Outline: Fusion reactions

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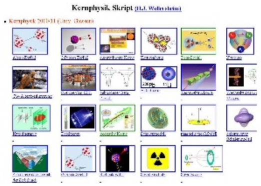
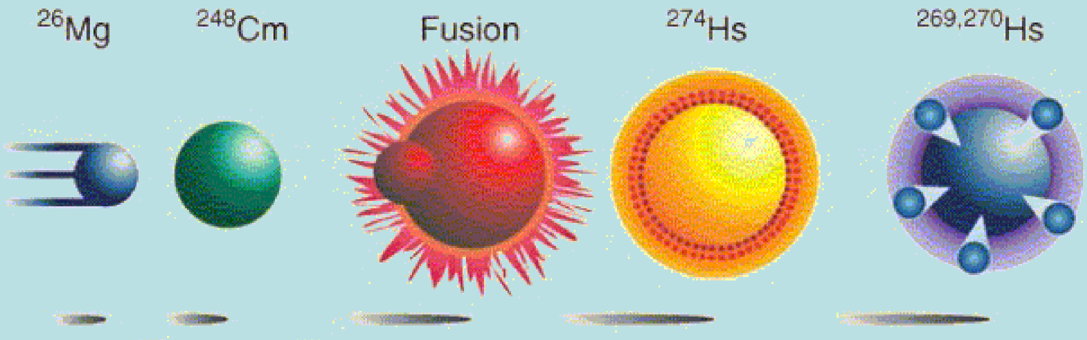


1. fusion cross section
2. de-excitation of a hot compound nucleus
3. nuclear temperature, level density, evaporation particles
4. limiting nuclear angular momentum
5. production of super heavy elements SHE

Fusion reactions

"Hot Fusion"



$5 \cdot 10^{15} \longrightarrow 5 \cdot 10^{8} \longrightarrow$

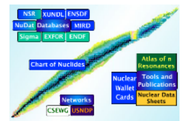
projectiles on target — compound nuclei — 1 atom

Hot fusion (~1952) successful up to element 106 (Seaborgium)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 126.2\ MeV \quad \left({}^{26}Mg + {}^{248}Cm\right)$$

- reaction: a + A → C* → B + b



https://www.nndc.bnl.gov/qcalc/

$$\Delta m = m_a + m_A - m_{CN}$$

$$\Delta m = (25.983 + 248.072 - 274.143) * 931.478\ MeV/c^2$$

$$= -82.153\ MeV/c^2$$

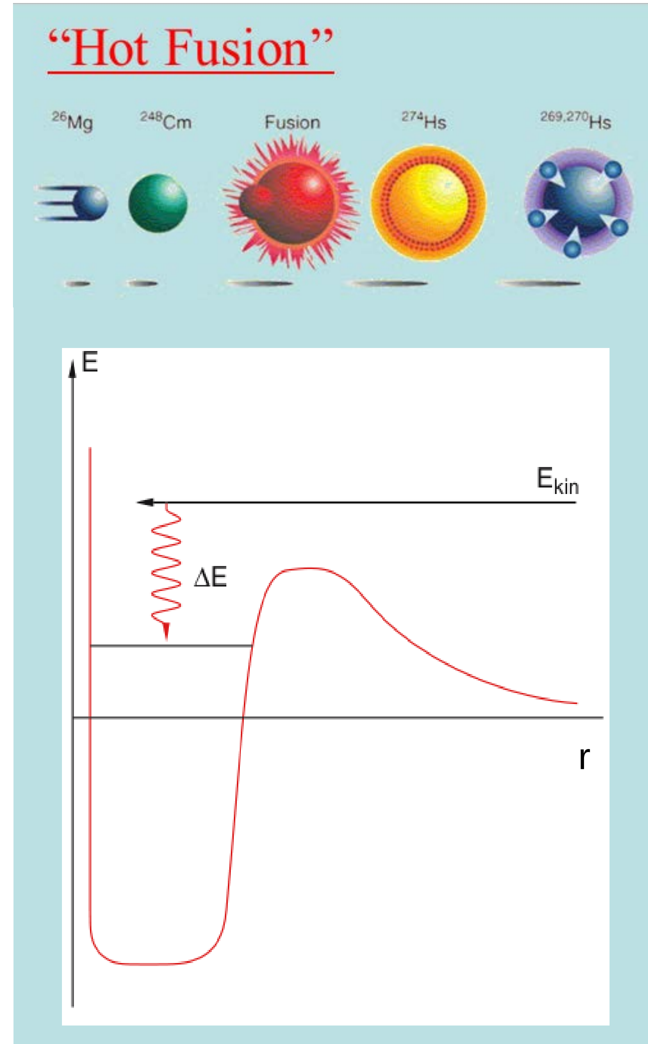
- excitation energy of compound nucleus

$$E^* = E_{kin} + \Delta m \cdot c^2$$

$$= 126.2\ MeV - 82.2\ MeV$$

$$\mathbf{= 44.0\ MeV}$$

- approximate 4 neutrons will be evaporated to avoid fission

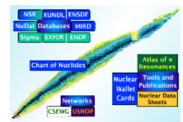


Cold fusion (1981-1996)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 223.3\ MeV \quad \left({}^{58}Fe + {}^{208}Pb\right)$$

- reaction: a + A → C* → B + b



https://www.nndc.bnl.gov/qcalc/

$$\Delta m = m_a + m_A - m_{CN}$$

$$\Delta m = (57.933 + 207.977 - 266.130) * 931.478\ MeV/c^2$$

$$= -205.092\ MeV/c^2$$

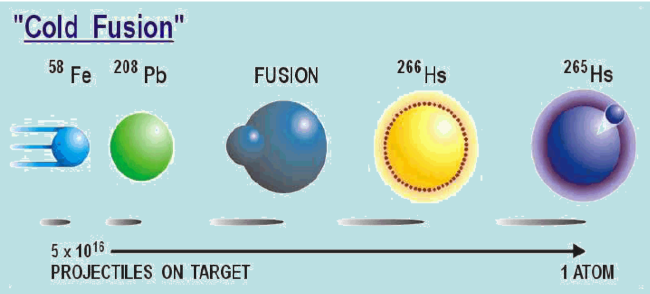
- excitation energy of compound nucleus

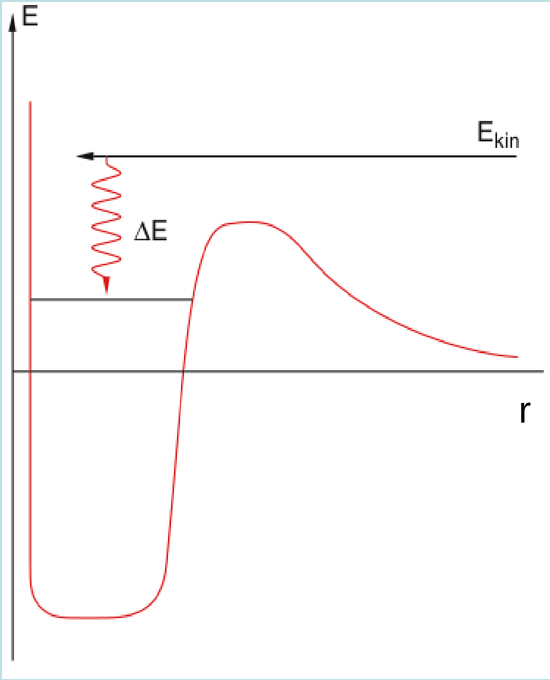
$$E^* = E_{kin} + \Delta m \cdot c^2$$

$$= 223.3\ MeV - 205.1\ MeV$$

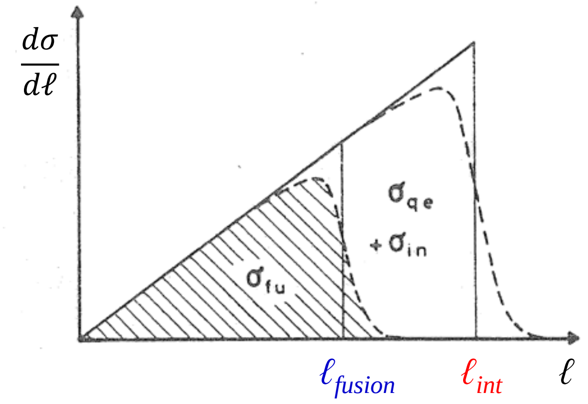
$$\mathbf{= 18.2\ MeV}$$

- approximate 1-2 neutrons will be evaporated to avoid fission





Fusion cross section



Radius for fusion barrier:

$$R_{fusion} = R_{int} - \begin{cases} 0.3117 \cdot (Z_p \cdot Z_t)^{0.2122} & Z_p \cdot Z_t < 500 \\ 1.096 + 1.391 \cdot Z_p \cdot Z_t / 1000 & Z_p \cdot Z_t \geq 500 \end{cases} \quad [fm]$$

