\infty \cdot a \qquad k_\infty = \frac{\sqrt{2 \cdot m \cdot E_{cm}}}{\hbar}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

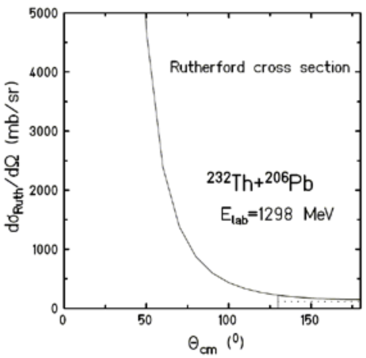
$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$

# Summary

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}$$

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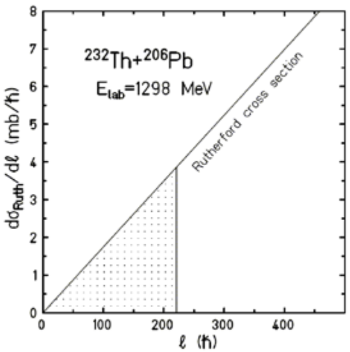


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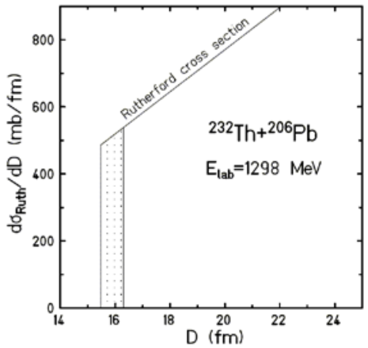
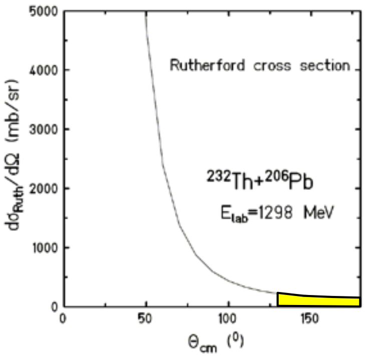


# Elastic scattering and nuclear reactions

❖ impact parameter and scattering angle:

$$b = a \cdot \cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot \sin^{-4}\frac{\theta}{2}$$

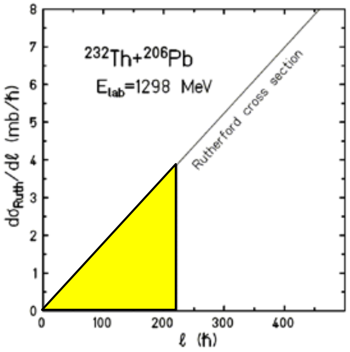



$\theta_{1/4} = 132^0$

❖ angular momentum and scattering angle:

$$\ell = \eta \cdot \cot\frac{\theta}{2}$$

$$\frac{d\sigma}{d\ell} = \frac{2\pi}{k_\infty^2} \cdot \ell$$

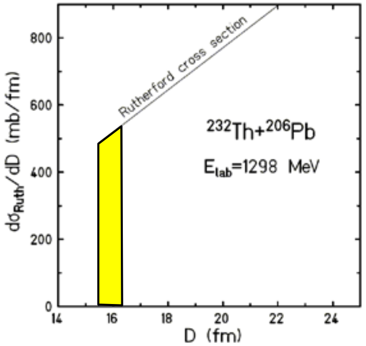



$\ell_{gr} = 206\ \hbar$

❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[\sin^{-1}\frac{\theta}{2} + 1\right]$$

$$\frac{d\sigma}{dD} = 2\pi \cdot (D - a)$$



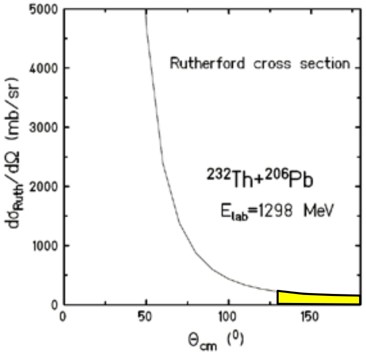

$R_{int} = 16.2$ fm

# Elastic scattering and nuclear reactions

❖ impact parameter and scattering angle:

$$b = a \cdot cot\frac{\theta}{2}$$

$$\sigma_{reaction} = 2\pi a^2 \cdot \left[\left(1 - cos\ \theta_{1/4}^{cm}\right)^{-1} - 0.5\right]$$

