Outline: Nuclear shell structure exotic nuclei

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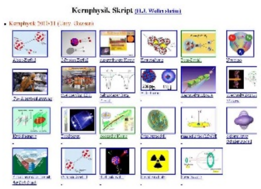
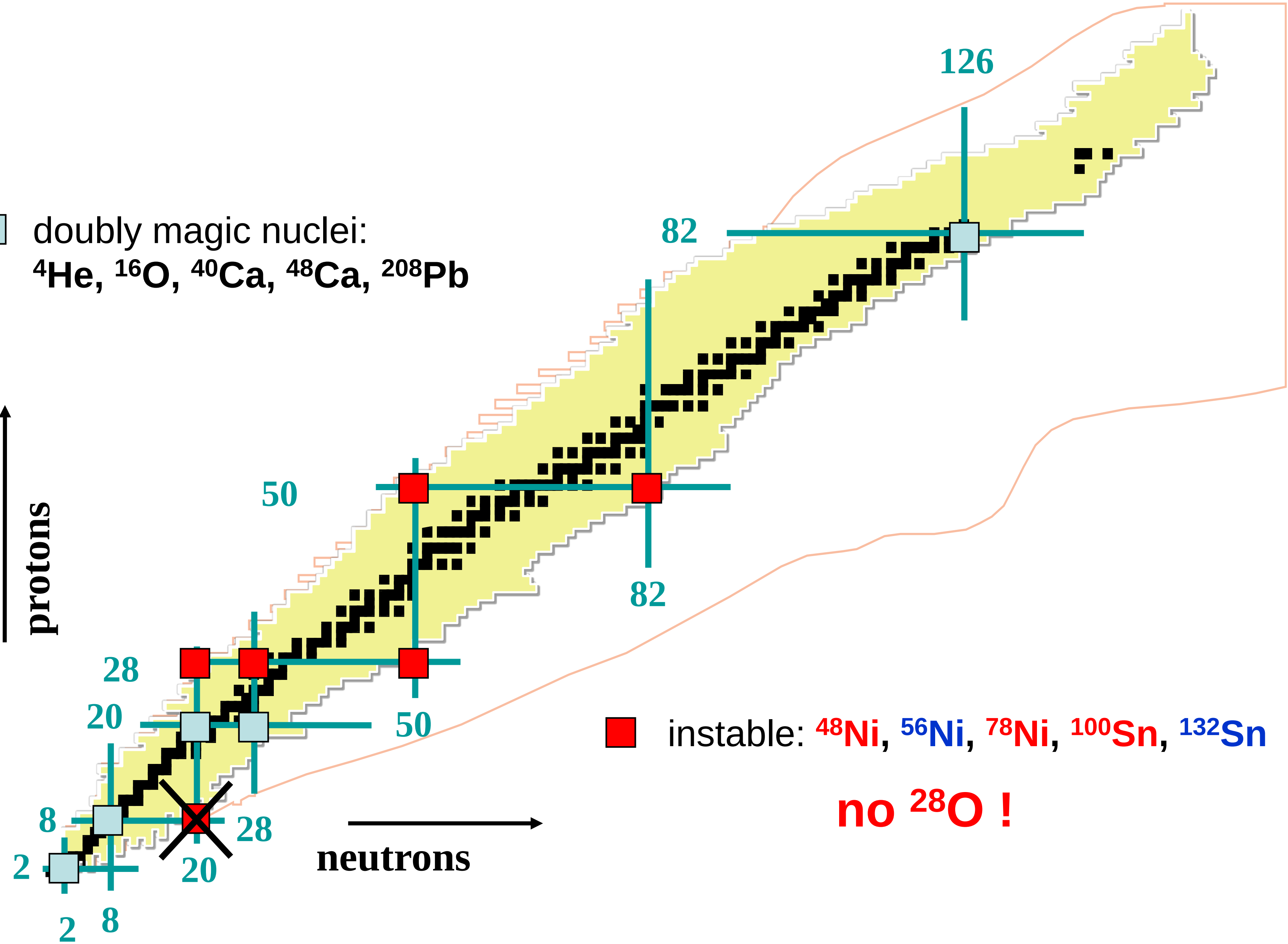
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. shell structure of super-heavy nuclei (^{250}Fm, ^{254}No)
2. classical anomalies: ^{11}Be, ^{11}Li
3. monopole interaction of the tensor force: N=20, N=28
4. neutron-proton pairing in ^{92}Pd