	R_i [fm]	C_i [fm]	R_{int} [fm]	$V_C(R_{int})$ [MeV]	R_{fusion} [fm]	$V_C(R_{fusion})$ [MeV]
^{58}Fe	4.40	4.17	13.75	223.3	12.36	248.4
^{208}Pb	6.96	6.82				

Total cross section for fusion:

$$\sigma_{fusion} = \pi R_{fusion}^2 \cdot \left[1 - \frac{V_C\left(R_{fusion}\right)}{E_{cm}}\right] \qquad with \quad E_{cm} = \frac{A_t}{A_t + A_p} \cdot E_{lab}$$

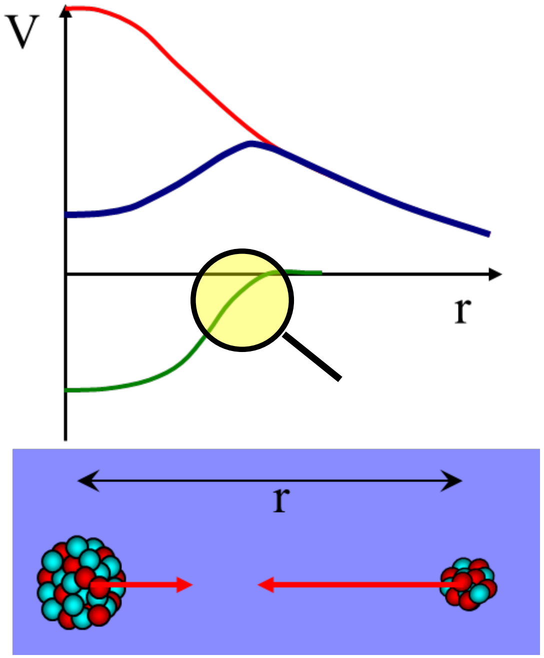
$$\sigma_{fusion} = \frac{\pi}{k_\infty^2} \cdot \ell_{fusion} \cdot \left(\ell_{fusion} + 1\right) \qquad with \quad k_\infty = 0.2187 \cdot \frac{A_t}{A_t + A_p} \cdot \sqrt{A_p \cdot E_{lab}} \quad [fm^{-1}]$$

Interaction potential

The potential between projectile and target nucleus is given by a function of the relative distance between them

$$V(r) = V_N(r) + V_C(r)$$

nuclear potential + Coulomb potential



$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2 \cdot R_C}\left(3 - \frac{r^2}{R_C^2}\right) & r < R_C \\ \frac{Z_1 Z_2 e^2}{r} & r \geq R_C \end{cases}$$

nuclear proximity potential:

$$V_N(r) = 4\pi \cdot \gamma \cdot \frac{C_p \cdot C_t}{C_p + C_t} \cdot b \cdot \Phi(\xi)$$

$$\Phi(\xi) = \begin{cases} -0.5 \cdot (\xi - 2.54)^2 - 0.0852 \cdot (\xi - 2.54)^3 & \xi \leq 1.2511 \\ -3.437 \cdot exp(-\xi/0.75) & \xi \geq 1.2511 \end{cases}$$

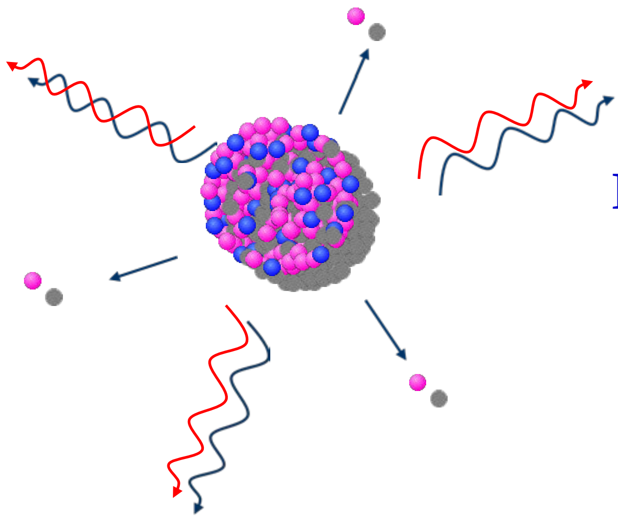
$$\xi = (r - C_p - C_t)/b$$

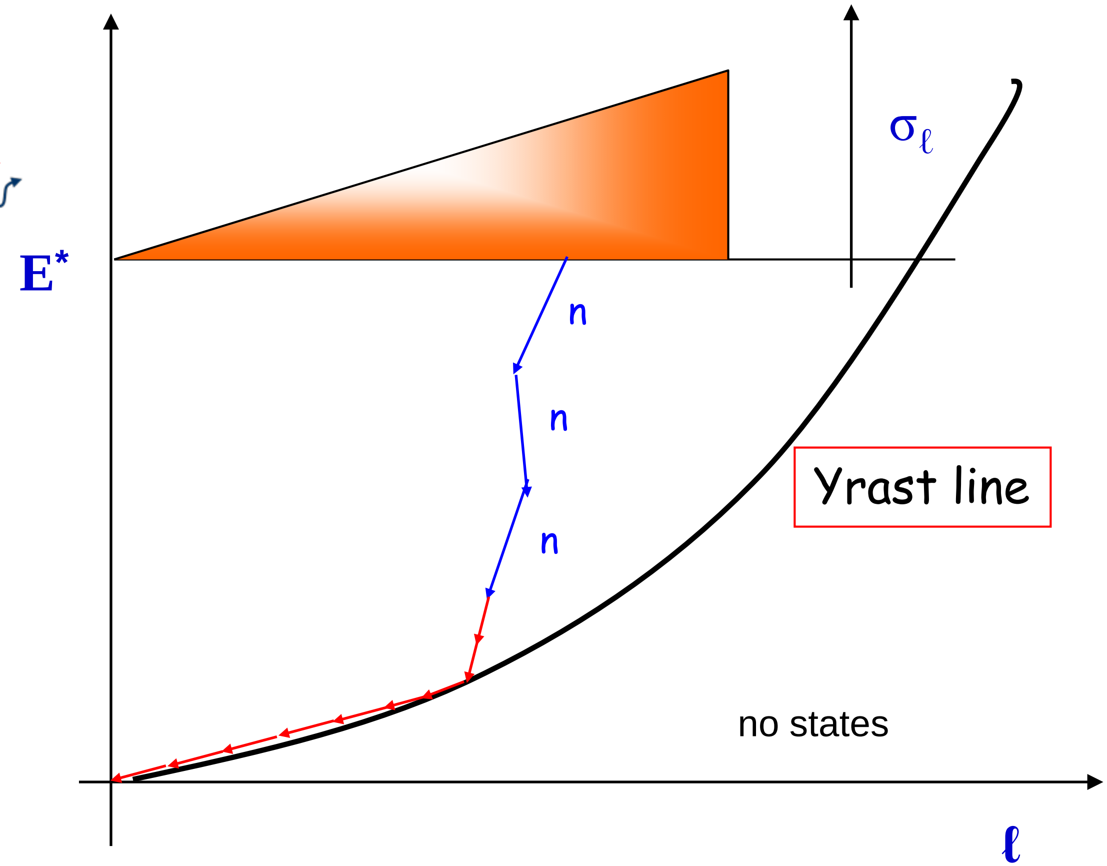
$$b = \frac{\pi}{\sqrt{3}} \cdot a \cong 1\, fm \quad with\ a = 0.55\, fm$$

$$\gamma = 0.9517 \cdot \left\{1 - 1.7826 \cdot \left(\frac{N_c - Z_c}{A_c}\right)^2\right\} \quad \frac{MeV}{fm^2}$$

$$C_i = R_i \cdot (1 - R_i^{-2}) \quad [fm] \qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