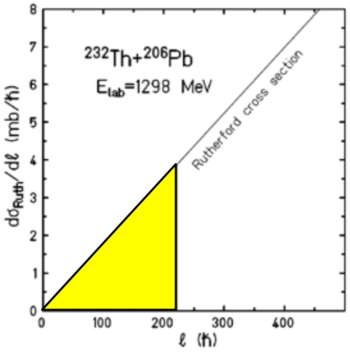



$\theta_{1/4} = 132^0$ a = 7.73 fm

❖ angular momentum and scattering angle:

$$\ell = \eta \cdot cot\frac{\theta}{2}$$

$$\sigma_{reaction} = \frac{\pi}{k_\infty^2} \cdot \ell_{gr}(\ell_{gr} + 1)$$



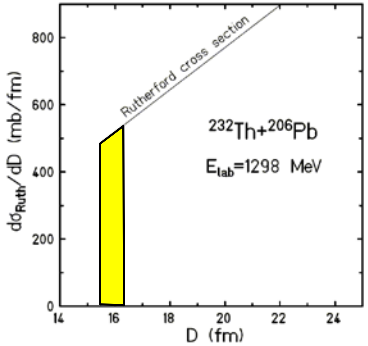

$\ell_{gr} = 206\ \hbar$ $k_\infty = 59.9\ fm^{-1}$

❖ distance of closest approach and scattering angle:

$$D = a \cdot \left[sin^{-1}\frac{\theta}{2} + 1\right]$$

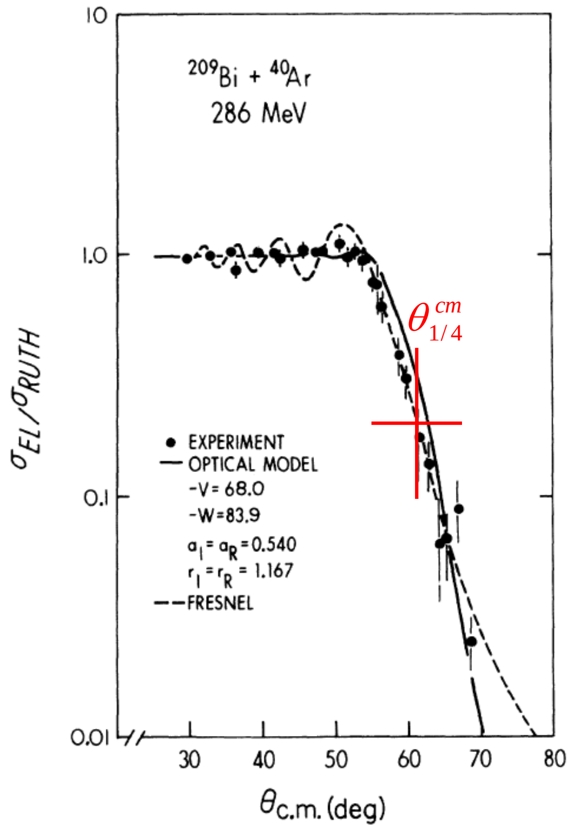
$$\sigma_{reaction} = \pi \cdot R_{int}^2 \cdot \left(1 - \frac{V_C(R_{int})}{E_{cm}}\right)$$

