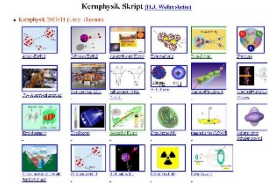


Outline: K-isomers in ^{178}Hf

Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

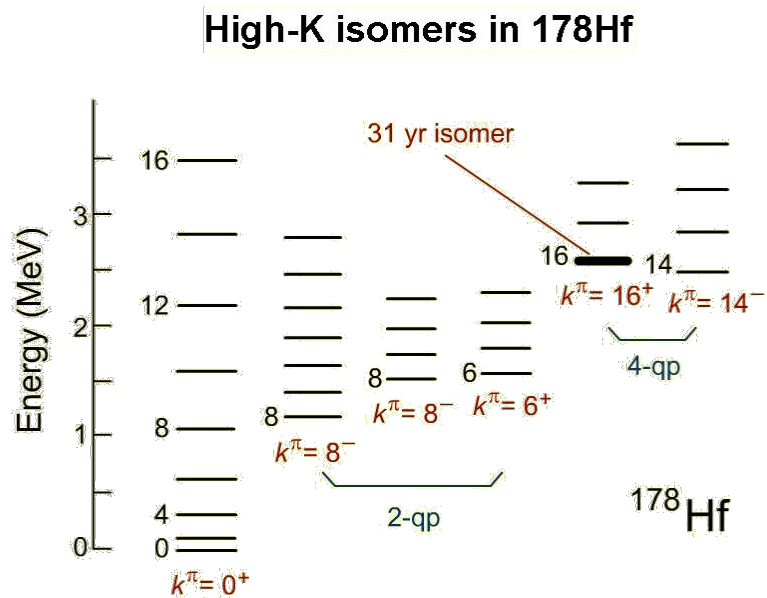
web-page: <https://web-docs.gsi.de/~wolle/> and click on



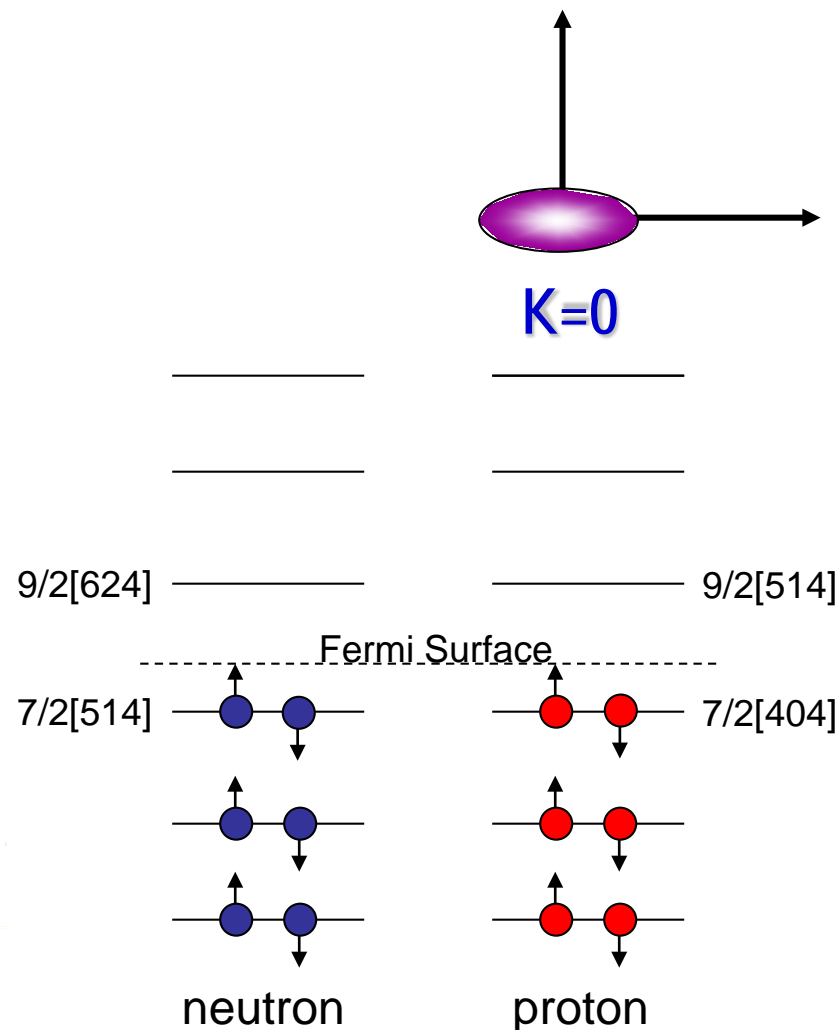
1. magnetic moments
2. K-selection rule
3. Coulomb excitation of the 8^- isomer in ^{178}Hf
4. investigation of the K=16 isomer in ^{178}Hf
5. deuteron and ^{208}Pb inelastic scattering, laser spectroscopy

Investigation of ^{178}Hf – K-isomers

- A well-known example:



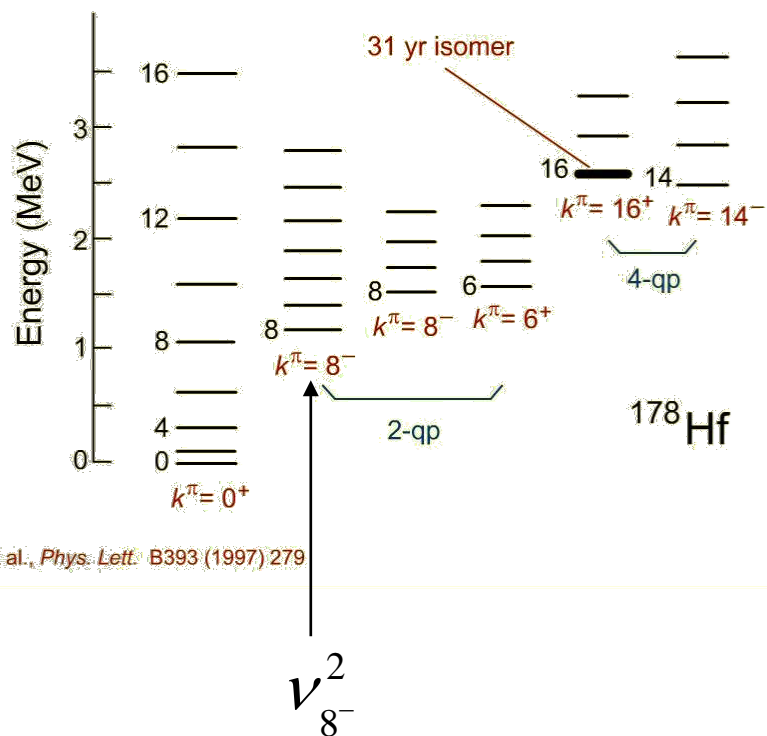
Mullins et al., *Phys. Lett. B* 393 (1997) 279



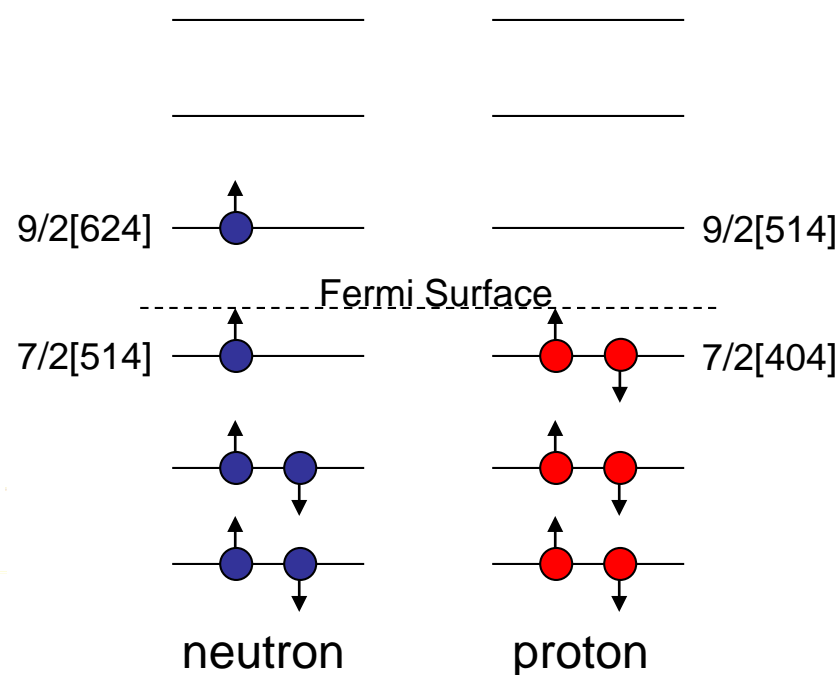
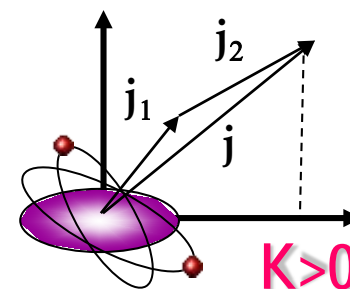
Investigation of ^{178}Hf – K-isomers

- A well-known example:

High-K isomers in ^{178}Hf

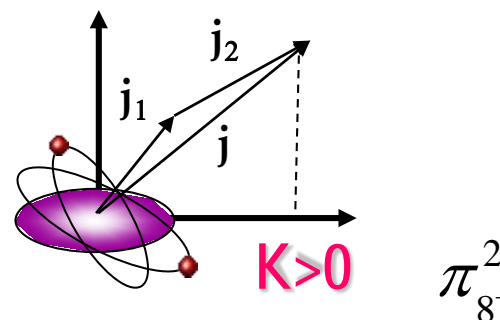


$\nu_{8^-} 2$

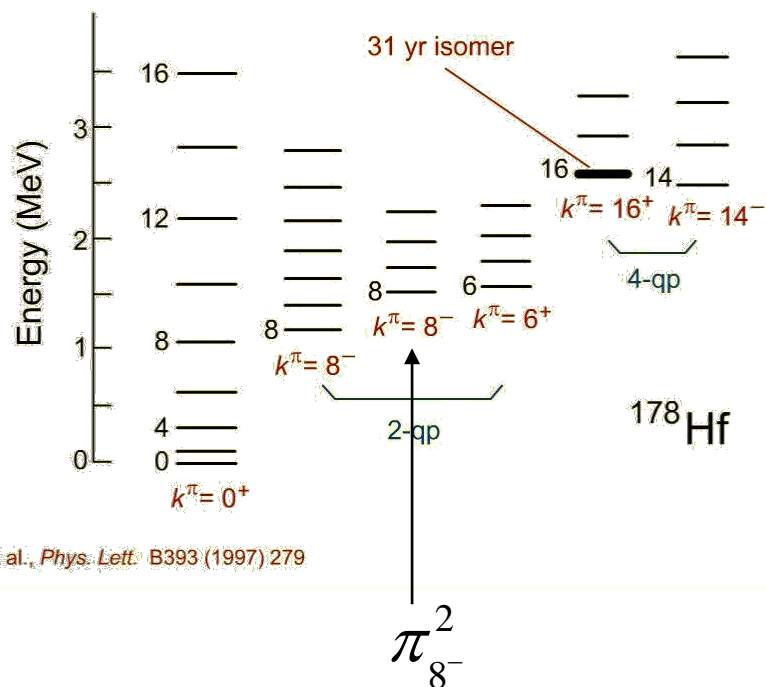


Investigation of ^{178}Hf – K-isomers

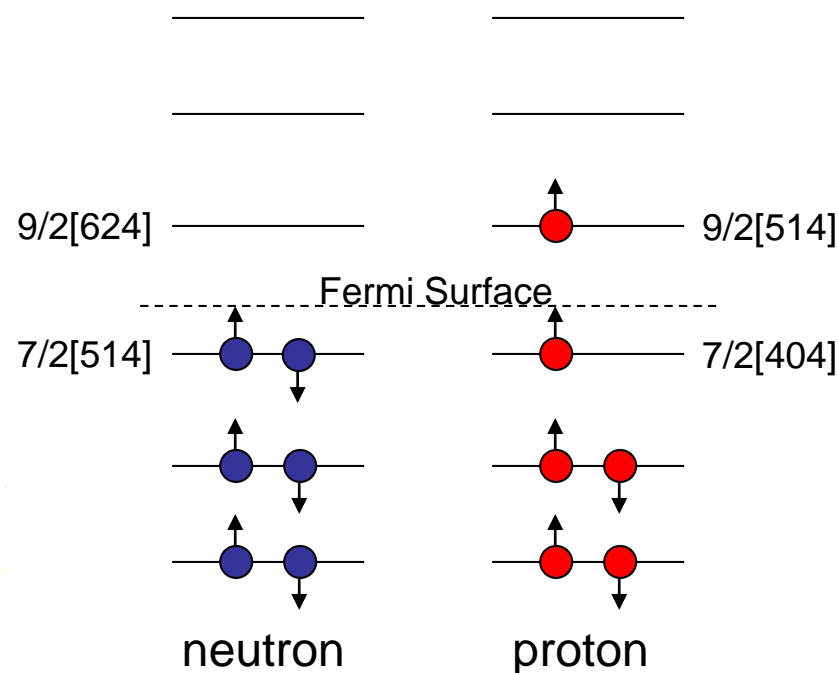
- A well-known example:



High-K isomers in ^{178}Hf



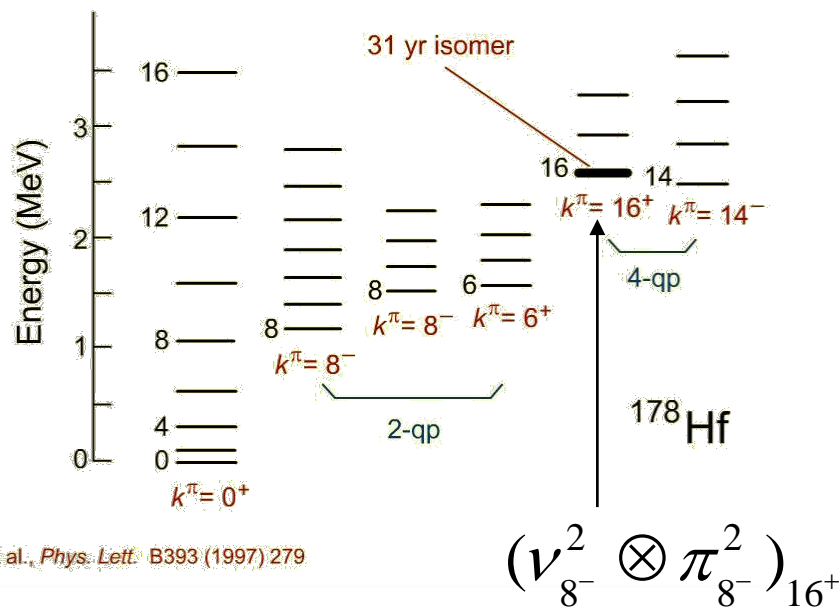
Mullins et al., *Phys. Lett. B* 393 (1997) 279