The statistical model de-excitation of the hot compound system

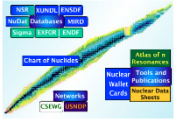




$$E^* = E_{kin} + \Delta m \cdot c^2$$

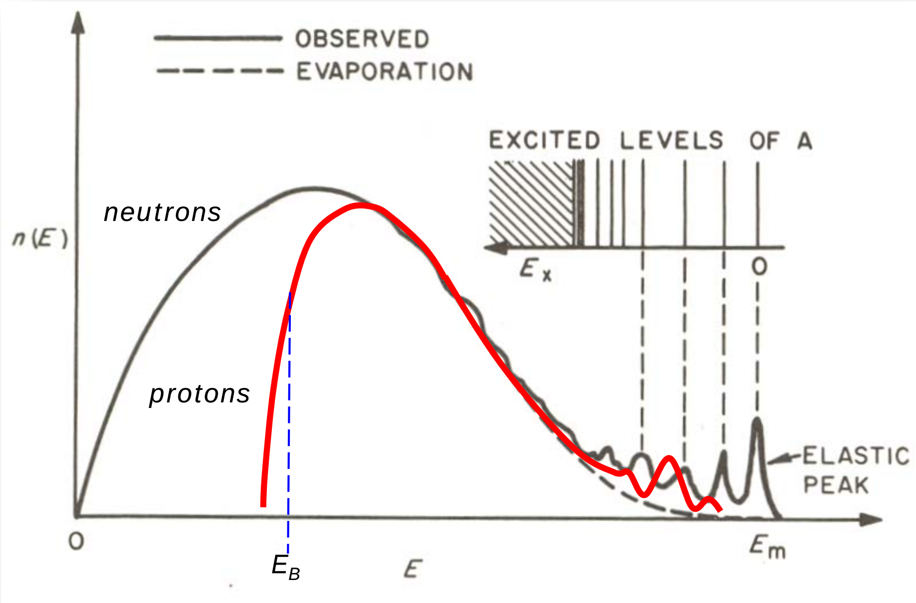
$$E_{kin} > V_C = \frac{Z_a \cdot Z_A \cdot e^2}{R_{int}}$$

$$\Delta m = m_a + m_A - m_{CN}$$



https://www.nndc.bnl.gov/qcalc/

Evaporation particles



compound nucleus reactions

direct reactions

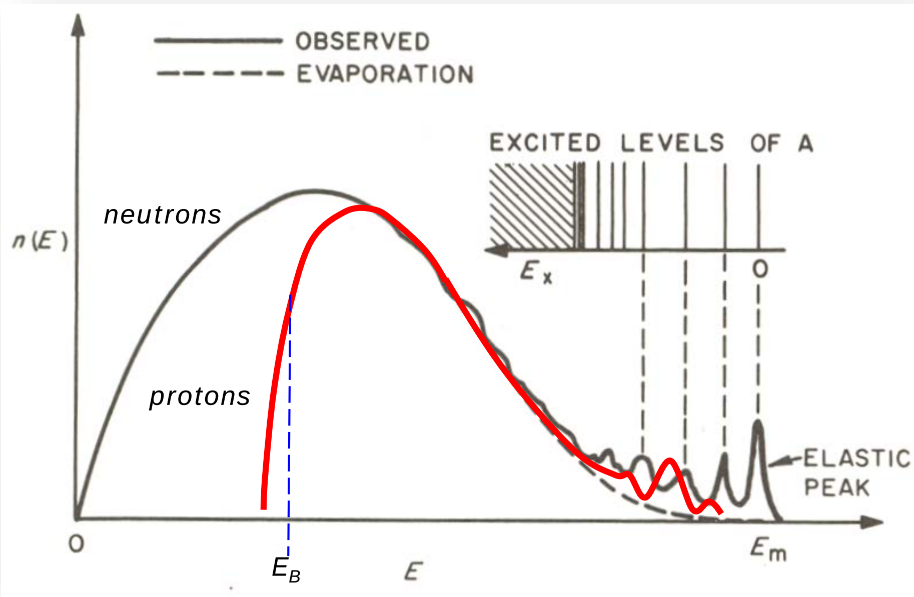
cm-spectra of particles statistically emitted from CN (evaporation) are of Maxwell Boltzmann type

$$\frac{dN}{dE} \propto (E - E_B) \cdot e^{-E/T}$$

E_B = Coulomb barrier
T = effective nuclear temperature

Typical energy spectrum of nucleons emitted at a fixed angle in inelastic nucleon-nucleon reactions.

Evaporation particles



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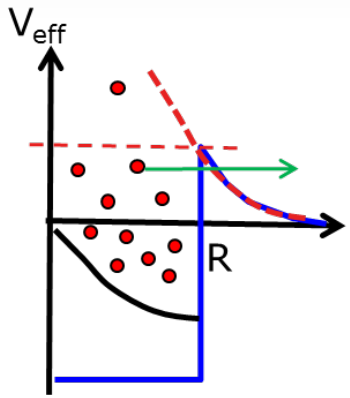
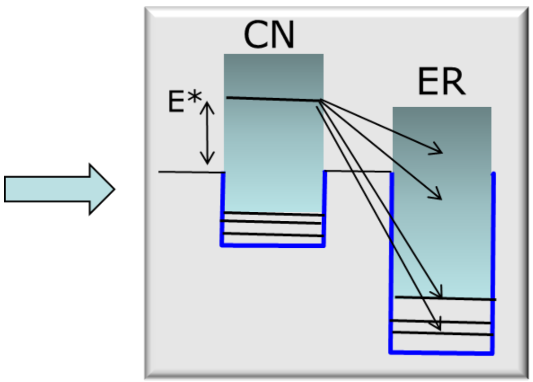
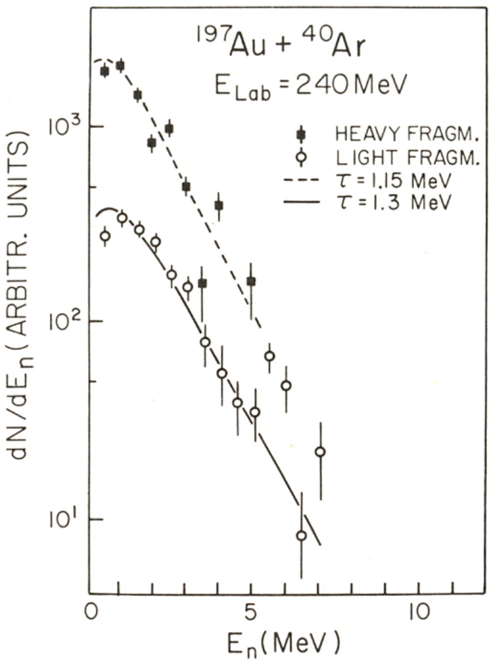




Even for fixed E^* the particle spectrum is continuous (Maxwell Boltzmann), except for transitions to discrete spectrum at low E_{ER}^*

Nuclear temperatures and level densities

de-excitation of the hot compound system

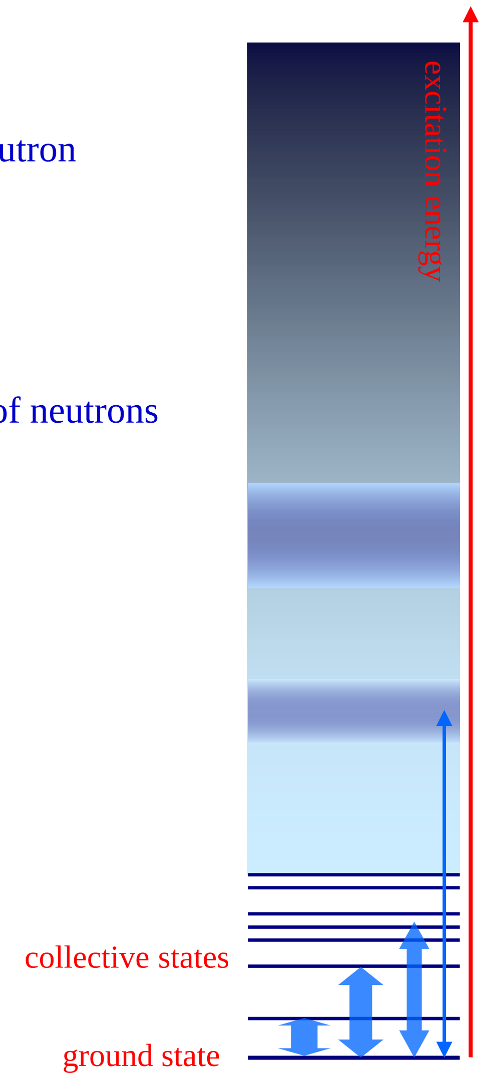


$$\frac{dN}{dE_n} \propto E_n \cdot e^{-E_n/T}$$ spectrum of single neutron

$$\langle E_n \rangle = 2T \qquad \max \frac{dN}{dE_n} \;@\; E_n = T$$

$$\frac{dN}{dE_n} \propto \sqrt{E_n} \cdot e^{-E_n/T_{eff}}$$ spectrum of cascade of neutrons

$$\langle E_n \rangle = 1.5T \qquad T_{eff} \approx 0.92 \cdot T \quad (1^{st}\ daughter)$$