New challenges in nuclear structure new magic numbers

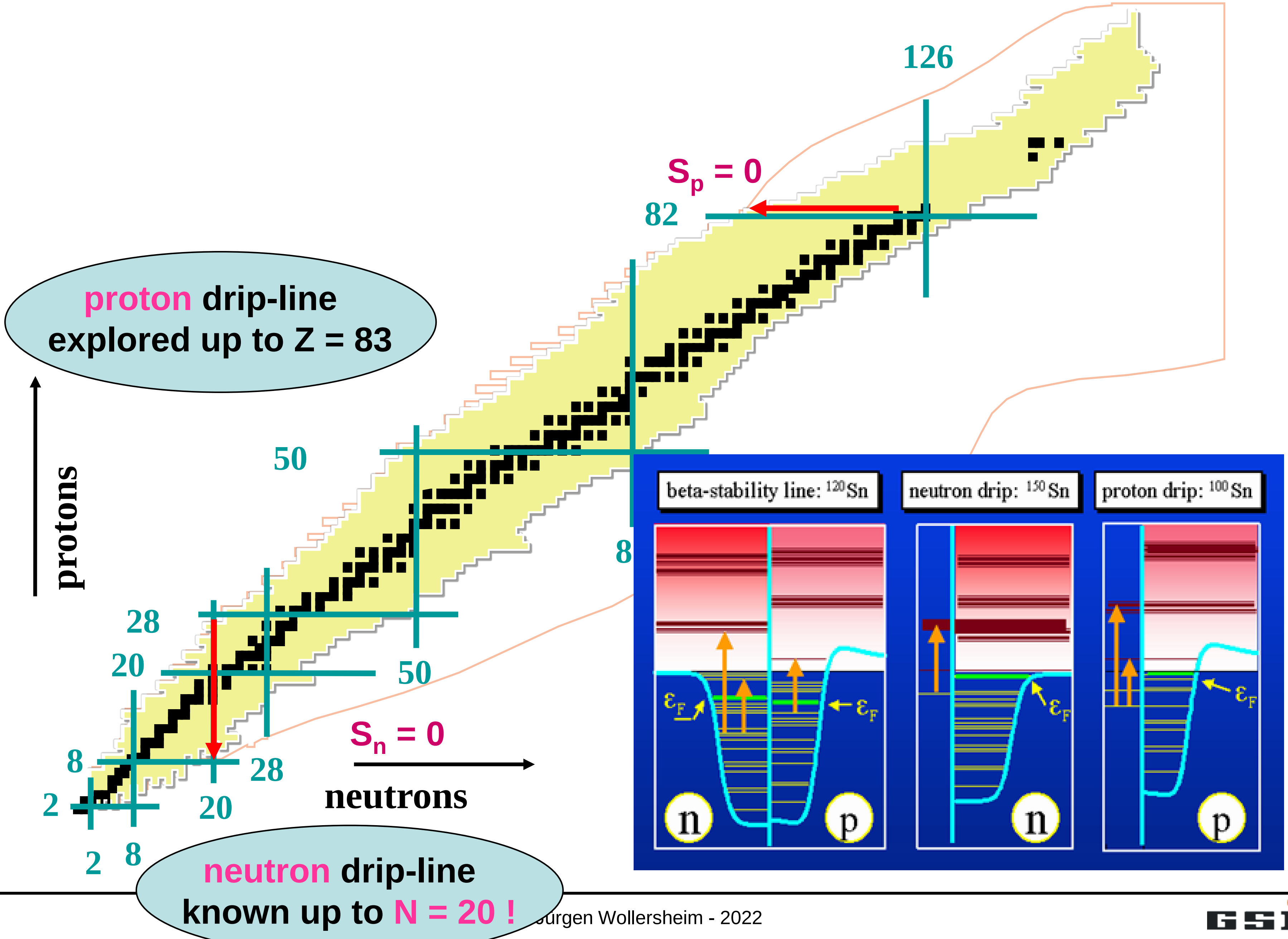
doubly magic nuclei:
^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, ^{208}Pb