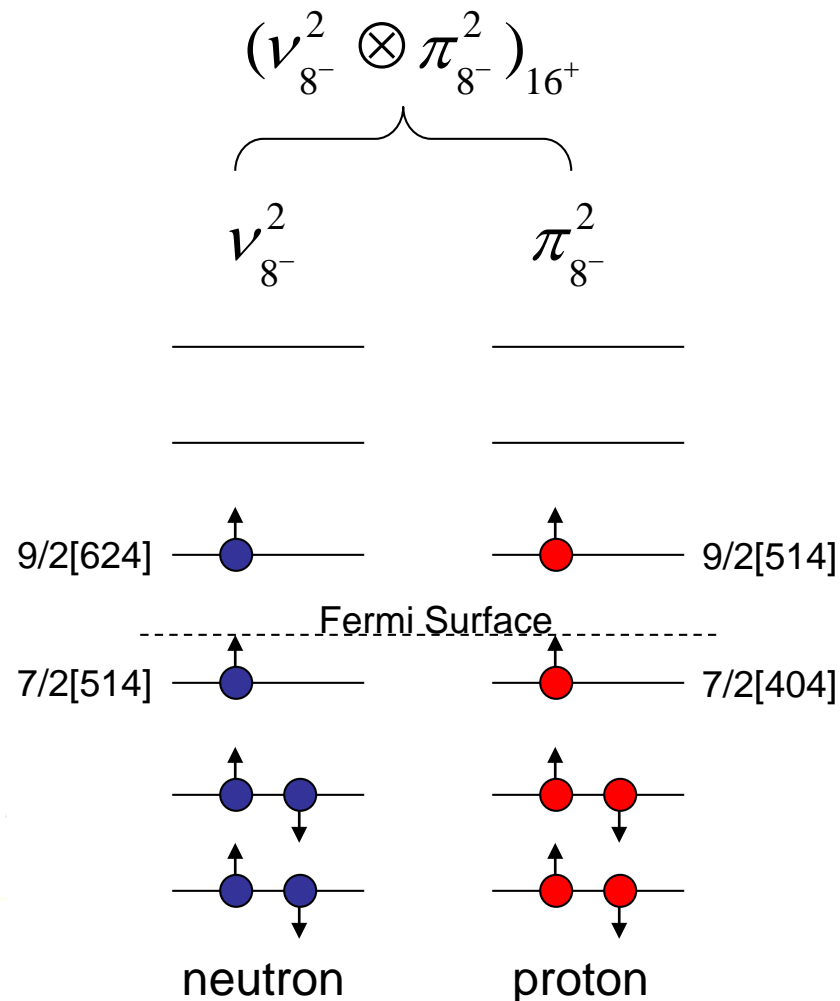
Investigation of ^{178}Hf – K-isomers

- A well-known example:

High-K isomers in ^{178}Hf



Mullins et al., *Phys. Lett. B* 393 (1997) 279



Magnetic moments in ^{178}Hf

$$g(j) = \begin{cases} \frac{2 \cdot \ell \cdot g_\ell + g_s}{2 \cdot \ell + 1} & \text{for } j = \ell + 1/2 \\ \frac{2 \cdot (\ell + 1) \cdot g_\ell - g_s}{2 \cdot \ell + 1} & \text{for } j = \ell - 1/2 \end{cases}$$

proton $g_\ell = 1$ $g_s = 5.59$
neutron $g_\ell = 0$ $g_s = -3.83$

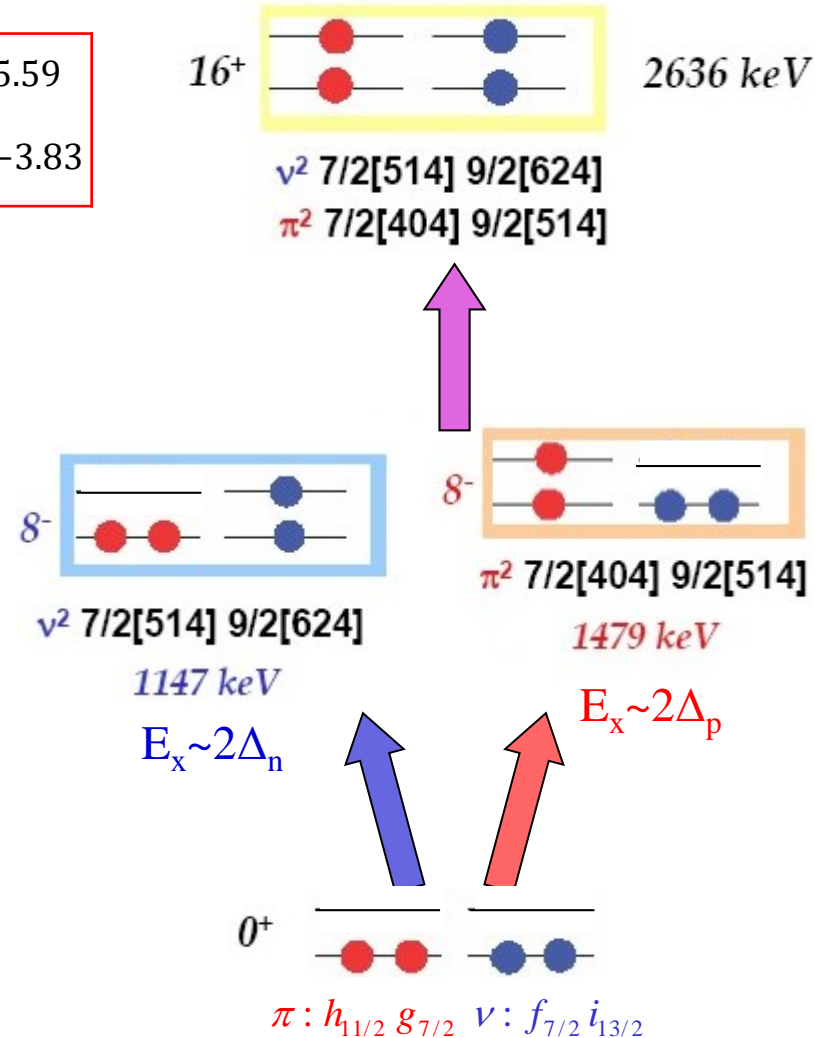
$$g(\mathbf{h}_{11/2}) = 1.42 \quad g(\mathbf{g}_{7/2}) = 0.49 \quad g(\mathbf{f}_{7/2}) = -0.55 \quad g(\mathbf{i}_{13/2}) = -0.29$$

$$g(j_1 \times j_2; J) = \frac{1}{2} \cdot (g_1 + g_2) + \frac{j_1 \cdot (j_1 + 1) - j_2 \cdot (j_2 + 1)}{2 \cdot J \cdot (J + 1)} \cdot (g_1 - g_2)$$

$$g(\mathbf{h}_{11/2} \times \mathbf{g}_{7/2}; 8^-) = 1.08 \quad g(\mathbf{f}_{7/2} \times \mathbf{i}_{13/2}) = -0.36$$

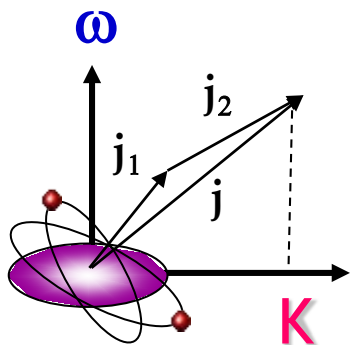
$g(8^- \times 8^-; 16^+) = 0.36 \quad \rightarrow \quad \mu = g \cdot I = 5.76 \text{ nm}$

$7.26 \pm 0.16 \text{ nm}$

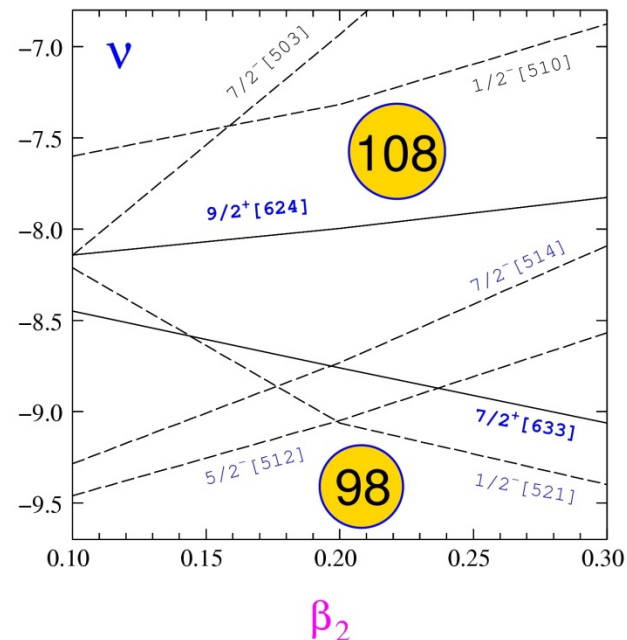
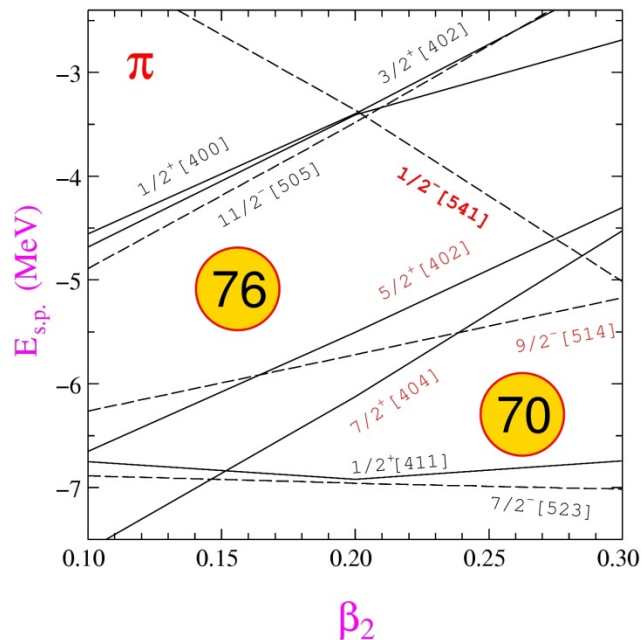


K-isomers: Where to find them?

- Deformed nuclei with axially-symmetric shape



Mass 180 region : Yb (Z=70) - Ir (Z=77)

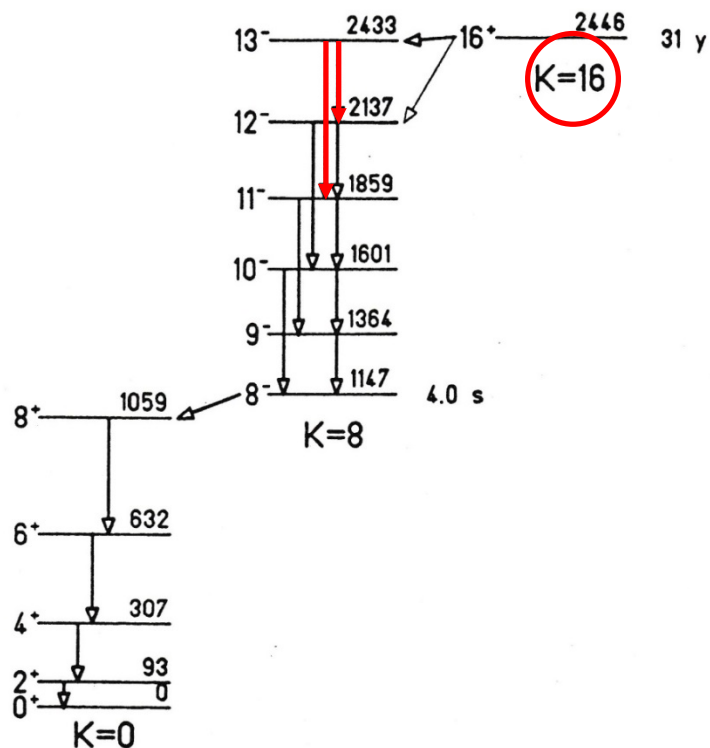


- High-K orbitals near the Fermi surface

π : 7/2[404], 9/2[514], 5/2[402]

ν : 7/2[514], 9/2[624], 5/2[512], 7/2[633]

Decay study of the $K = 16$ isomeric state



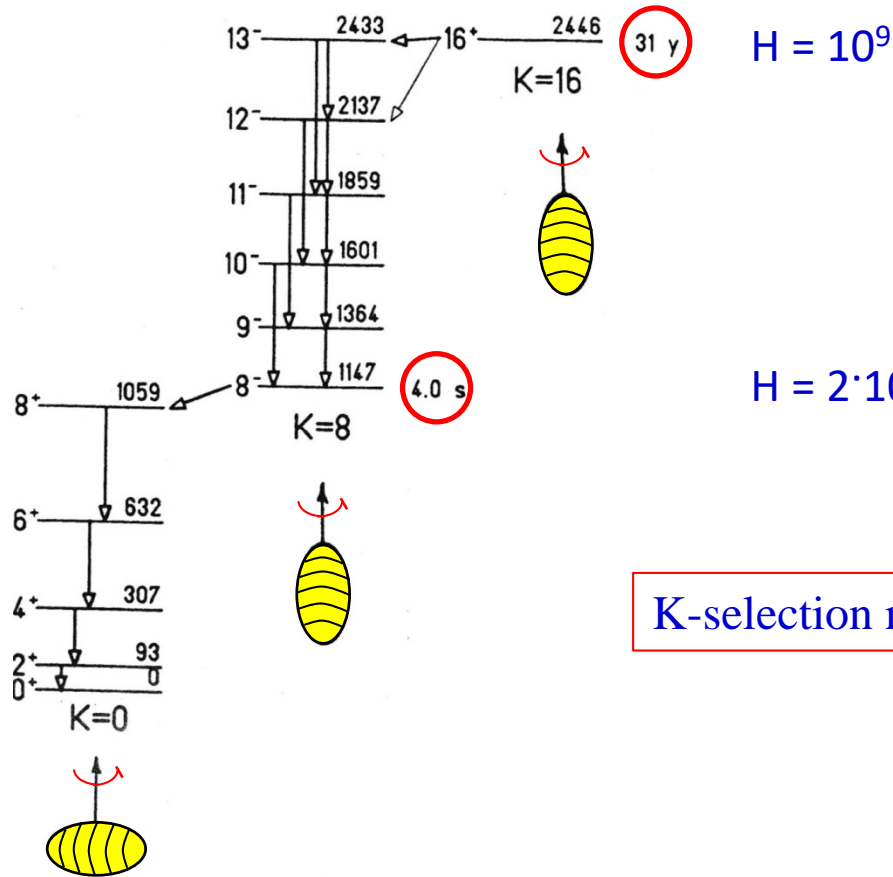
	$\frac{B(E2, I \rightarrow I - 2)}{B(E2, I \rightarrow I - 1)}$	rigid rotor $K=8$
10 ⁻	0.104±0.012	0.0769
11 ⁻	0.171±0.009	0.1607
12 ⁻	0.230±0.017	0.2517
13 ⁻	0.313±0.016	0.3500