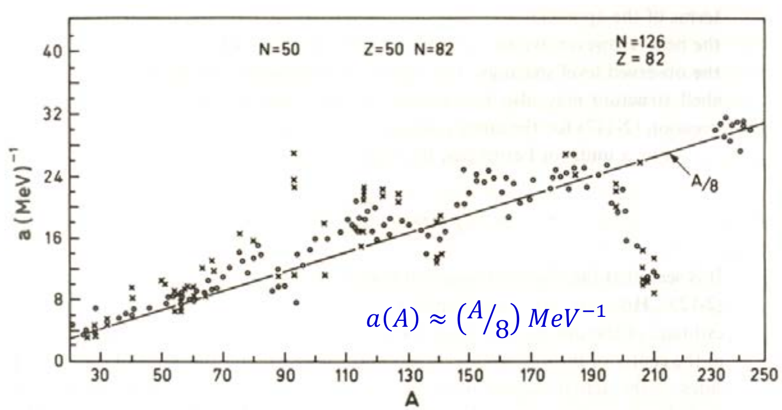
Fermi gas relations:

$$E^* = a \cdot T^2 \quad \text{"little} - a\text{"}$$

$$S = \int \frac{dE^*}{E^*} = 2\sqrt{a \cdot E^*}$$

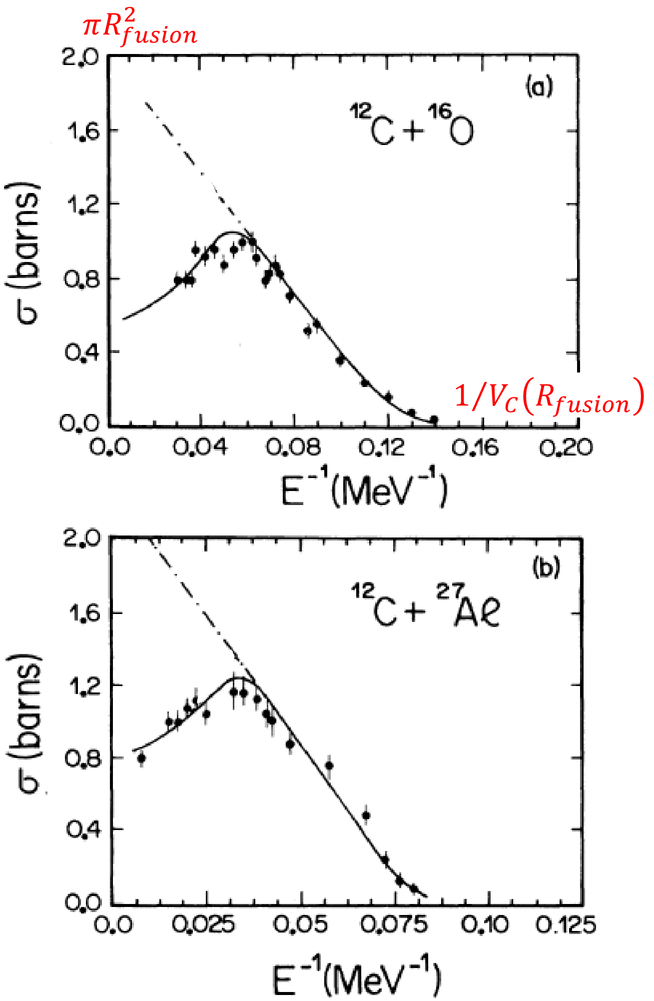
$$\rho\left(E^*\right) = \rho_0 \cdot e^{2\sqrt{a \cdot E^*}}$$

deviations at shell closures

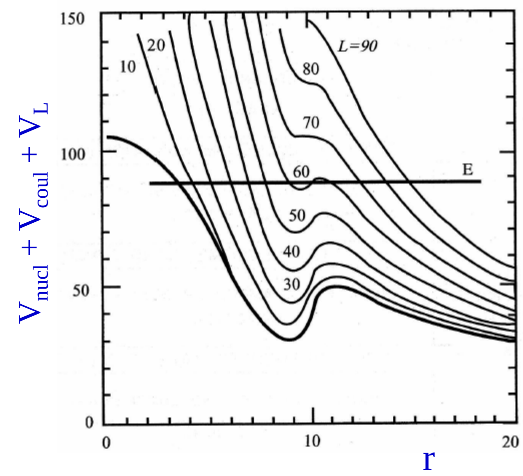


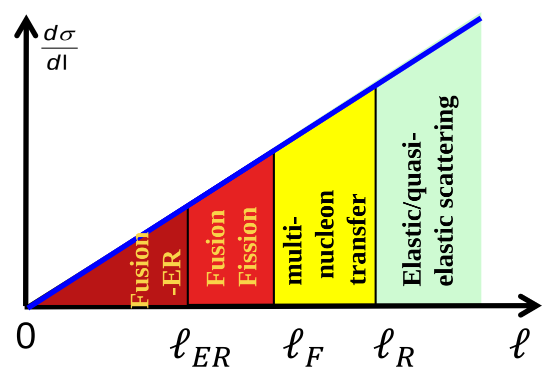
$$a(A) \approx \left({}^{A}/_{8}\right) MeV^{-1}$$

Fusion excitation functions



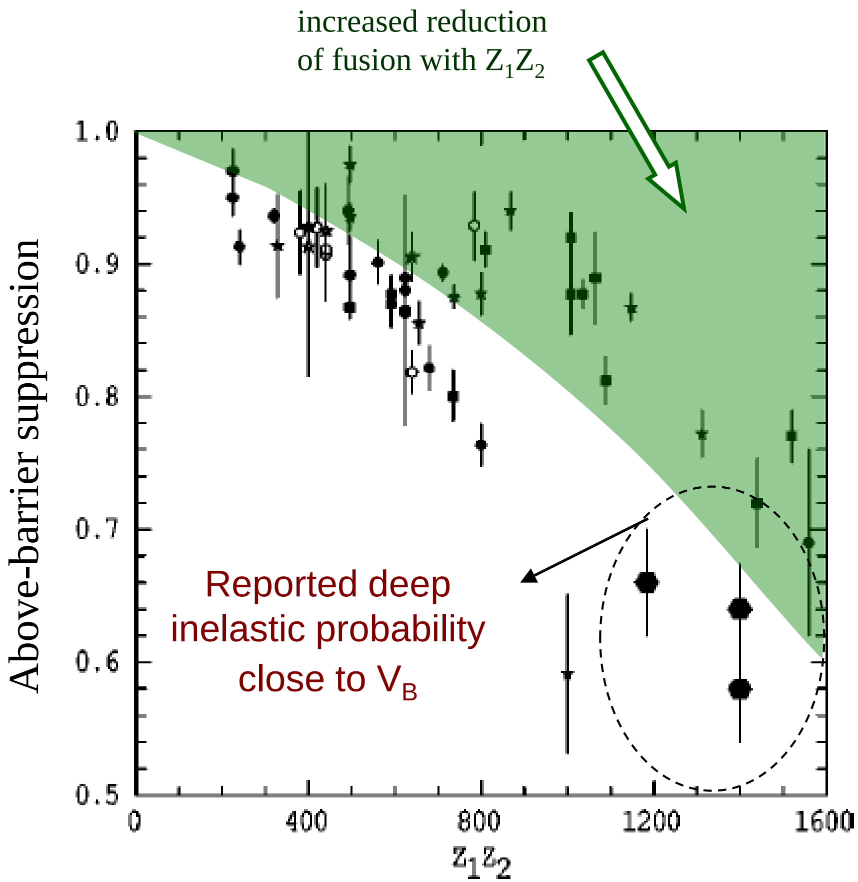
maximum ℓ_{fusion} due to nuclear centrifugal stability