$R_{int}$ can be measured for $E_{cm} >> 0$

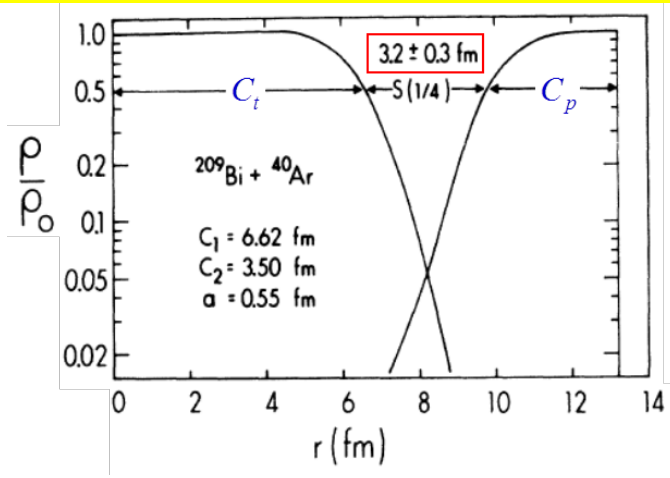



$R_{int}$ = 16.2 fm $V_C(R_{int})$ = 656 MeV

# Elastic scattering and the nuclear radius




Nuclear density distributions at the nuclear interaction radius




$$R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35} \quad [fm]$$

$C_i = R_i \cdot (1 - R_i^{-2}) \quad [fm] \qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$

$\theta_{1/4} = 60^0 \rightarrow R_{int} = 13.4$ [fm]

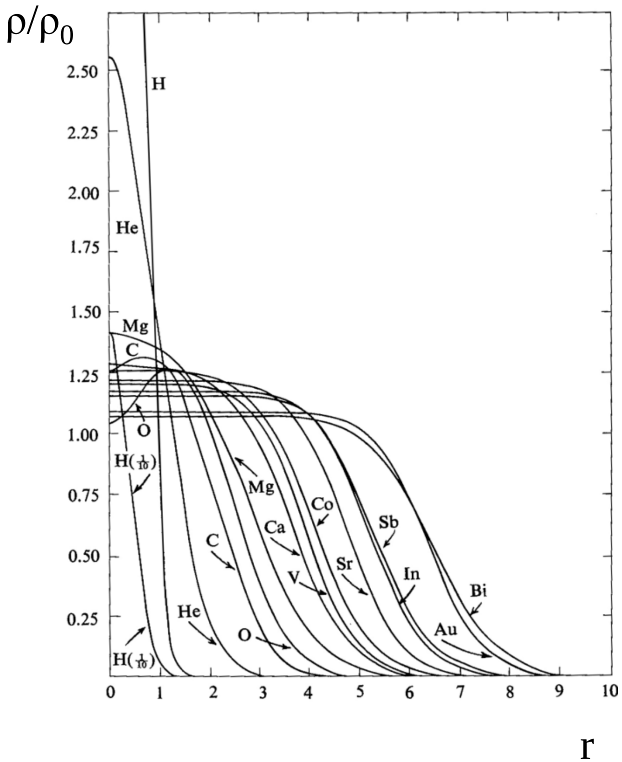
$\rightarrow \ell_{gr} = 152$ [ħ]

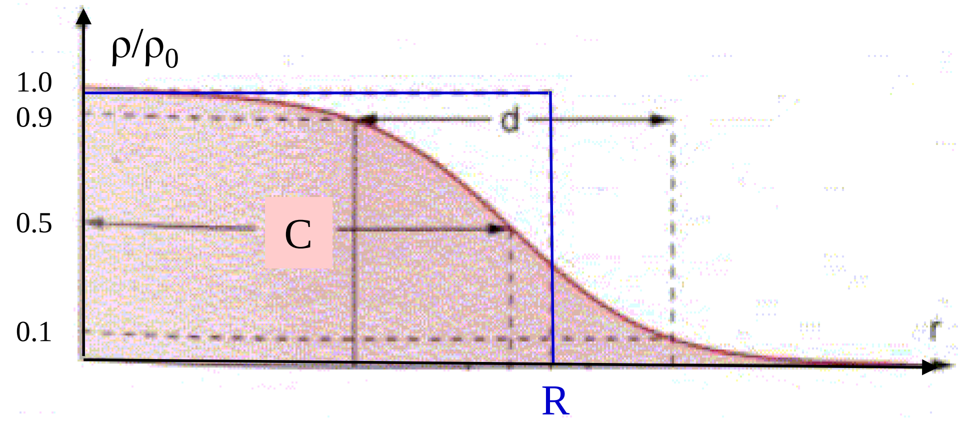
Nuclear interaction radius: (distance of closest approach)