E2/M1 mixing ratios are related to $\frac{g_K - g_R}{Q_{20}}$

mixed 2-quasiparticle configuration: $g_K = 0.36(2) > 0$

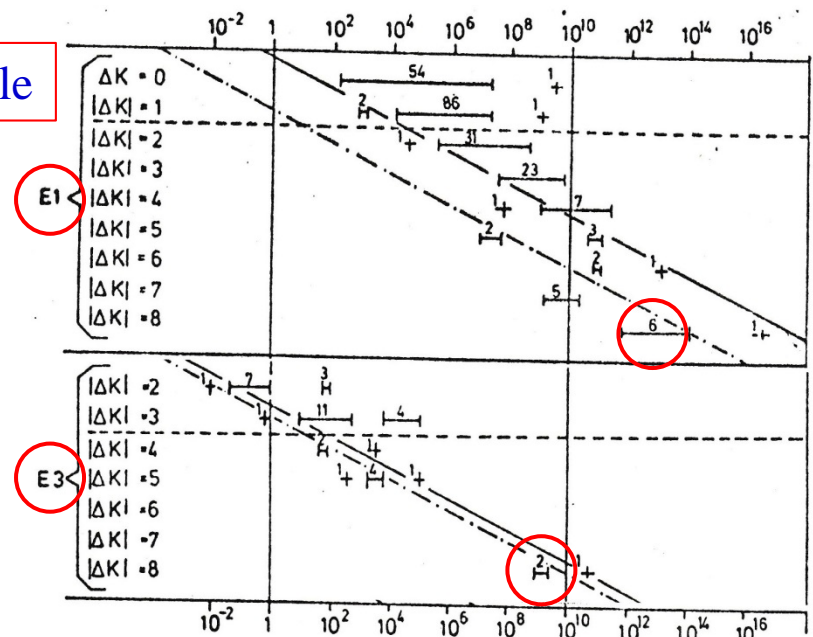
$\{\pi[514]9/2 + \pi[404]7/2\}8^-$ and $\{v[514]7/2 + v[624]9/2\}8^-$

K - isomerism



$$H = \frac{t_{1/2}(\text{experiment})}{t_{1/2}(\text{Weisskopf estimate})}$$

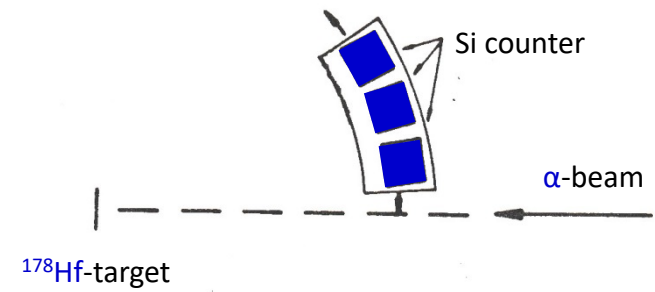
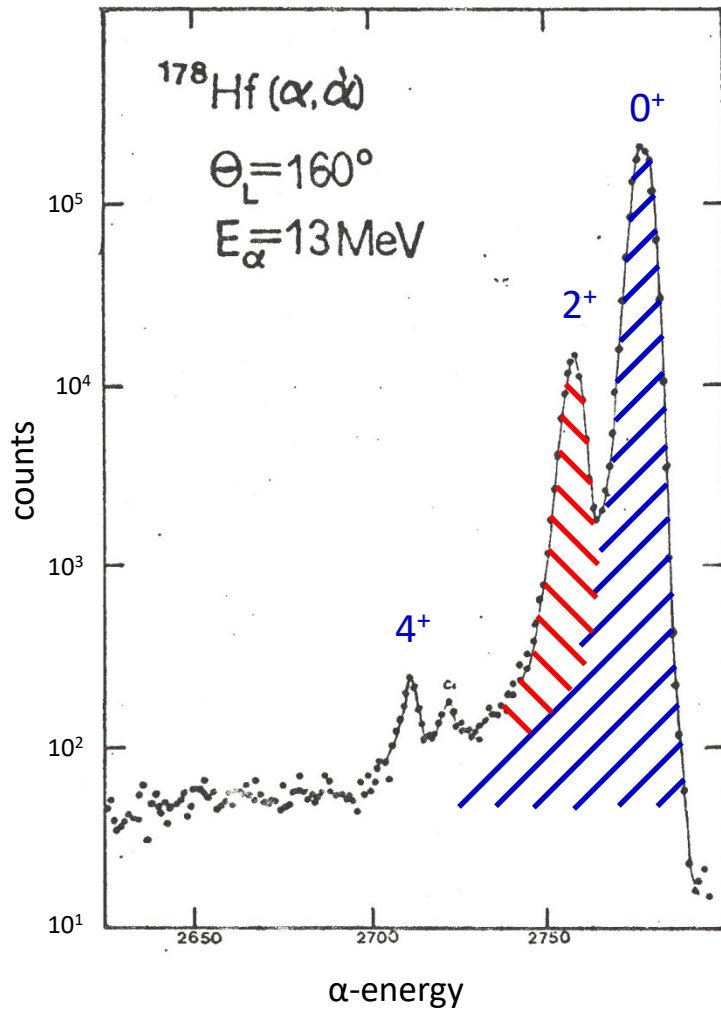
K-selection rule



Helmer & Reich; Nucl. Phys. A114 (1968), 649

K.E.G. Löbner; Phys. Lett 26B (1968), 369

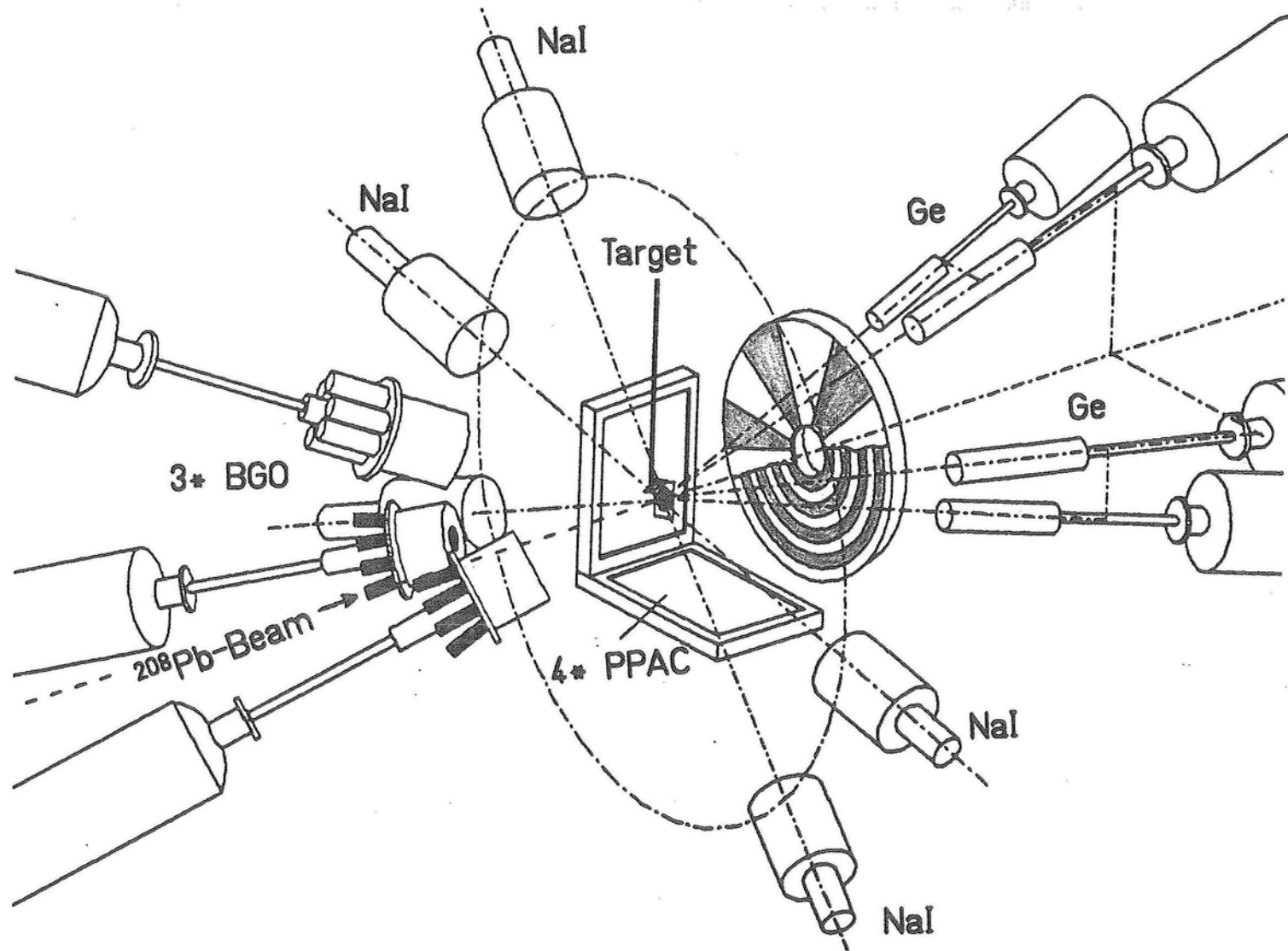
^{178}Hf : particle spectroscopy



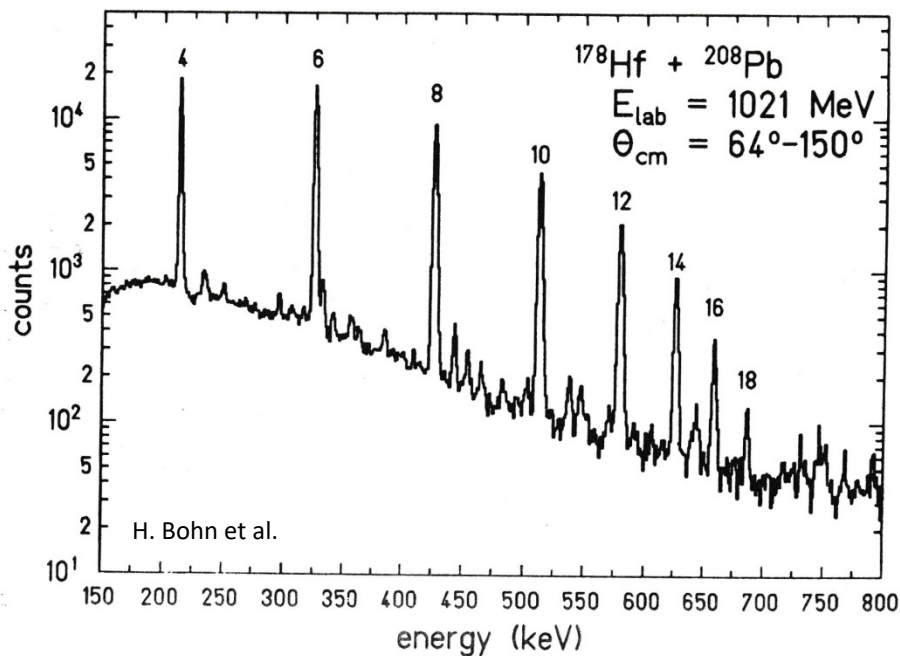
$$\begin{aligned}
 d\sigma_{C.E.}(E, \theta_{cm}) &= P_I(E, \theta_{cm}) \cdot d\sigma_{Ruth}(E, \theta_{cm}) \\
 &= 4.82 \left(1 + \frac{A_p}{A_t}\right)^{-2} \frac{A_p}{Z_t^2} (E_{lab} - \Delta E') \cdot B(E2, 0^+ \rightarrow 2^+) \cdot df_{E2}(\xi, \theta_{cm})
 \end{aligned}$$

I	$P_I/P_{0^+} (160^\circ)$	
2	$6.8 \cdot 10^{-2}$	
4	$7.6 \cdot 10^{-4}$	
17	$4.0 \cdot 10^{-4}$	$\sigma_{m2}/\sigma_{gs} = 0.05$

Coulomb excitation of ^{178}Hf with ^{208}Pb ions



Coulomb excitation of ^{178}Hf with ^{208}Pb ions



E2, E3 excitation of collective states:

ground state band $K = 0$

γ -vibrational band $K = 2$

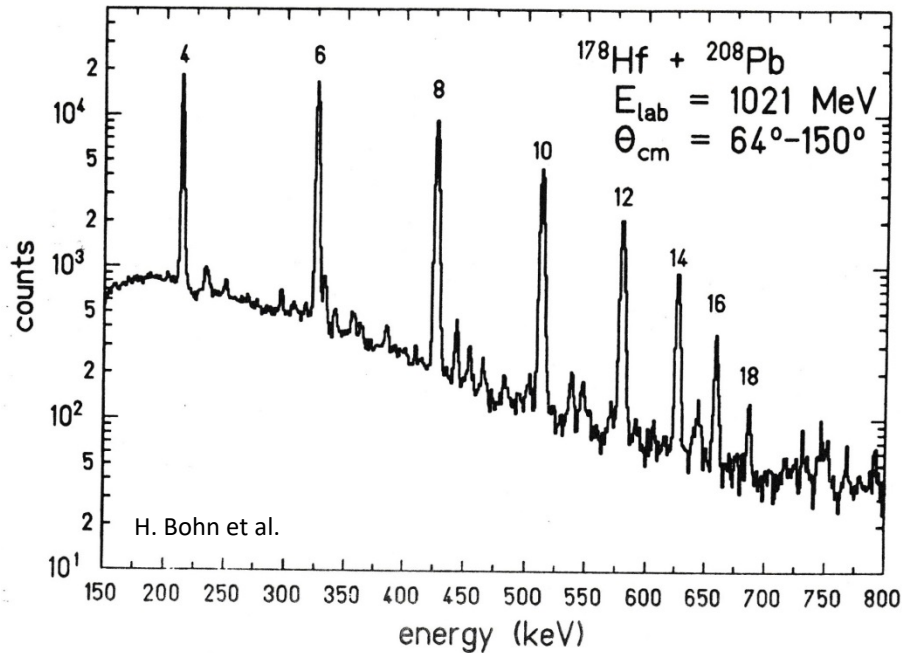
determination of K-quantum number:

$$B(E2; I \rightarrow I - 2) = \frac{[(I - 1)^2 - K^2][I^2 - K^2]}{(4I^2 - 1)I(I - 1)} \frac{15}{32\pi} \cdot Q_{20}^2$$

$$Q_{20} = 6.99(4) b \quad Q_{40} = 0.54 \begin{pmatrix} 26 \\ 47 \end{pmatrix} b^2 \text{ or } -1.58(48) b^2$$

$$\beta_2 = 0.29(1) \quad \beta_4 = -0.20(4)$$

Coulomb excitation of ^{178}Hf with ^{208}Pb ions



E2, E3 excitation of collective states:

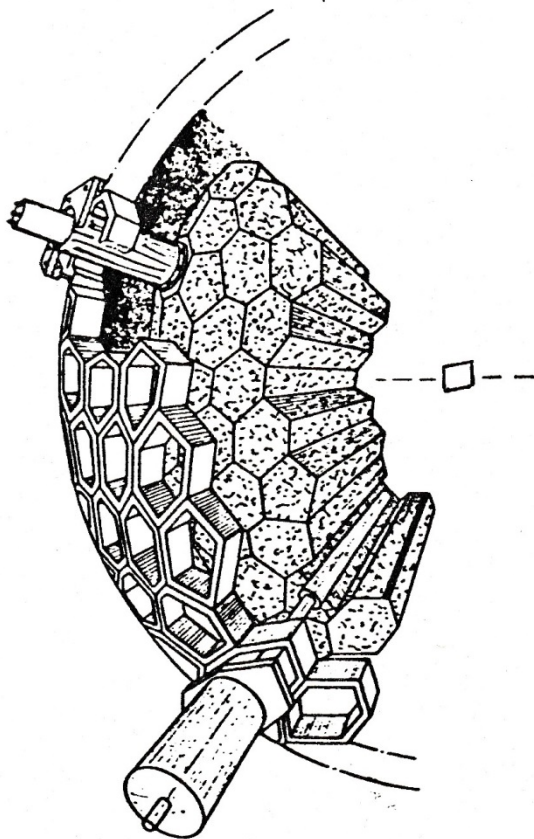
ground state band $K = 0$

γ -vibrational band $K = 2$

➤ excitation of the $K = 8$ isomeric state !
observation of delayed γ -rays in the $K = 0$ ground state band

➤ $^{178}\text{Hf} + ^{86}\text{Kr}$, $^{178}\text{Hf} + ^{136}\text{Xe}$

Coulomb excitation of the $K = 8$ isomer in ^{178}Hf



162 NaI detectors
6 Ge detector

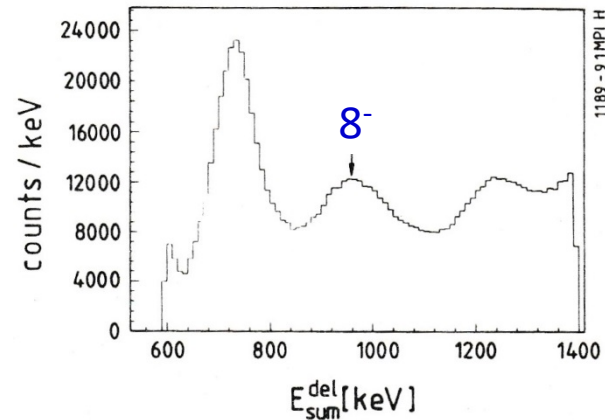
parameters and resolutions FWHM

E_i	5.5% at 1332 keV
t_i	2.8 ns
$W(\theta)$	14°
E_{total}	18-22% for $M_\gamma=20$
M_γ	25-30% for $M_\gamma=20$

The two basic observables which can be measured for the resulting γ -ray shower are the total energy emitted as γ -radiation and the number of γ -rays.