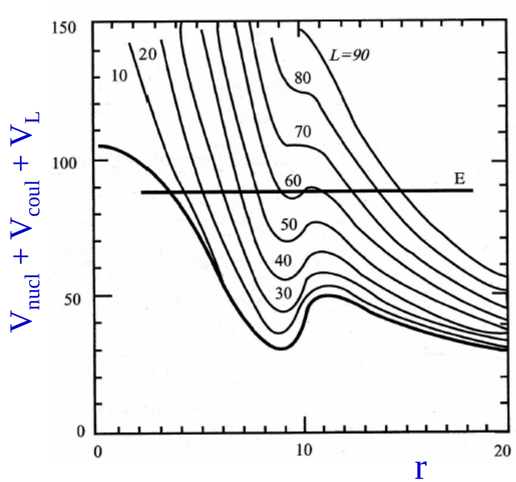


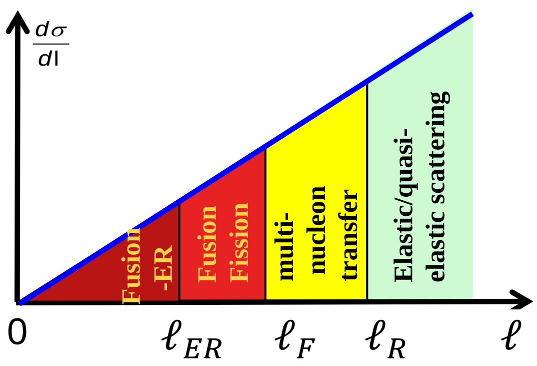


P. Sperr et al., PRL37, 321(1976)

Reduction in fusion at above barrier energies







P. Sperr et al., PRL37, 321(1976)

A limiting nuclear angular momentum rotating charged liquid drop

surface energy:

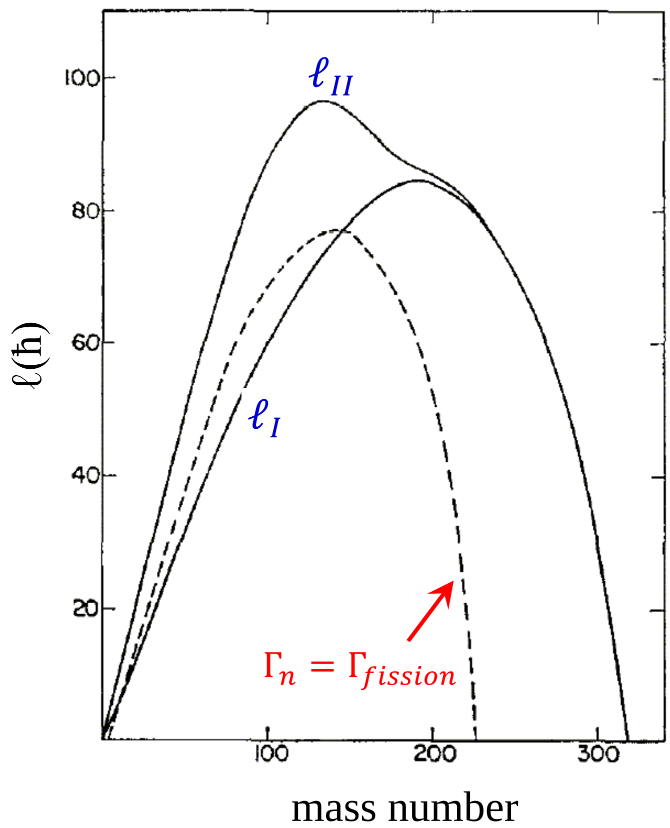
$$E_S^{(0)} = 17.9439 \cdot \left[1 - 1.7826 \cdot \left(\frac{N-Z}{A}\right)^2\right] \cdot A^{2/3} \quad [MeV]$$

Coulomb energy:

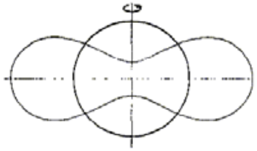
$$E_{Coul}^{(0)} = 0.7053 \cdot \left(Z^2/A^{1/3}\right) \quad [MeV]$$

rotational energy:

$$E_{Rot}^{(0)} = \frac{1}{2}\frac{\hbar^2 \cdot \ell^2}{(2/5) \cdot A \cdot m \cdot R^2} = 34.54 \cdot \frac{\ell^2}{A^{5/3}} \quad [MeV]$$



change of the nuclear shape



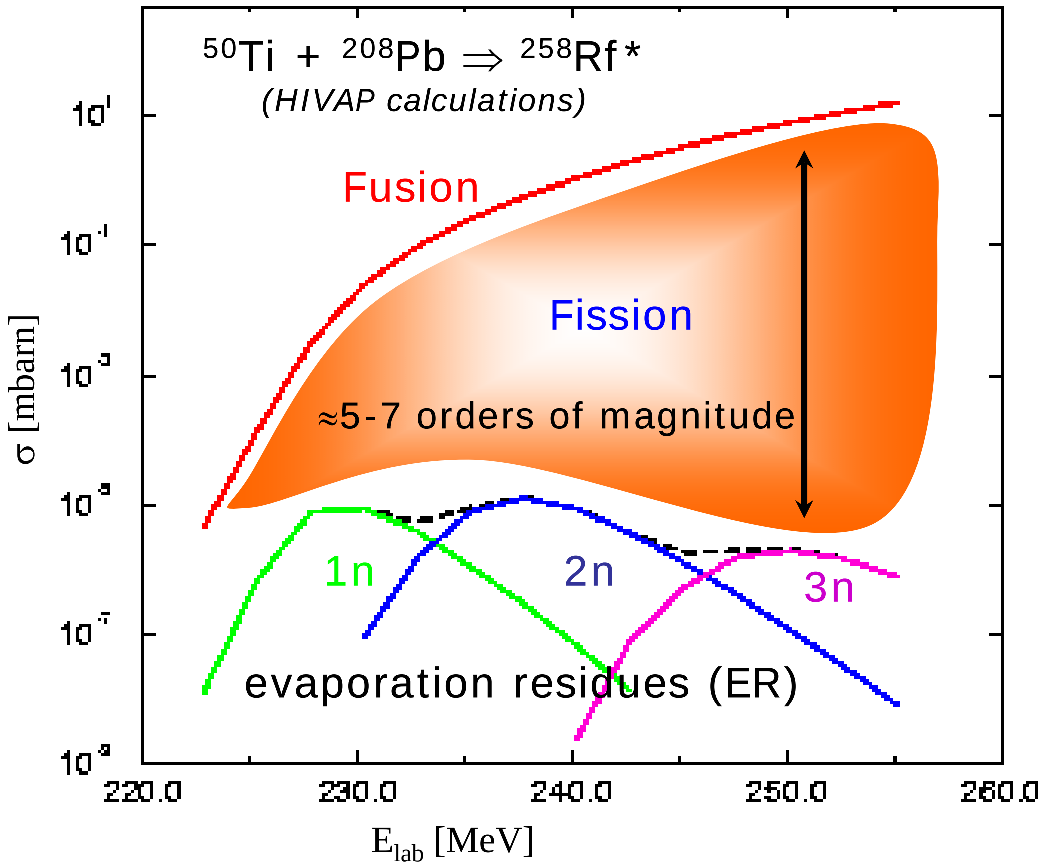
$$\frac{E_{Rot}^{(0)}}{E_S^{(0)}} = \begin{cases} 0.2829 - 0.3475 \cdot X - 0.0016 \cdot X^2 + 0.0501 \cdot X^3 & 0 \leq X \leq 0.75 \\ (7/5) \cdot (1-X)^2 - 4.5660 \cdot (1-X)^3 + 6.7443 \cdot (1-X)^4 & 0.75 \leq X \leq 1.0 \end{cases}$$