$$R_{int} = D = a \cdot \left[ sin^{-1}\frac{\theta_{1/4}}{2} + 1 \right]$$

$C_i = R_i \cdot (1 - R_i^{-2})$ [fm]

# Nuclear radius



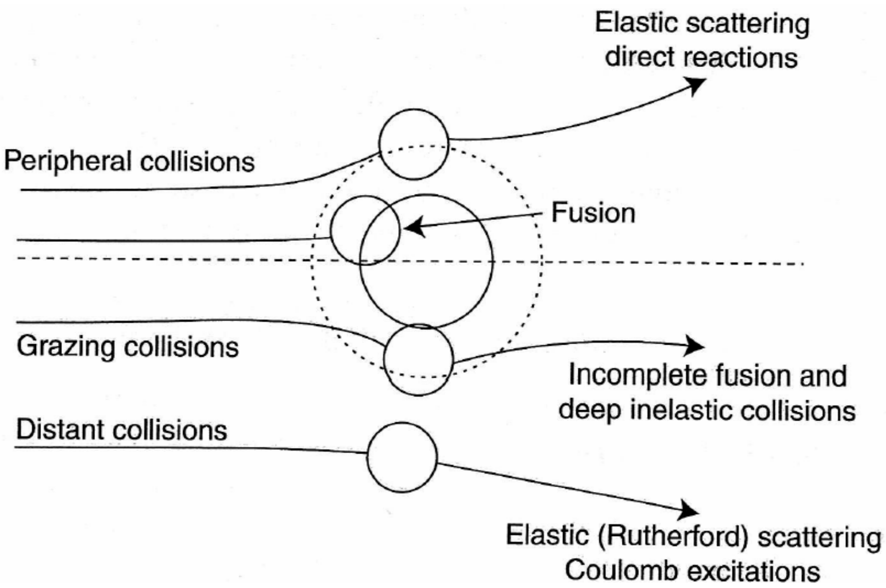

nuclear radius of a homogenous charge distribution:

$$R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

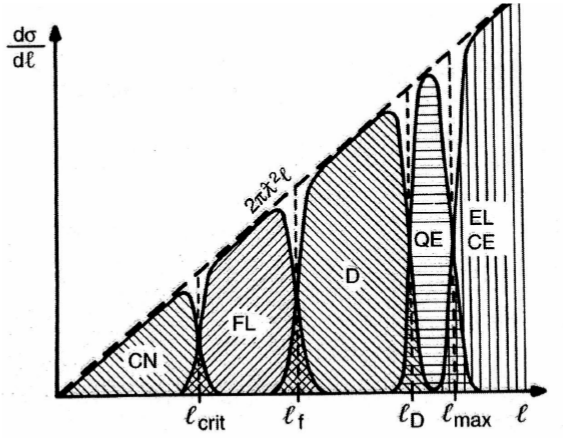
nuclear radius of a Fermi charge distribution:

$$C_i = R_i \cdot \left(1 - R_i^{-2}\right) \quad [fm]$$

# Classification of heavy ion collisions




partial cross section vs. angular momentum




- CN: compound nucleus
- FL: fusion-like
- D: deep inelastic
- QE: quasi elastic
- CE: Coulomb excitation
- EL: elastic