➤ $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV

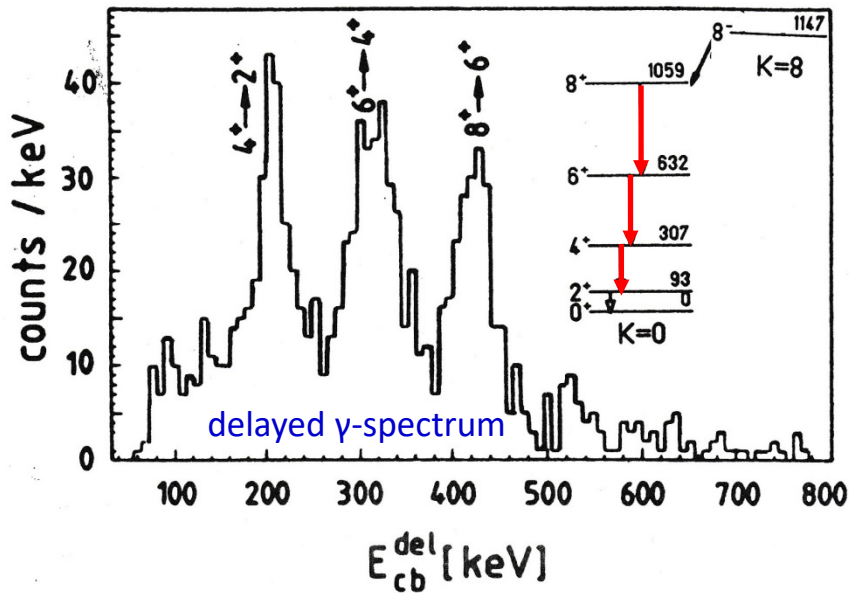
particle detection at $\sim 180^\circ$, Pb catcher (0.5 mm thickness) was positioned 1cm downstream to stop the recoiling ^{178}Hf ions.



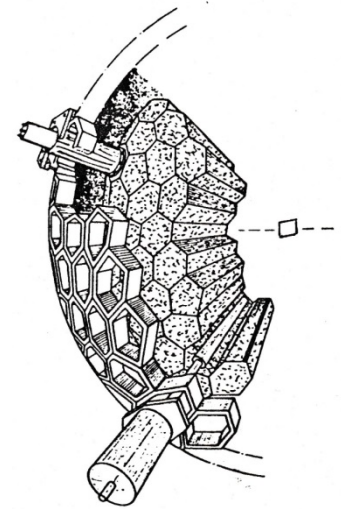
Delayed sum-energy spectrum taken at 590 MeV. (delayed time window 20-65ns with respect to beam pulse) A peak associated to the $K^\pi=8^-$ shows up. The other peaks correspond to isomers of fusion products from target contaminants and β -decay.

Coulomb excitation of the $K = 8$ isomer in ^{178}Hf

➤ $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV

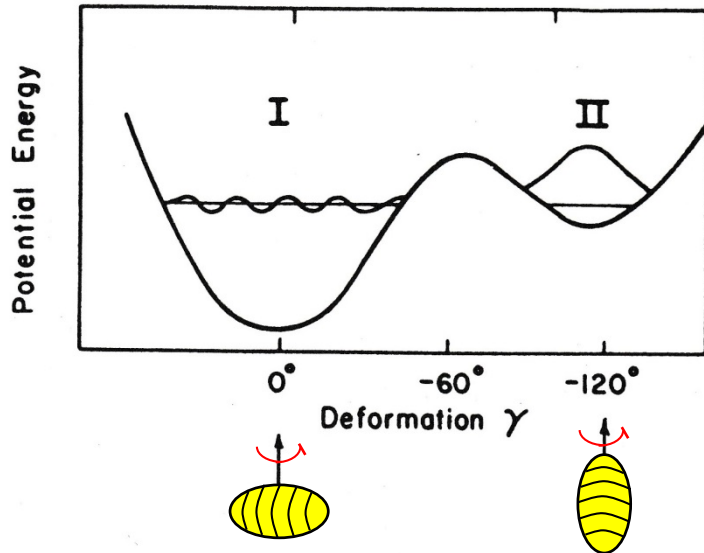


4 s $\rightarrow M_{30} (E3) \leq 0.01$



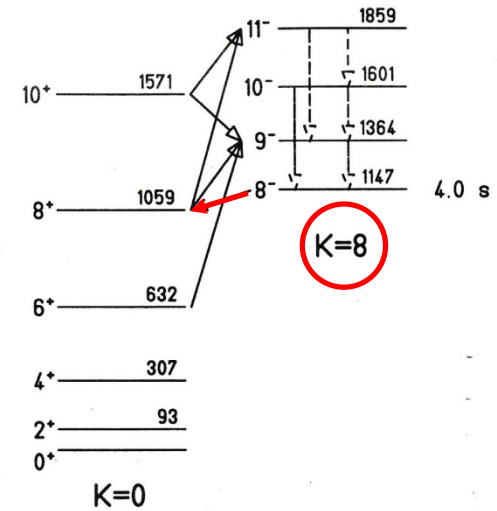
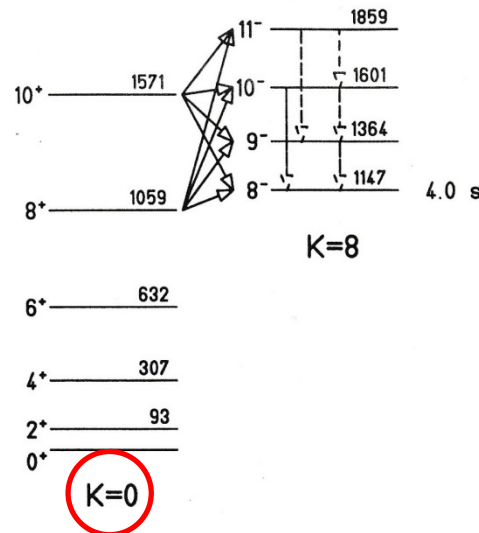
Delayed γ -ray spectrum of the Crystal Ball with $850\text{keV} \leq E_{sum}^{del} \leq 1100\text{keV}$ and $3 \leq N_{det} \leq 6$. In addition at least one of the delayed γ -rays must have been detected in one of the Ge-detectors.

Decay of the isomer by barrier penetration



+ small K=8 admixture

+ small K=0 admixture



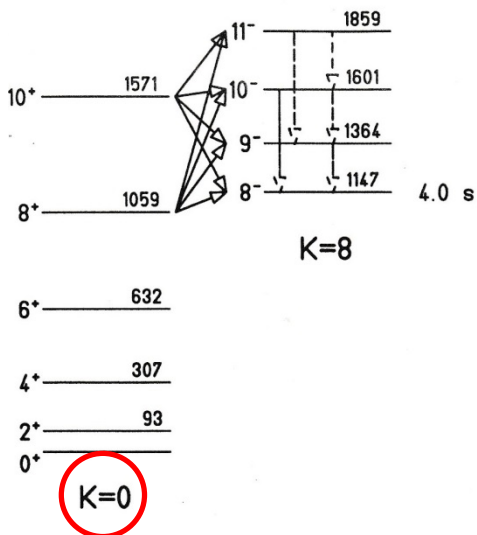
rigid rotor model:

$$\langle I_f || M(E2) || I_i \rangle = \sqrt{2I_i + 1} \cdot (I_i 3K0 | I_f K) \cdot M_{30}$$

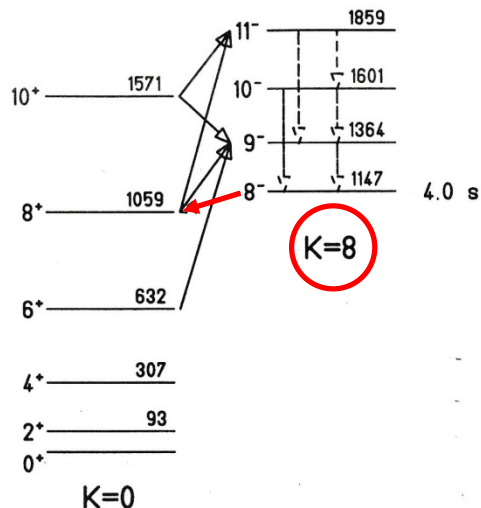
8⁻ lifetime is independent from excitation

2-band K-mixing model

+ small K=8 admixture



+ small K=0 admixture



8- lifetime is independent from excitation

rigid rotor model:

$$\langle I_f \| M(E3) \| I_i \rangle = \sqrt{2I_i + 1} \cdot (I_i 3K0 | I_f K) \cdot M_{30}$$

Clebsch Gordan coefficient:

$$\langle I3K0 | (I-3)K \rangle = -\sqrt{\frac{5(I+K-2)(I+K-1)(I+K)(I-K-2)(I-K-1)(I-K)}{2(I-2)(I-1)I(2I-3)(2I-1)(2I+1)}}$$

$$\langle I3K0 | (I-2)K \rangle = \sqrt{\frac{15(I+K-1)(I+K)(I-K-1)(I-K)}{(I-2)(I-1)I(2I-1)(2I+1)(2I+2)}} * K$$

$$\langle I3K0 | (I-1)K \rangle = -\sqrt{\frac{3(I+K)(I-K)}{(I-1)I(2I-3)(2I+1)(2I+2)(2I+3)}} * (5K^2 - I^2 + 1)$$

$$\langle I3K0 | IK \rangle = \frac{5K^2 - 3I^2 - 3I + 1}{\sqrt{(I-1)I(I+1)(I+2)(2I-1)(2I+3)}} * K$$

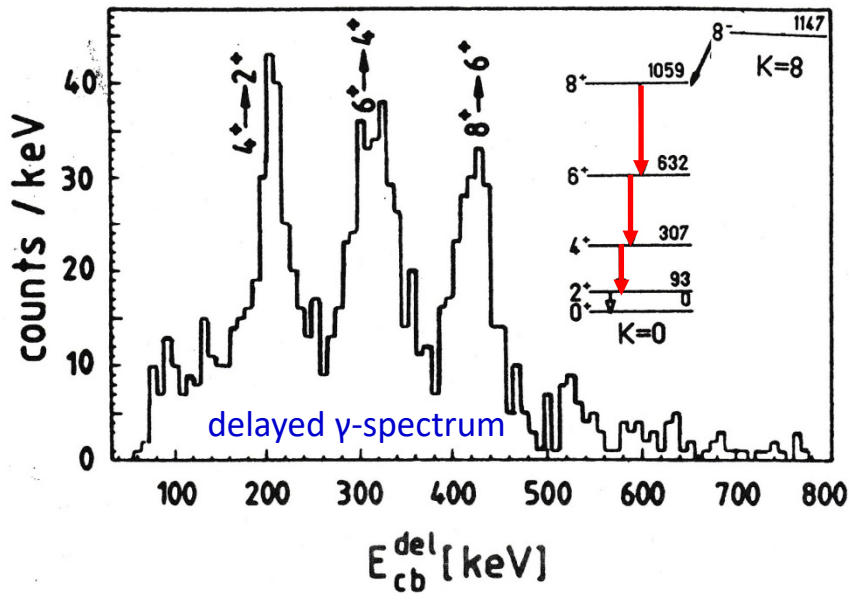
$$\langle I3K0 | (I+1)K \rangle = \sqrt{\frac{3(I+K+1)(I-K+1)}{I(I+1)(2I-1)(2I+1)(2I+4)(2I+5)}} * (5K^2 - I^2 - 2I)$$

$$\langle I3K0 | (I+2)K \rangle = \sqrt{\frac{15(I+K+1)(I+K+2)(I-K+1)(I-K+2)}{I(I+1)(I+2)(2I+1)(2I+3)(2I+6)}} * K$$

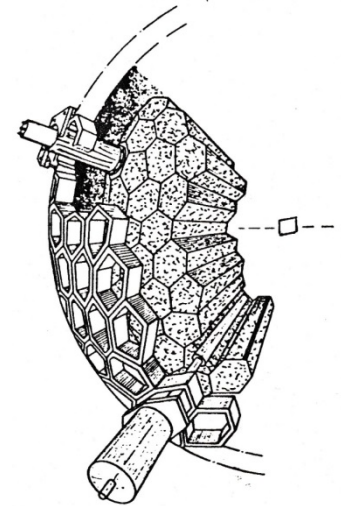
$$\langle I3K0 | (I+3)K \rangle = \sqrt{\frac{5(I+K+1)(I+K+2)(I+K+3)(I-K+1)(I-K+2)(I-K+3)}{2(I+1)(I+2)(I+3)(2I+1)(2I+3)(2I+5)}}$$