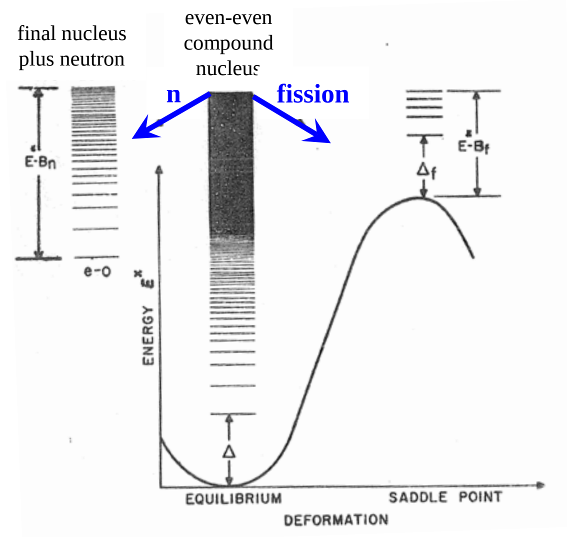
with $X = \dfrac{E_{Coul}^{(0)}}{2 \cdot E_S^{(0)}}$ "fissility parameter"

example: ${}^{127}_{57}La$ $E_S^{(0)} = 444.9\ [MeV]$ $E_{Coul}^{(0)} = 455.9\ [MeV]$ $X = 0.512$ $E_{Rot}^{(0)}/E_S^{(0)} = 0.1112$ $E_{Rot}^{(0)} = 49.48\ [MeV]$ $\ell_I = 67.8\ [\hbar]$

Cohen, Plasil, Swiatecki; Ann. Phys. 82, 557 (1974)

Fusion and evaporation cold fusion





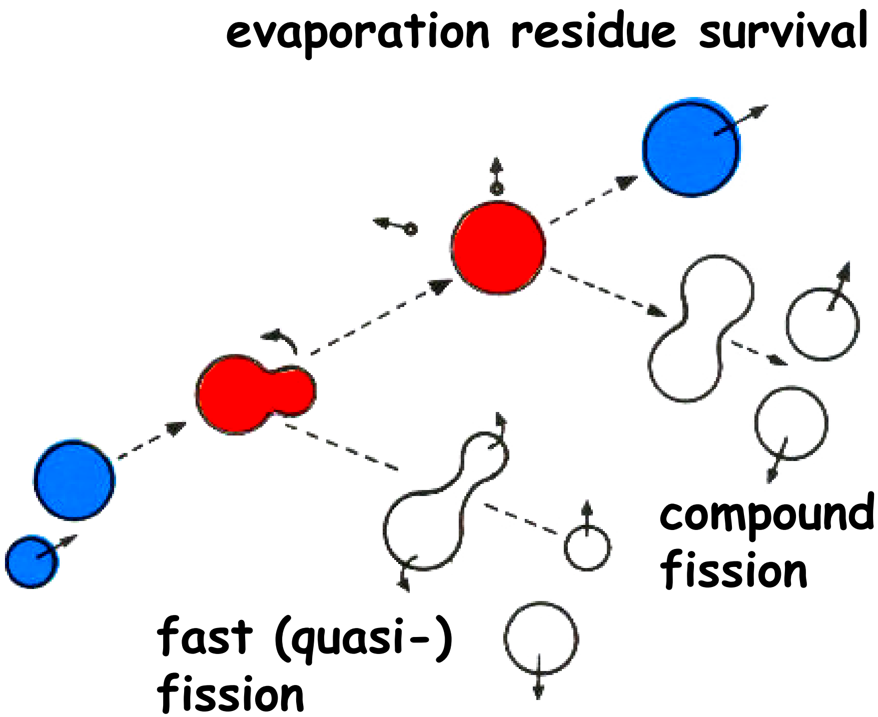
Both decay processes are determined by the level density, either from the residual nucleus or at the saddle point.

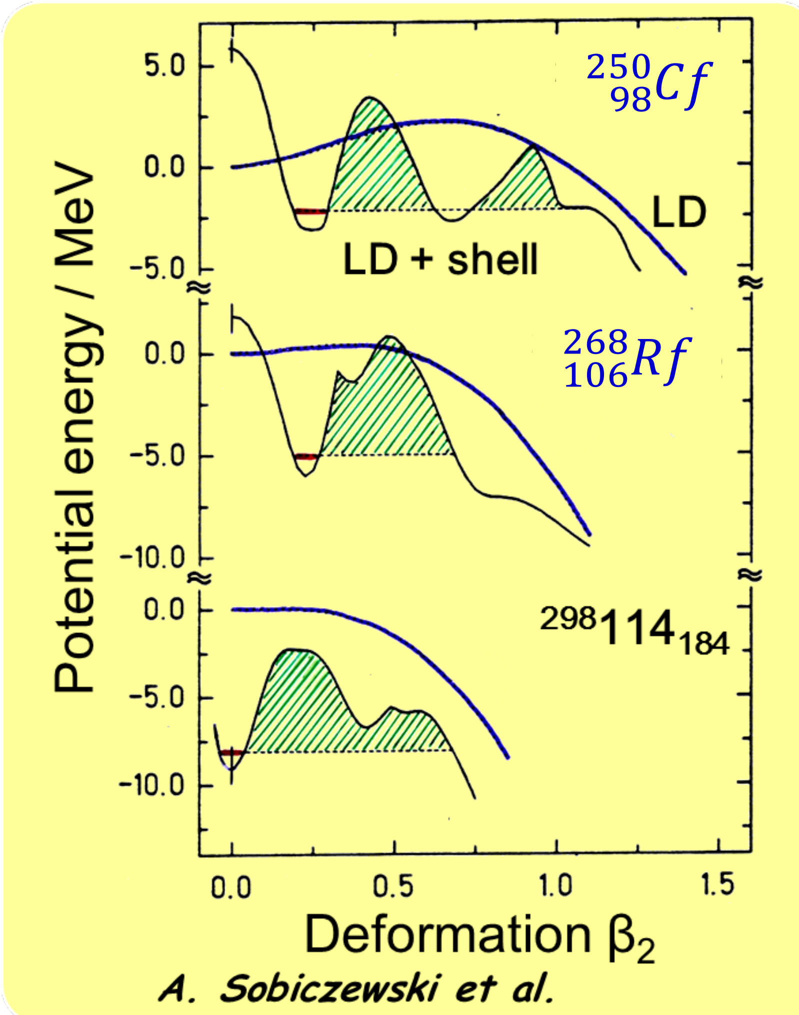
level density: $\rho(E^*) = const \cdot \exp(E^*/T)$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{2 \cdot T \cdot A_{CN}^{2/3}}{K_0} \cdot \exp\left[\left(B_f - B_n\right)/T\right]$$

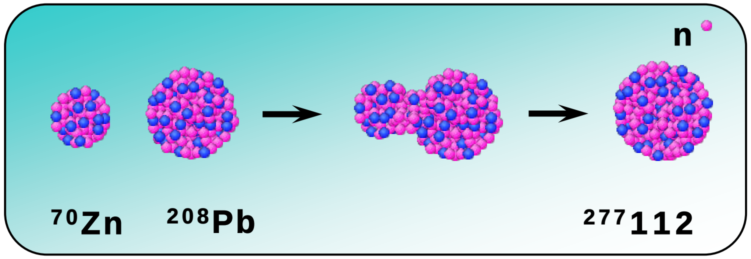
$K_0 = \hbar^2 / 2 \cdot m \cdot r_0^2 \approx 11.4 MeV$ $\qquad T = \sqrt{8 \cdot E^* / A_{CN}}$