Coulomb excitation of the $K = 8$ isomer in ^{178}Hf

➤ $^{178}\text{Hf} + ^{130}\text{Te}$ at 560, 590, 620 MeV

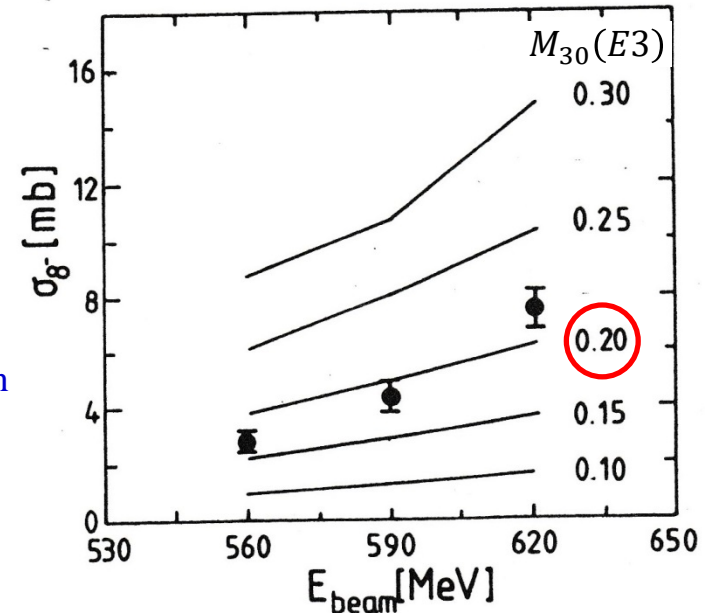


4 s → $M_{30}(E3) \leq 0.01$



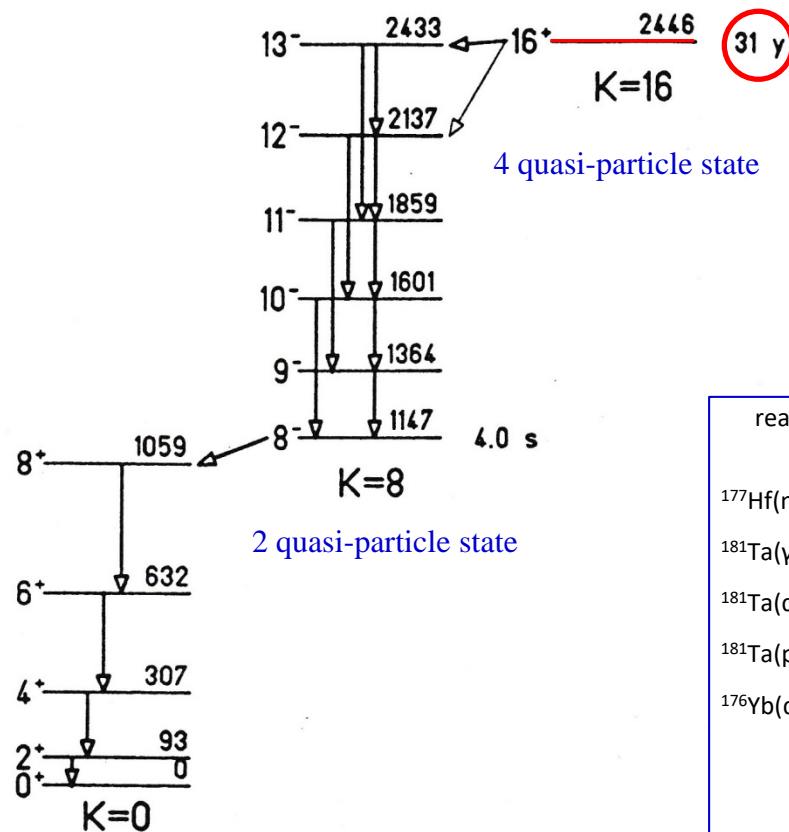
coupling between rotational motion and single particle excitation

$$\psi(8^-) \cong |K = 8\rangle + \alpha |K = 0\rangle$$



Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$

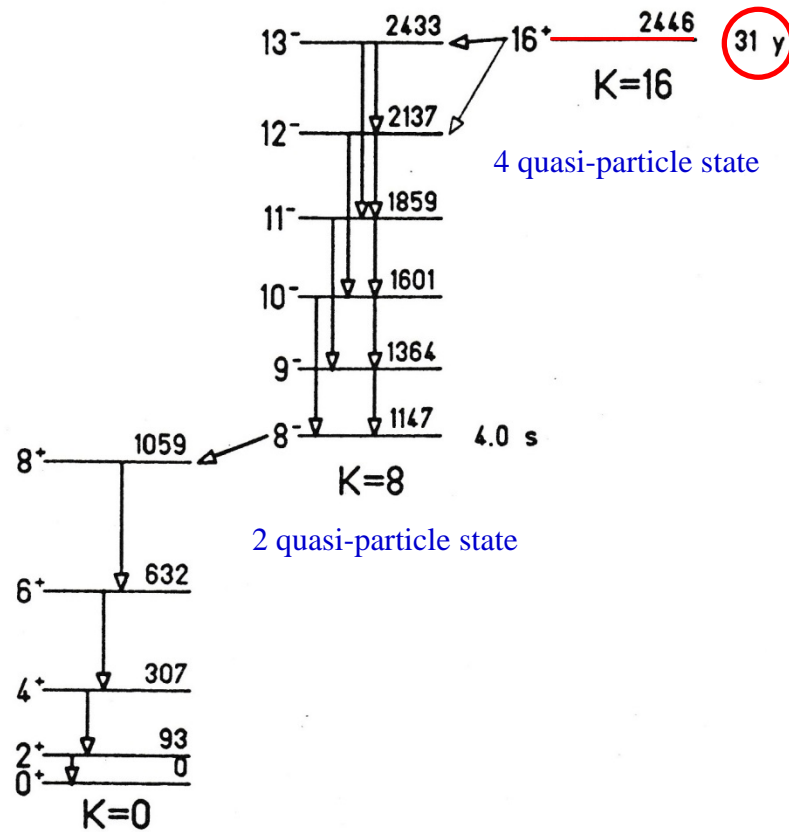


Production of $^{178\text{m}2}\text{Hf}$ in the past

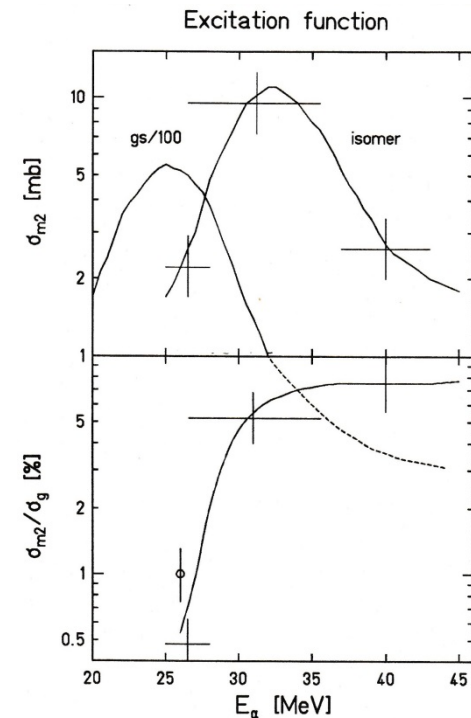
reaction	projectile energy (MeV)	σ_m/σ_g	reference
$^{177}\text{Hf}(n,\gamma)$	thermal	10^{-9}	R.G. Helmer et al.; Nucl. Phys. A211, 1 (1973)
$^{181}\text{Ta}(\gamma,p2n)$	≤ 85	low	F.W.N. de Boer et al.; Nucl. Phys. A263,1 (1976)
$^{181}\text{Ta}(\alpha,\alpha p2n)$	120	-	J. Van Klinken et al.; Nucl. Phys. A339, 189 (1980)
$^{181}\text{Ta}(p,\alpha)$	92.5	10^{-3}	W. Kutschera et al.; Radio. Beam Conf., 345 (1989)
$^{176}\text{Yb}(\alpha,2n)$	26	0.01	T. Khoo et al.; Phys. Lett. 67B, 271 (1977)
	27	0.005	A. Kugler et al.; priv. communication
	32	0.05	NUPECC News 1991
	40	0.08	

Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$

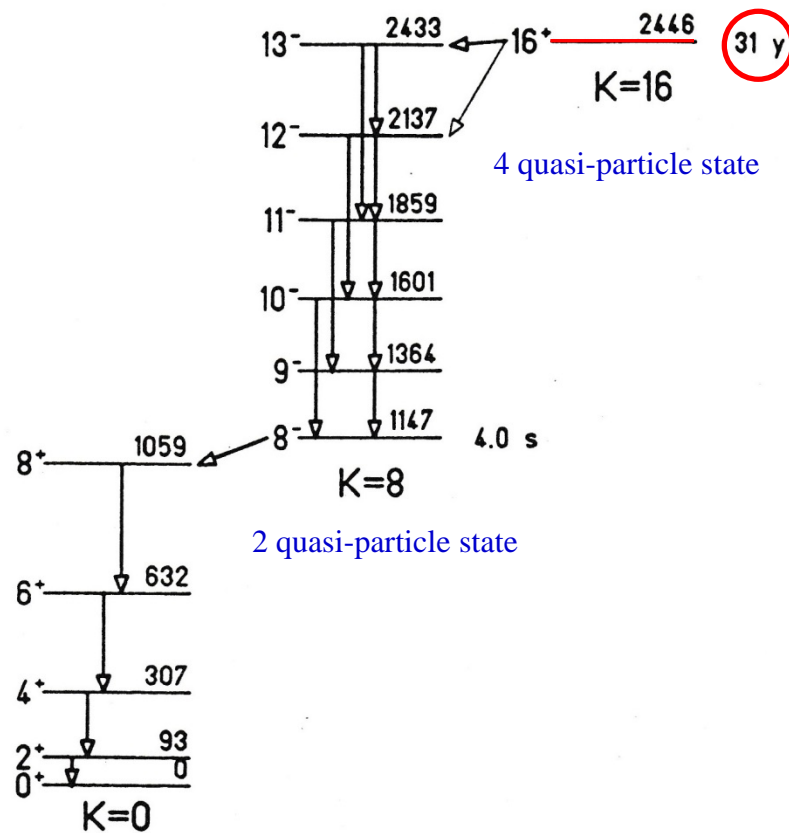


Production of $^{178\text{m}2}\text{Hf}$ using $^{176}\text{Yb}(\alpha, 2n)$



Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$

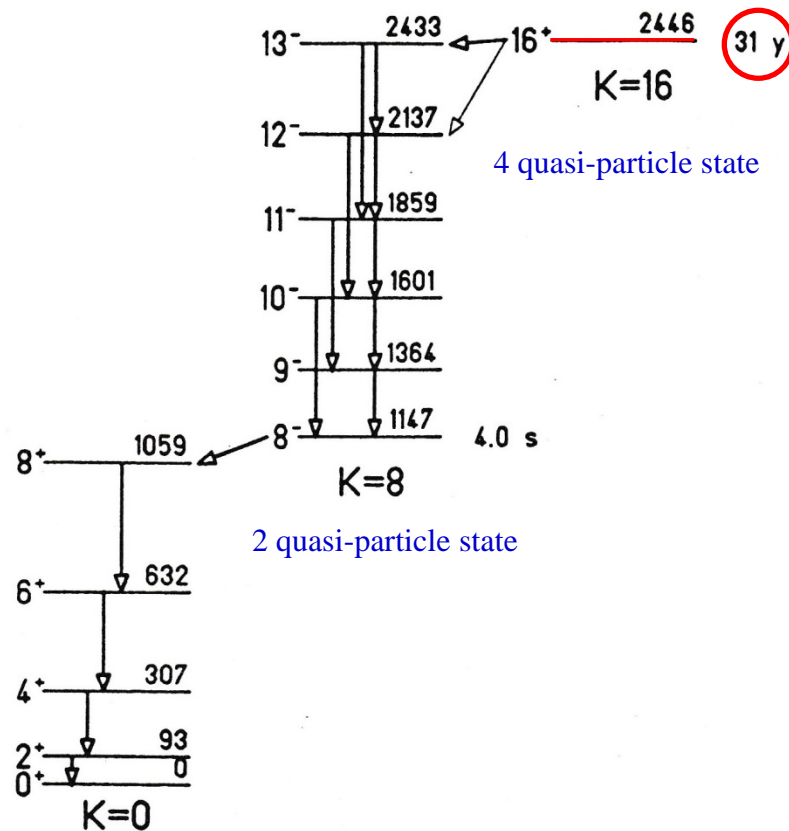


The isotopic distribution of radioactive Hf nuclei produced in the irradiation $^4\text{He} + ^{176}\text{Yb}$

Mass	170	171	172	173	175	178m	178g	179m
$T_{1/2}$	15.9h	12.2h	1.9y	24h	70d	31y	stable	24.8d
Experiment	5.10^{10}	$1.5 \cdot 10^{11}$	$4.5 \cdot 10^{12}$	$1.3 \cdot 10^{13}$	$2.2 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	--	$7.6 \cdot 10^{11}$
Calculation	8.10^{10}	$1.5 \cdot 10^{12}$	$4.4 \cdot 10^{12}$	$0.8 \cdot 10^{13}$	$2.2 \cdot 10^{13}$	--	$2.9 \cdot 10^{14}$	--

Investigation of the K=16 isomer in ^{178}Hf

decay scheme of $^{178\text{m}2}\text{Hf}$



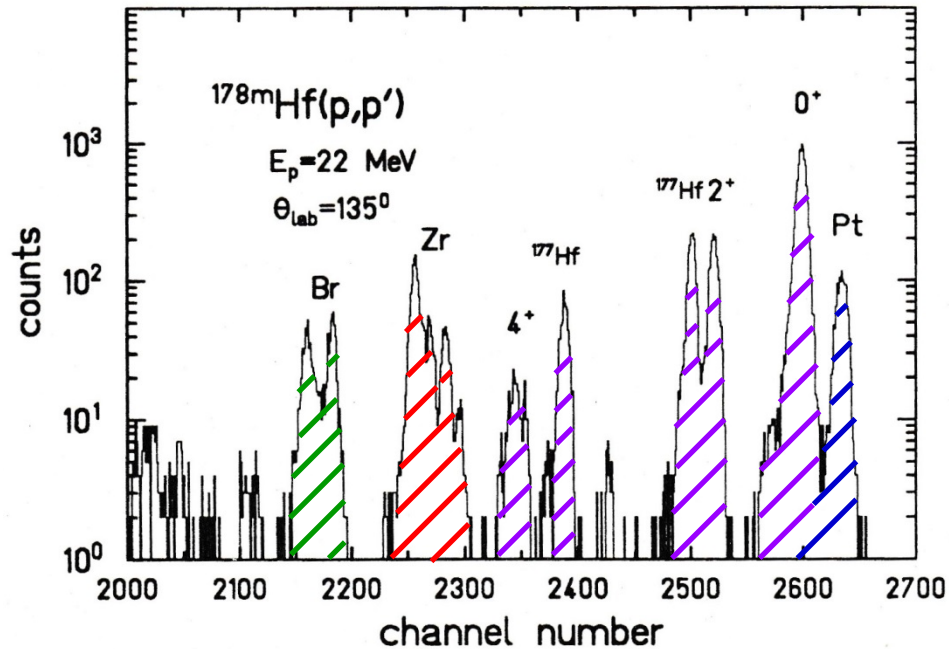
- magnet moment $\mu(16^+) = 7.3 \pm 0.2 \text{ nm}$
- 4-qp state: $\pi[514]9/2 + \pi[404]7/2 + \nu[514]7/2 + \nu[624]9/2$
- production: $^{176}\text{Yb}(\alpha, 2n)^{178}\text{Hf}$ Dubna
- chemical separation Orsay, Dubna
- mass separation Orsay
- target preparation Orsay

chemistry by M. Hussonnois (IN2P3-CNRS Orsay, France)

Yu. Ts. Oganessian; J. Phys. G: Nucl. Part. Phys. 18 (1992), 393

Target analysis

- inelastic p-scattering experiment

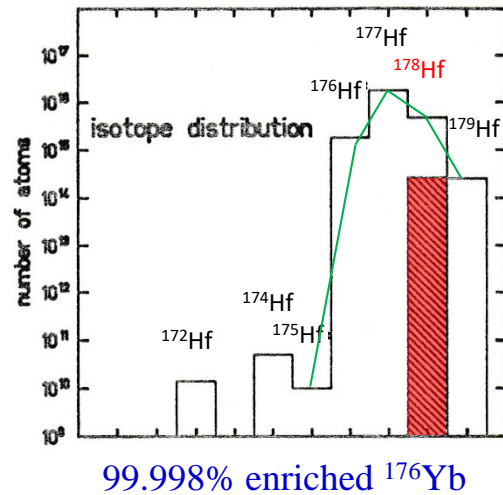
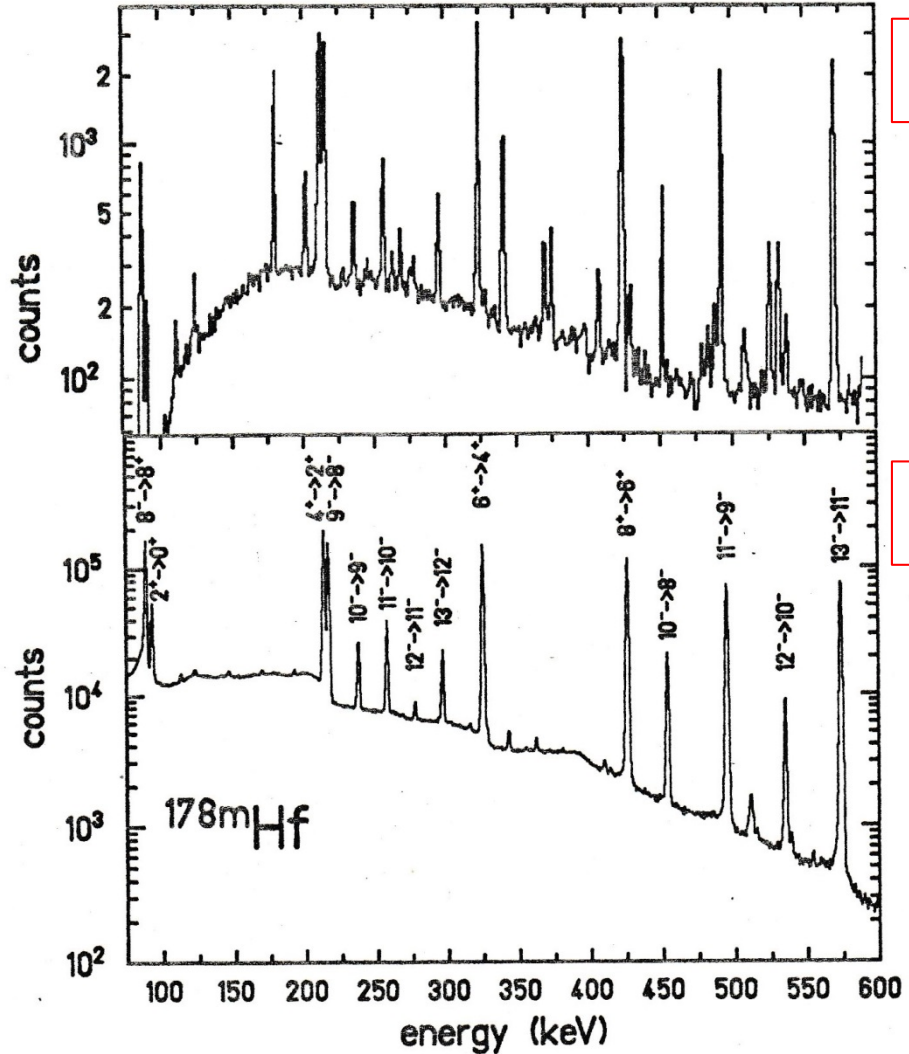
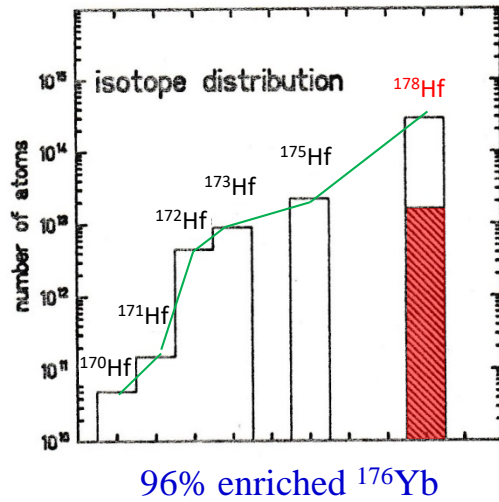


- X-ray analysis

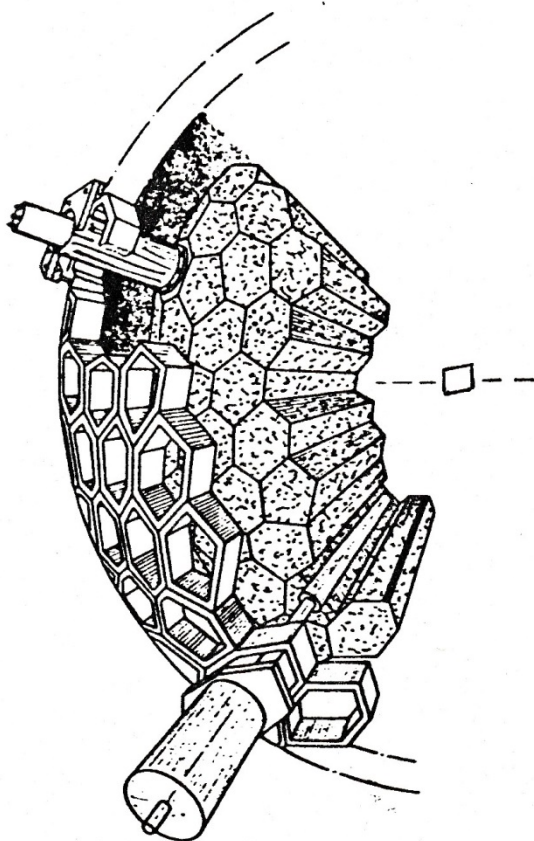
		Br	Zr	Hf	Pt	
$^{178m2}\text{Hf}$	15 μg	107	320	1278	33	counts/s
$^{\text{nat}}\text{Hf}$	40 μg	-	223	3645	-	counts/s

- isomer-to ground state ratio: $\sigma_{m2}/\sigma_{gs} = 2\%$

The $^{176}\text{Yb}(\alpha,2n)$ reaction



Decay study with the GSI-MPI Crystal Ball



162 NaI detectors
1 Ge detector

parameters and resolutions FWHM

E_i 5.5% at 1332 keV

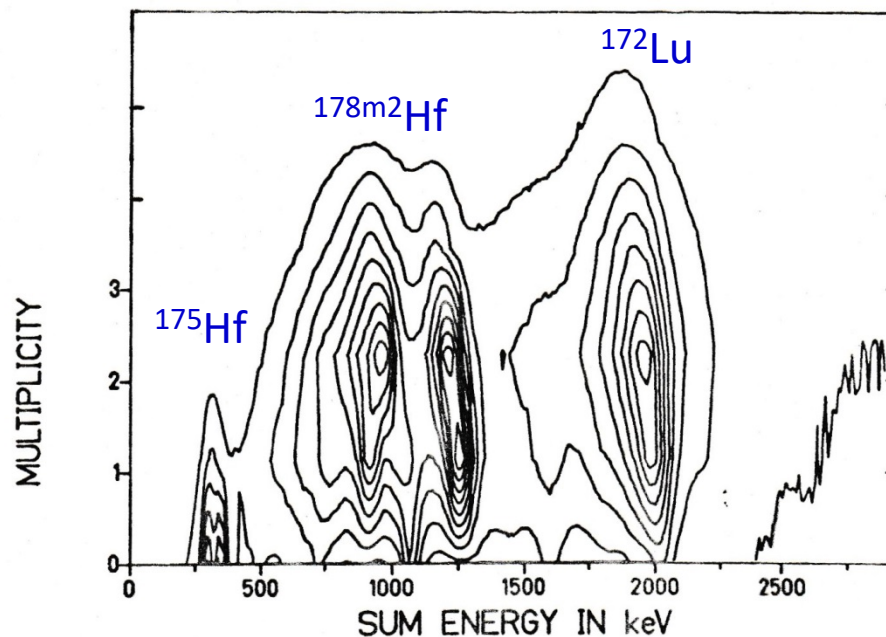
t_i 2.8 ns

$W(\theta)$ 14°

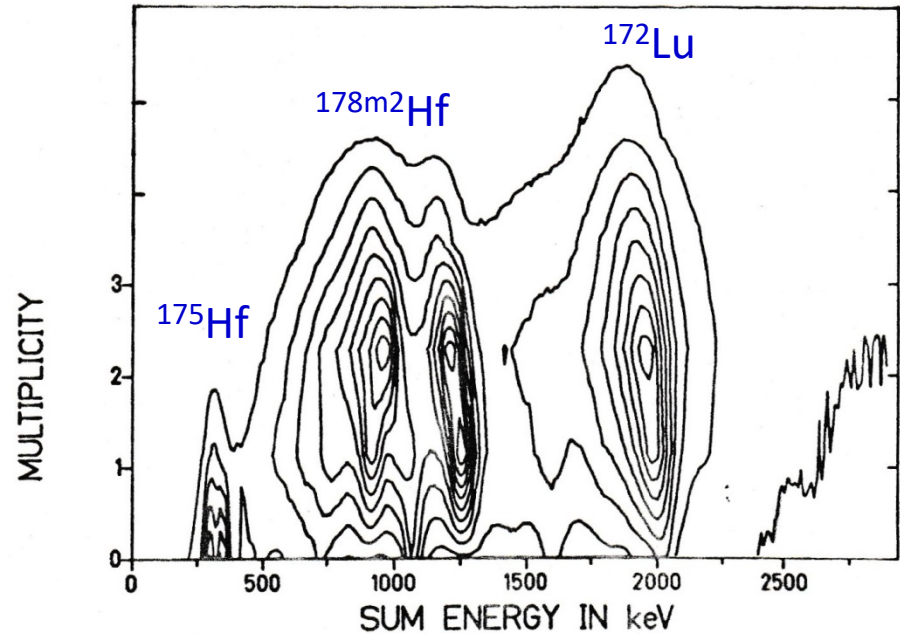
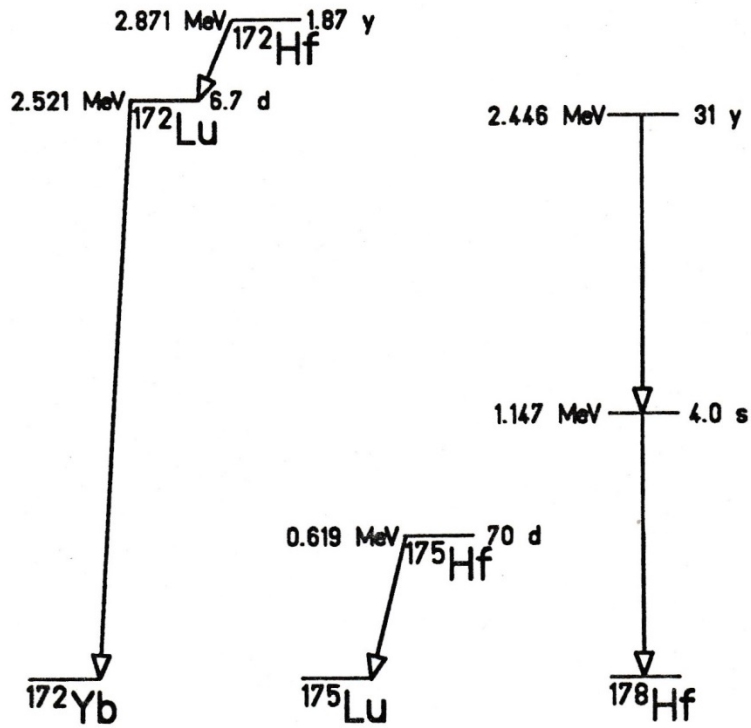
E_{total} 18-22% for $M_\gamma=20$

M_γ 25-30% for $M_\gamma=20$

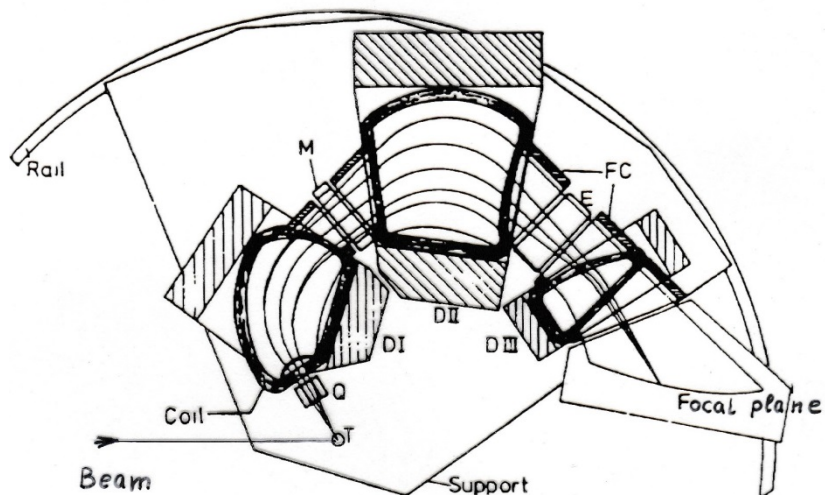
The two basic observables which can be measured for the resulting γ -ray shower are the total energy emitted as γ -radiation and the number of γ -rays.