Fusion/Fission competition for SHE liquid drop + shell corrections





Synthesis of heavy elements



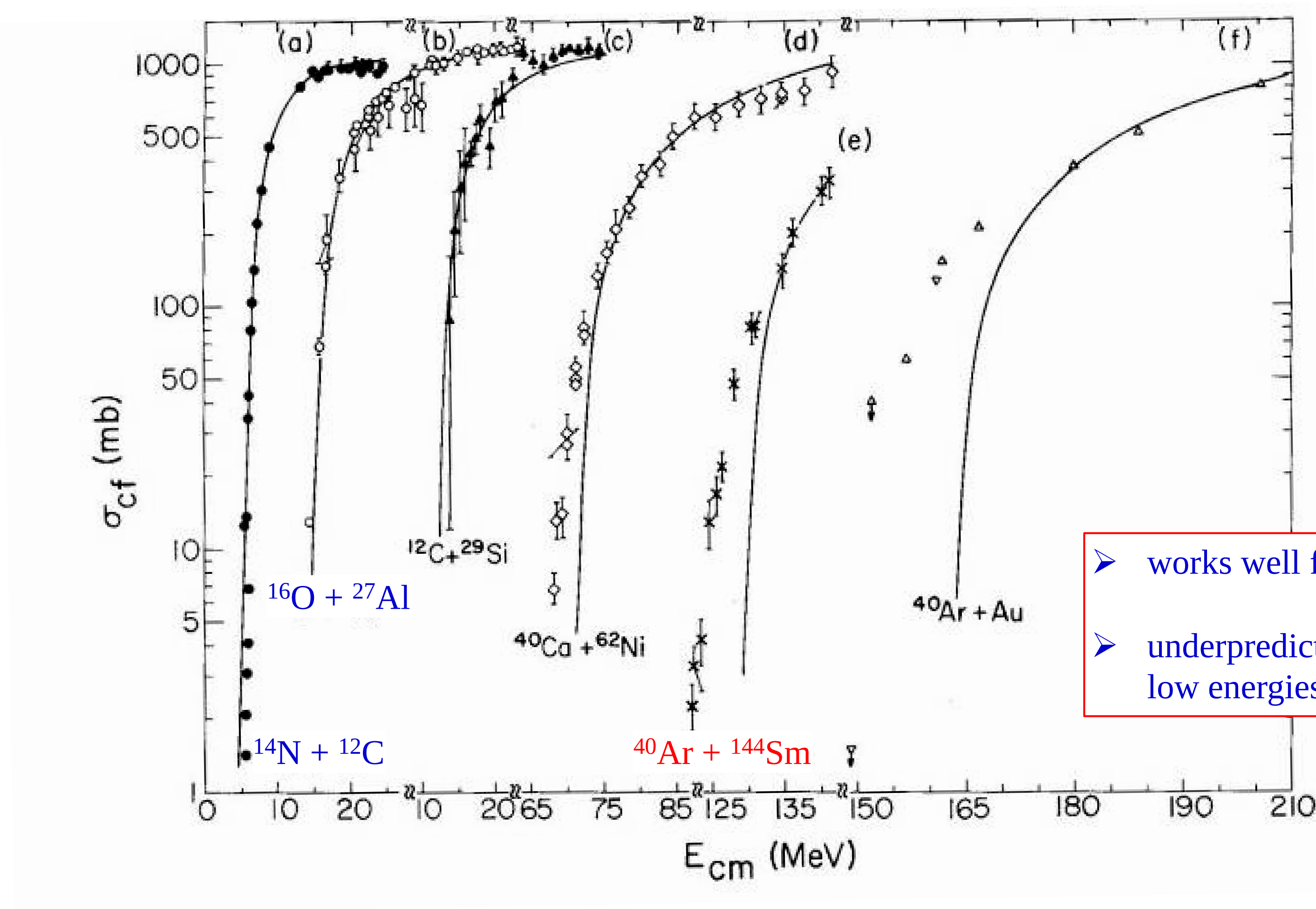
Fusion

$\frac{1}{10^{12}}$

Compound Fission

Comparison between potential model and experimental data

Fusion cross sections calculated with a static, energy independent potential

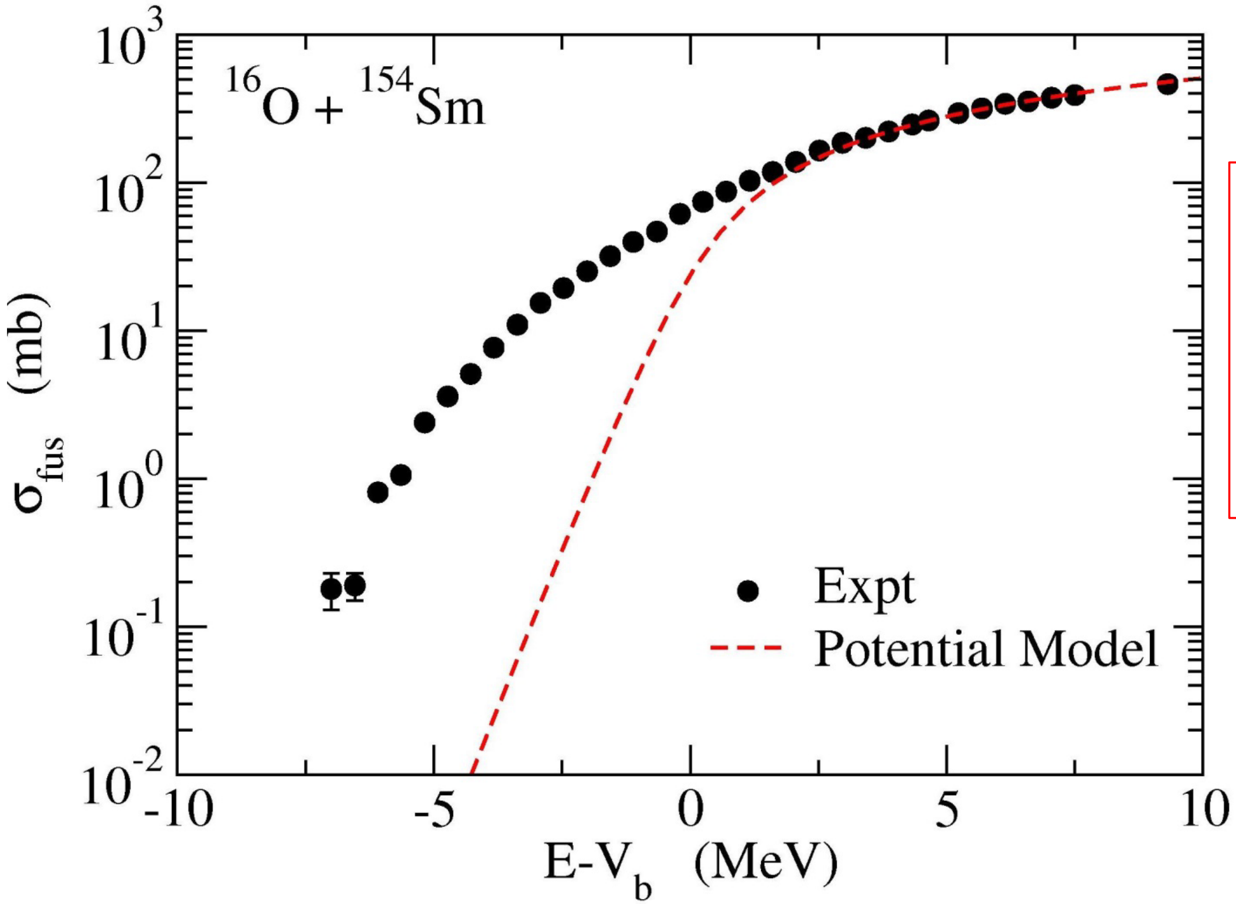


- works well for relatively light systems
- underpredicts σ_{fus} for heavy systems at low energies

L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Comparison between potential model and exp. data

Fusion cross sections calculated with a static, energy independent potential



Potential model:

reproduces the data reasonably well for $E > V_B$

underpredicts σ_{fus} for $E < V_B$

L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Comparison between potential model and exp. data