Decay scheme of $^{175,178m2}\text{Hf}$, ^{172}Lu

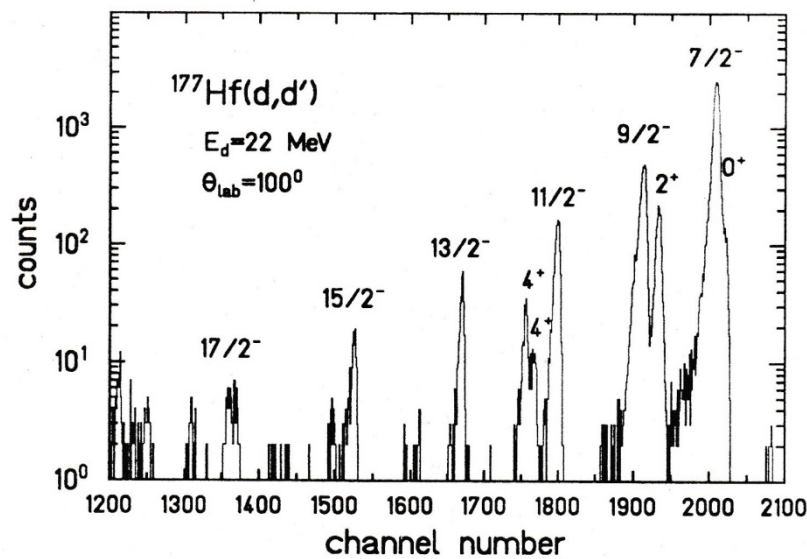


Inelastic Deuteron scattering experiment

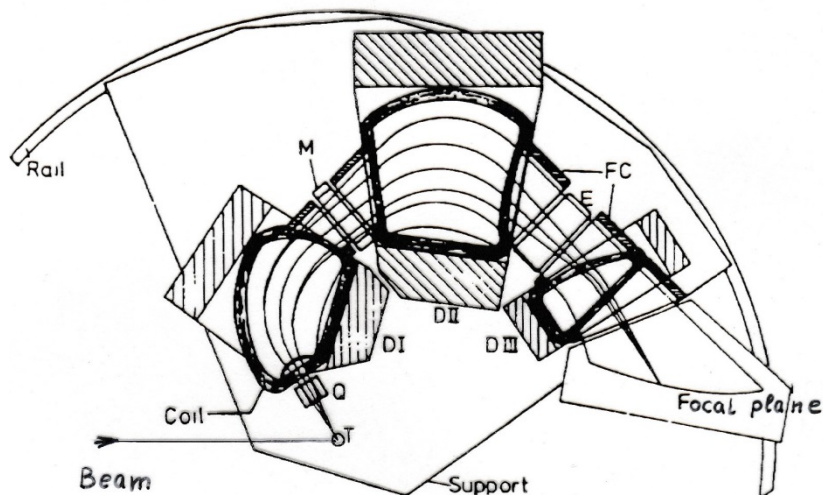


Q3D measurement at the Munich tandem

energy resolution: 12 keV



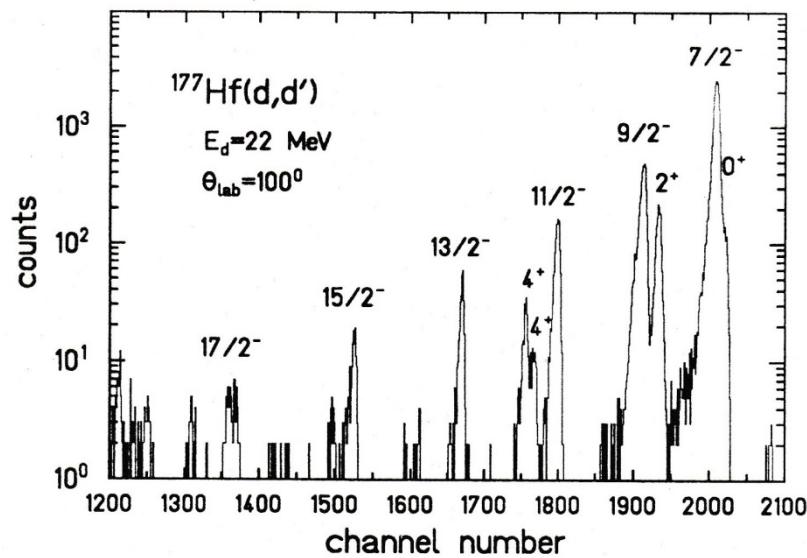
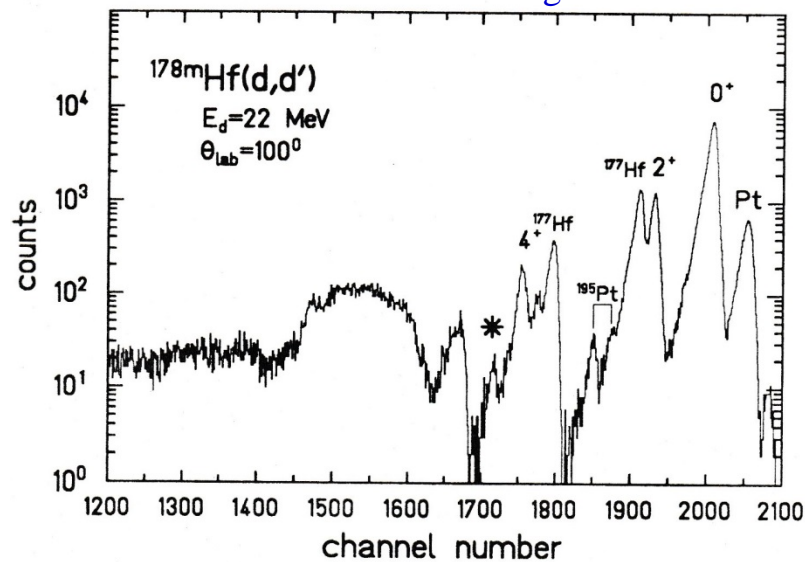
Inelastic Deuteron scattering experiment



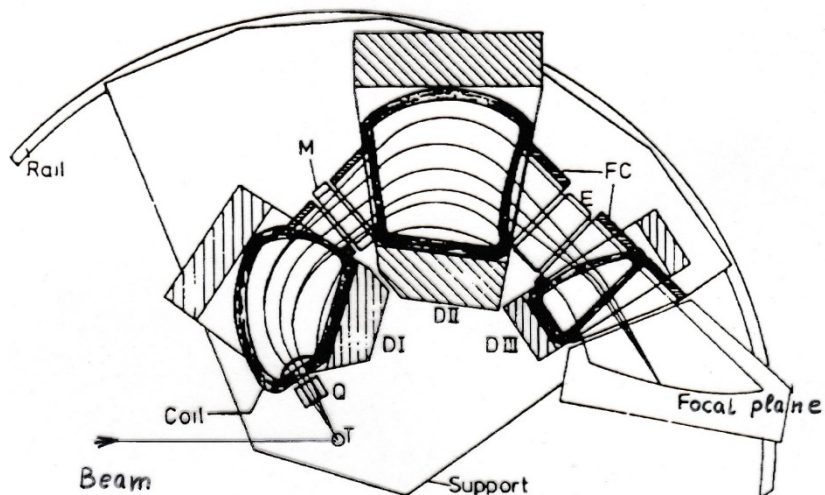
Q3D measurement at the Munich tandem

energy resolution: 12 keV

isomeric ^{178}Hf target

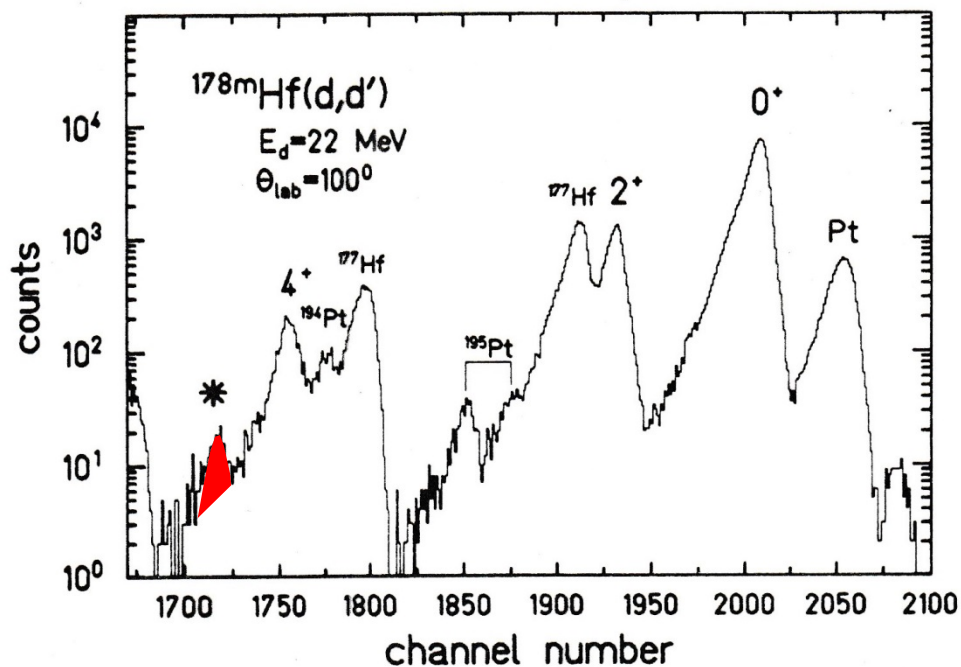


Inelastic Deuteron scattering experiment

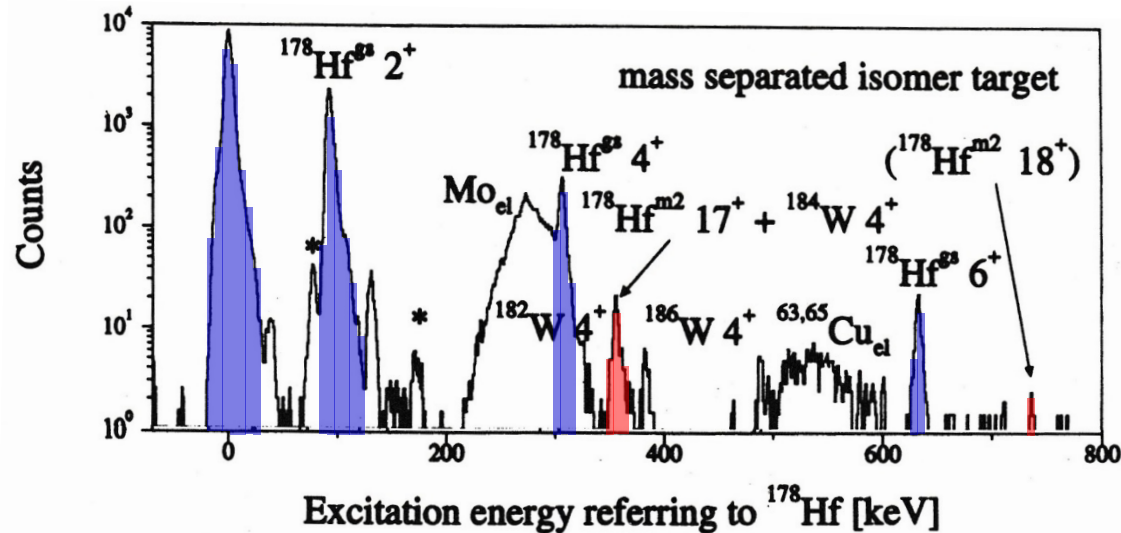


Q3D measurement at the Munich tandem

energy resolution: 12 keV



Inelastic Deuteron scattering from K=16 isomer



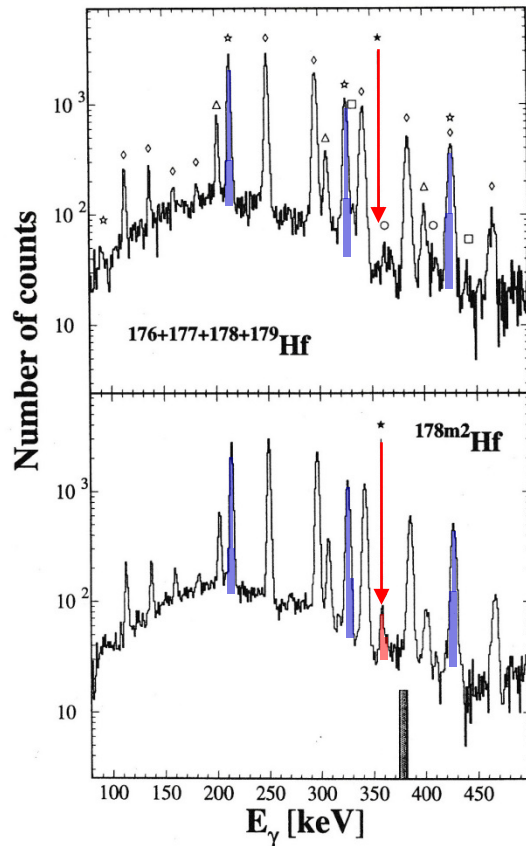
$$\frac{{}^{178m2}\text{Hf}}{{}^{178}\text{Hf}} \cong 3\%$$

experiment performed at Munich tandem accelerator with Q3D spectrograph (45⁰ – 100⁰)
 W, Ta, Mo, Cu result from sputtering and migration process in the mass-separator facility

❖ $E_x = 356.5 \pm 0.4 \text{ keV}$ 17⁺ member of rotational band

$$\frac{\mathfrak{I}({}^{178m2}\text{Hf})}{\mathfrak{I}({}^{178}\text{Hf})} = 1.48$$

Coulomb excitation of the K=16 isomer



$^{208}\text{Pb} \rightarrow \text{Hf}; 4.8 \text{ MeV/u}$

“artificial” γ -spectrum

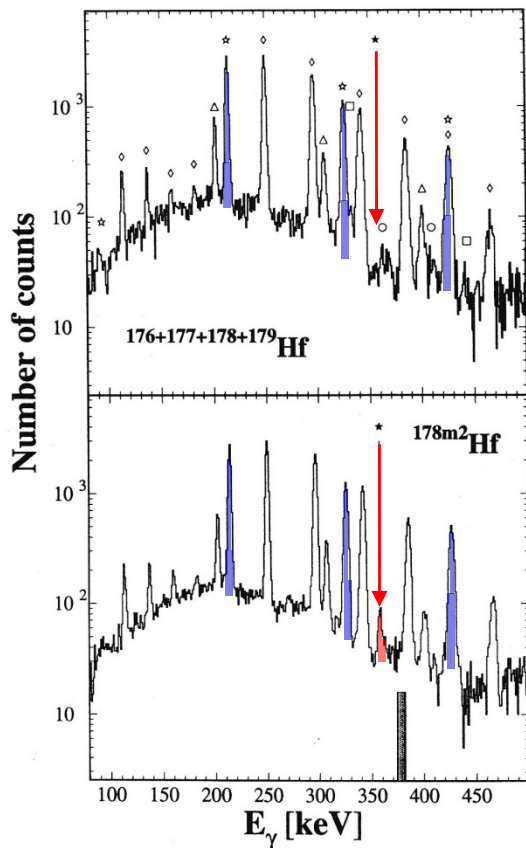
γ -spectrum with isomeric target

$$\frac{^{178m2}\text{Hf}}{\text{Hf}} = 0.6\% \quad 10^{14} \text{ } ^{178m2}\text{Hf atoms}$$

experiment performed at UNILAC accelerator

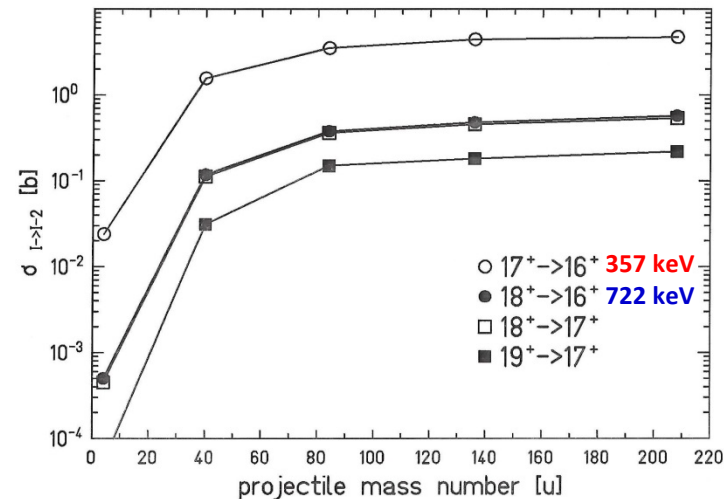
- ❖ $E_\gamma = 357.4 \pm 0.3 \text{ keV}$ $17^+ \rightarrow 16^+$ transition
- ❖ $Q_0 (K=16) = 8.2 \pm 1.1 \text{ b}$ rigid rotor model