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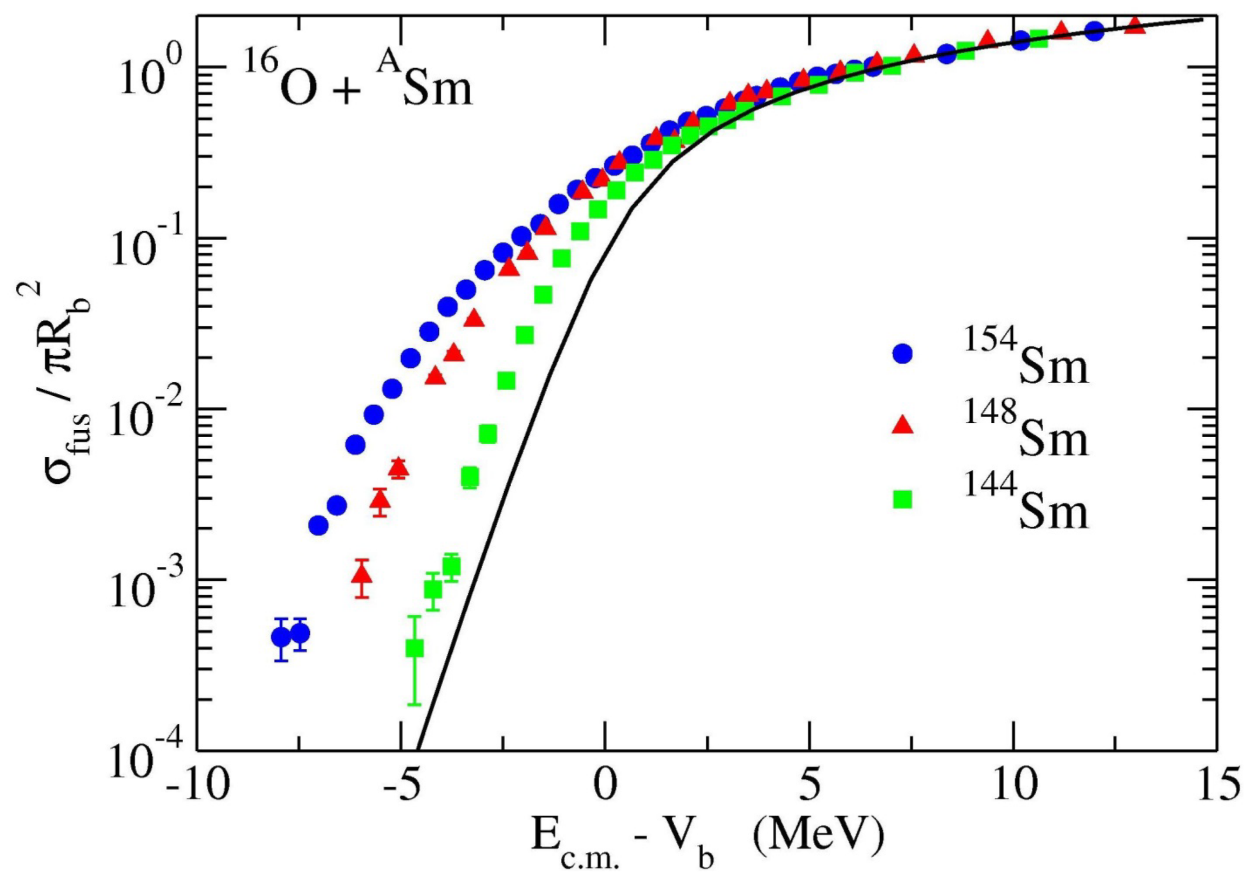
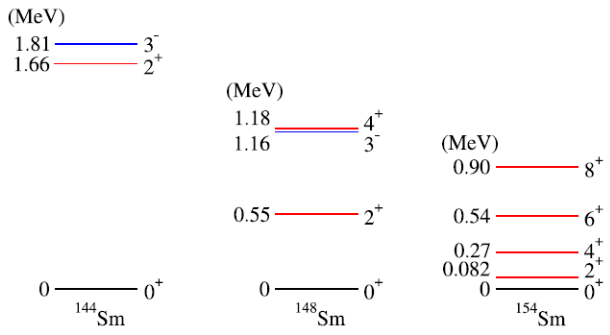


Effect of collective excitation

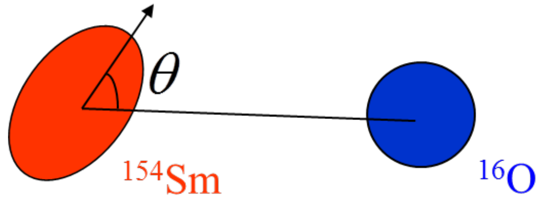
$$\beta_2 \cong \frac{4\pi}{3 \cdot Z_t \cdot R_t^2} \cdot \sqrt{\frac{B(E2\uparrow)}{e^2}}$$

$$\beta_4 \cong \frac{4\pi}{3 \cdot Z_t \cdot R_t^4} \cdot \sqrt{\frac{B(E4\uparrow)}{e^2}}$$

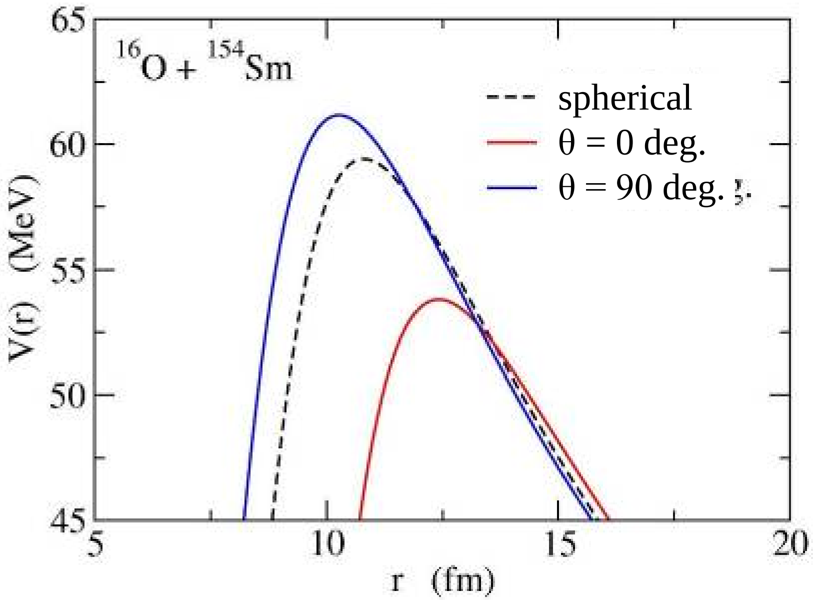


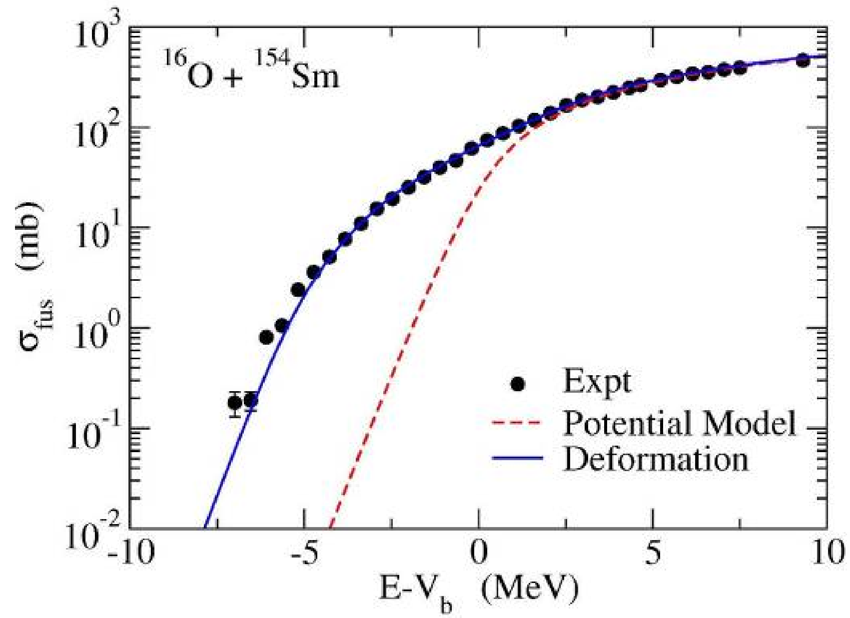
L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Fusion for a deformed nucleus



$$\sigma_{fus}(E) = \int_0^1 \sigma_{fus}(E,\theta)d(cos\theta)$$





- The barrier is lower for θ = 0
- The barrier is higher for θ = π/2

Deformation enhances σ_{fus} by a factor of 10 - 100

Fusion below and above the barrier inconsistent

■ $^{16}O + ^{208}Pb$

● $^{16}O + ^{204}Pb$

a = 0.66 fm

a = 1.18 fm

a = 1.65 fm

σ(mb)

a = 0.66 fm

a = 1.18 fm

a = 1.65 fm

$E_{c.m.} - V_B$ (MeV)

$E_{c.m.} - V_B$ (MeV)

M. Dasgupta et al., Phys Rev Lett 99 (2007) 192701