Coulomb excitation of the K=16 isomer



$^{208}\text{Pb} \rightarrow \text{Hf}; 4.8 \text{ MeV/u}$

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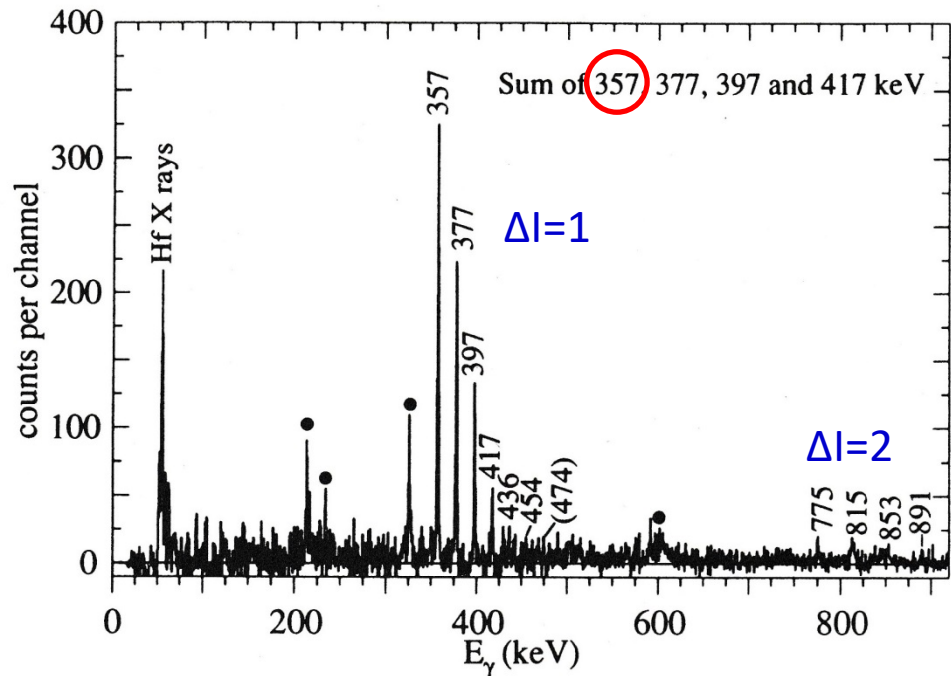
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Rotational band on the K=16 isomer in ^{178}Hf



Summed coincidence spectrum from projections made on transitions in the rotational band assigned to $^{178m2}\text{Hf}$.

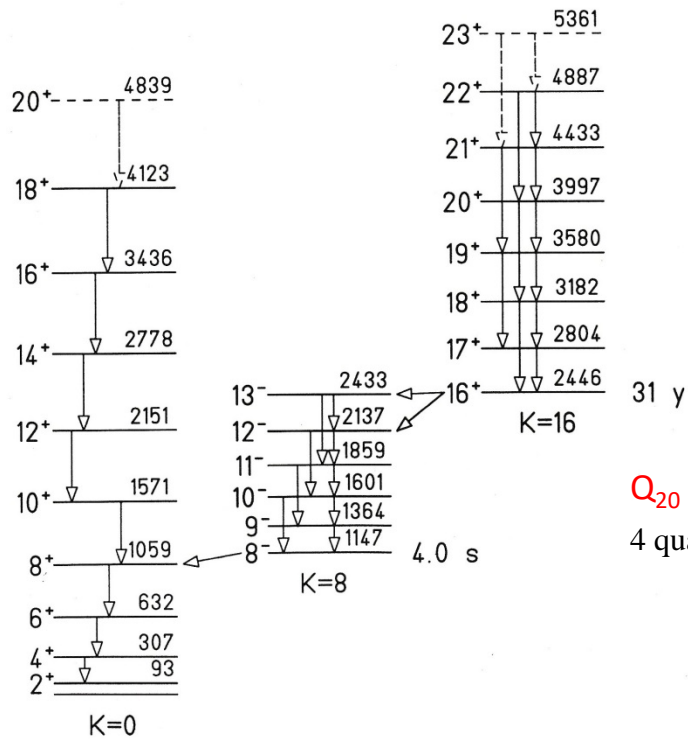
$^{176}\text{Yb}(^9\text{Be}, \alpha 3n) ^{178}\text{Hf}$ 55, 60 MeV

incomplete fusion reaction

14 charged-particle detectors cover 85% of 4π

E2/M1 mixing ratios $g_K=0.52(6)$ $\nu^2\pi^2 - qp$ band

Partial level scheme of ^{178}Hf



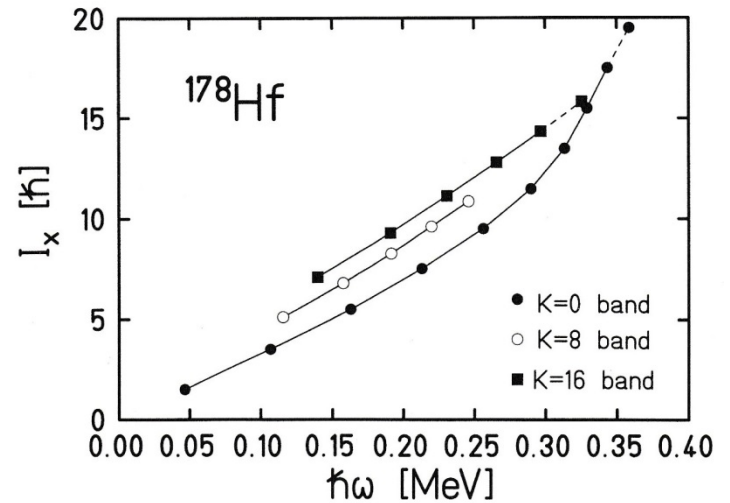
$Q_{20} = 7.0 \text{ b}$

$Q_{20} = 8.2 \pm 1.1 \text{ b}$

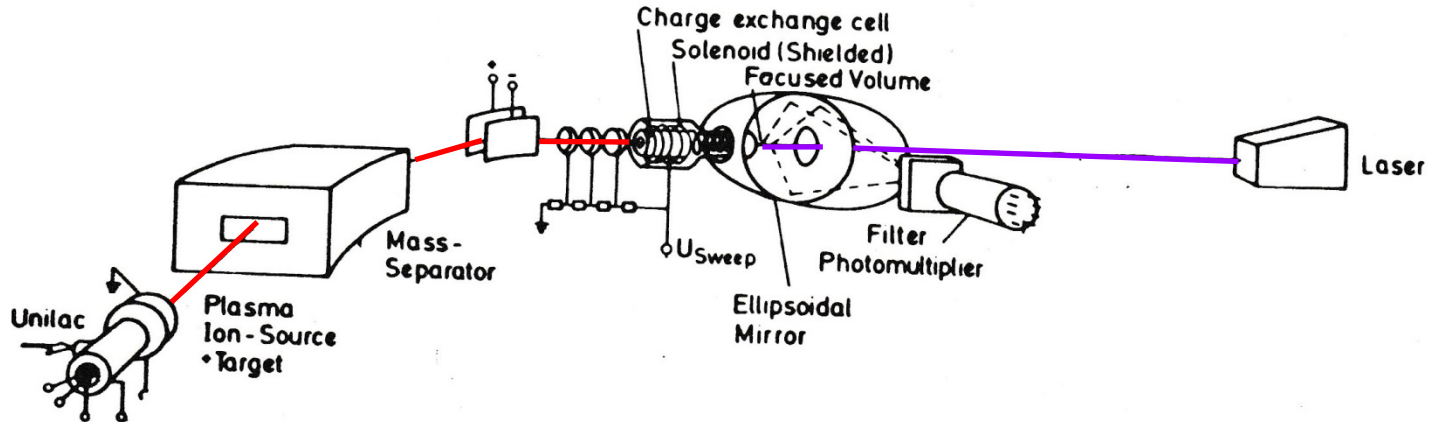
4 quasi-particle state

$$I_x = \{(I + 1/2)^2 - K^2\}^{1/2}$$

$$\hbar\omega = \frac{E(I + 1) - E(I - 1)}{I_x(I + 1) - I_x(I - 1)}$$

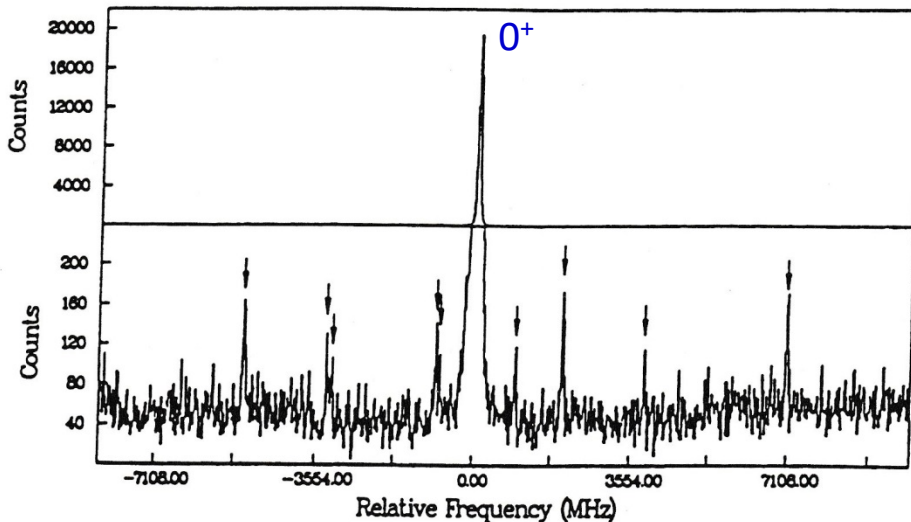


Collinear laser spectroscopy



kinetic energy spread in ion source: $\Delta E = mv \cdot \Delta v = \text{few eV}$

residual Doppler width after acceleration: $\Delta v = \frac{v_0 \Delta E}{\sqrt{2eU \cdot mc^2}}$ $\Delta v = 60 \text{ MHz for } 60 \text{ kV}$



$$\Delta W_{HFS} = \frac{A}{2}K + B \frac{3}{4} \frac{K(K+1) - I(I+1)J(J+1)}{2I(2I-1) \cdot J(2J-1)}$$

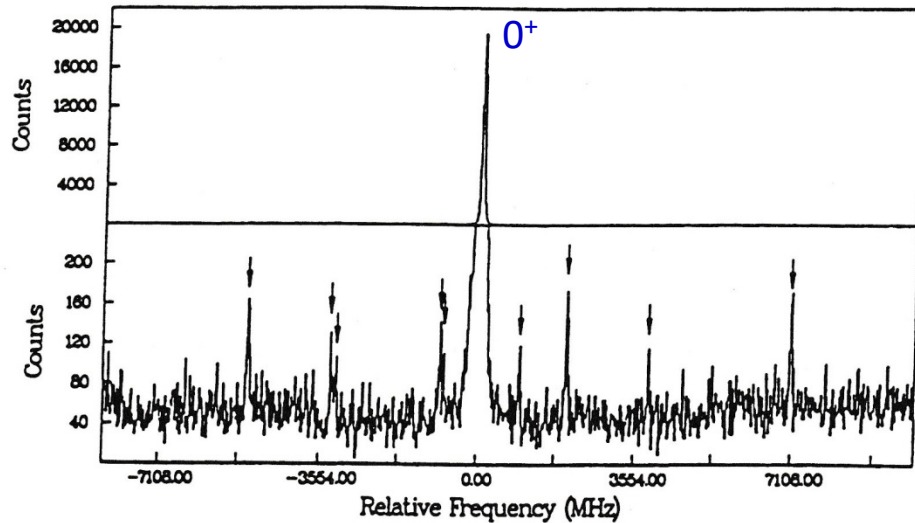
$$K = F(F+1) - I(I+1) - J(J+1)$$

$$A = \frac{\mu_I \cdot \langle H(0) \rangle}{I \cdot J}$$

$$B = eQ_s \cdot \langle V_{zz}(0) \rangle$$

The single peak at 0 MHz corresponds to the ground state ^{178}Hf .
The nine hyperfine resonances of $^{178\text{m}2}\text{Hf}$ are marked by arrows.

Collinear laser spectroscopy



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The single peak at 0 MHz corresponds to the ground state ^{178}Hf .
The nine hyperfine resonances of $^{178m2}\text{Hf}$ are marked by arrows.

$$\mu_I^{178m2} = +8.16(4) \text{ nm}$$

$$Q_s^{178m2} = +6.00(7) \text{ b} \quad \rightarrow \quad Q_0 = 7.2(1) \text{ b}$$

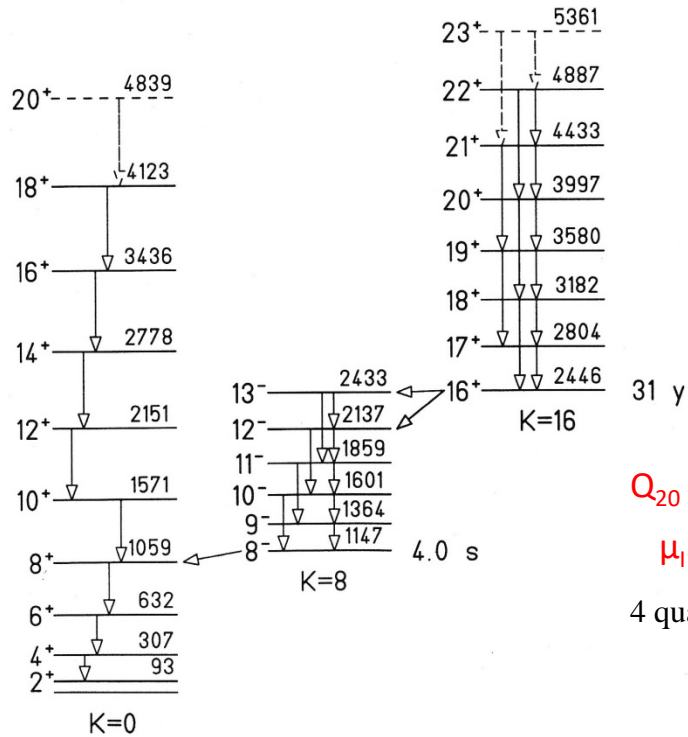
$$\delta \langle r^2 \rangle^{178,178m2} = -0.059(9) \text{ fm}^2$$

strong coupling scheme:

$$Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \cdot Q_0 \quad \rightarrow \quad Q_s (K=I) = \frac{I \cdot (2I-1)}{(I+1)(2I+3)} \cdot Q_0$$

$$\delta \langle r^2 \rangle = \delta \langle r^2 \rangle_0 + \frac{5}{4\pi} \langle r^2 \rangle_0 \sum_{\lambda} \delta \langle \beta_{\lambda}^2 \rangle$$

Partial level scheme of ^{178}Hf



$Q_{20} = 7.0 \text{ b}$

$Q_{20} = 7.2 \pm 0.1 \text{ b}$

$\mu_1 = +8.16(4) \text{ n.m.}$

4 quasi-particle state

$Q_{20} = 8.2 \pm 1.1 \text{ b}$ (Coulomb excitation)

$$I_x = \{(I + 1/2)^2 - K^2\}^{1/2}$$

$$\hbar\omega = \frac{E(I + 1) - E(I - 1)}{I_x(I + 1) - I_x(I - 1)}$$