instable: ^{48}Ni, ^{56}Ni, ^{78}Ni, ^{100}Sn, ^{132}Sn

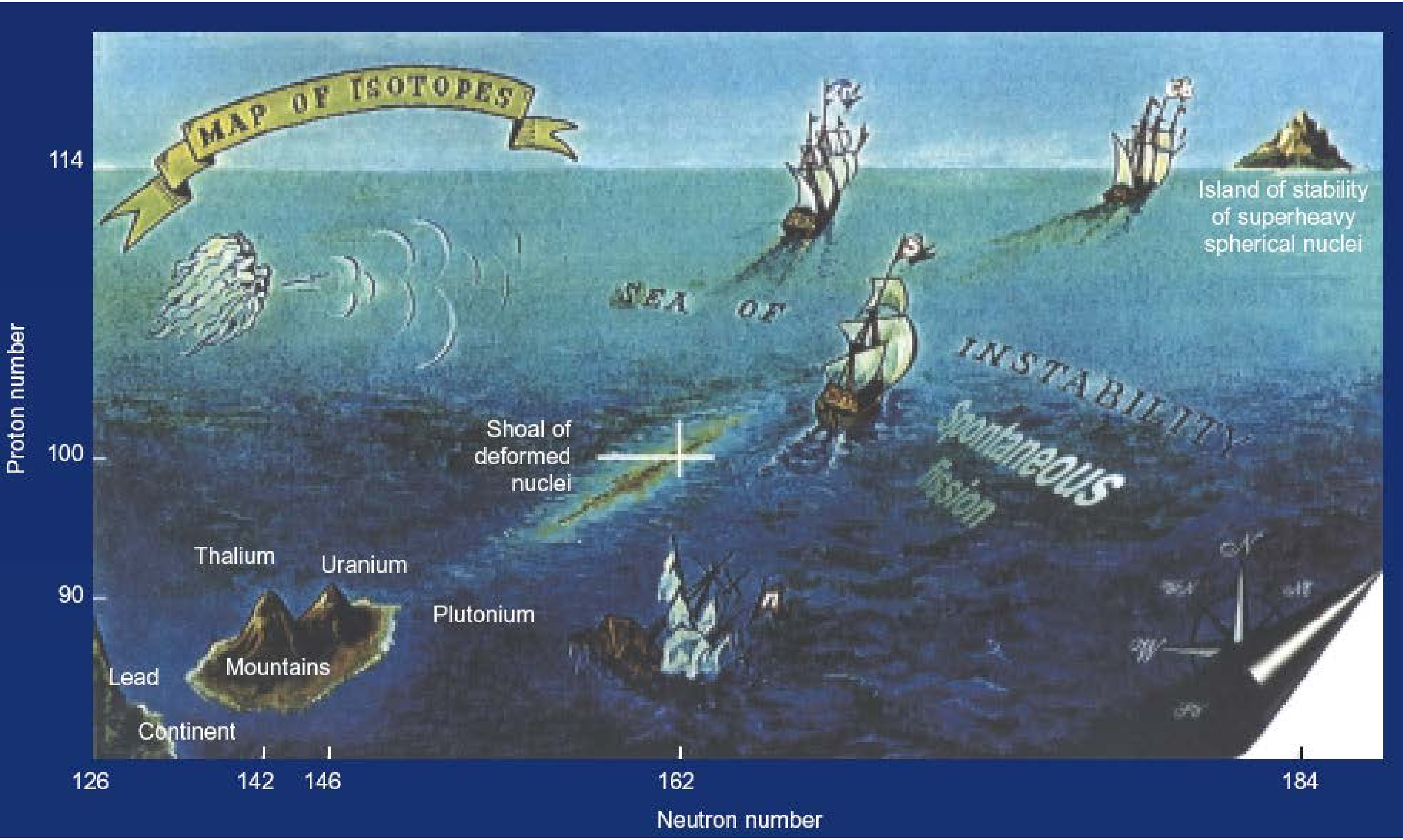
no ^{28}O !

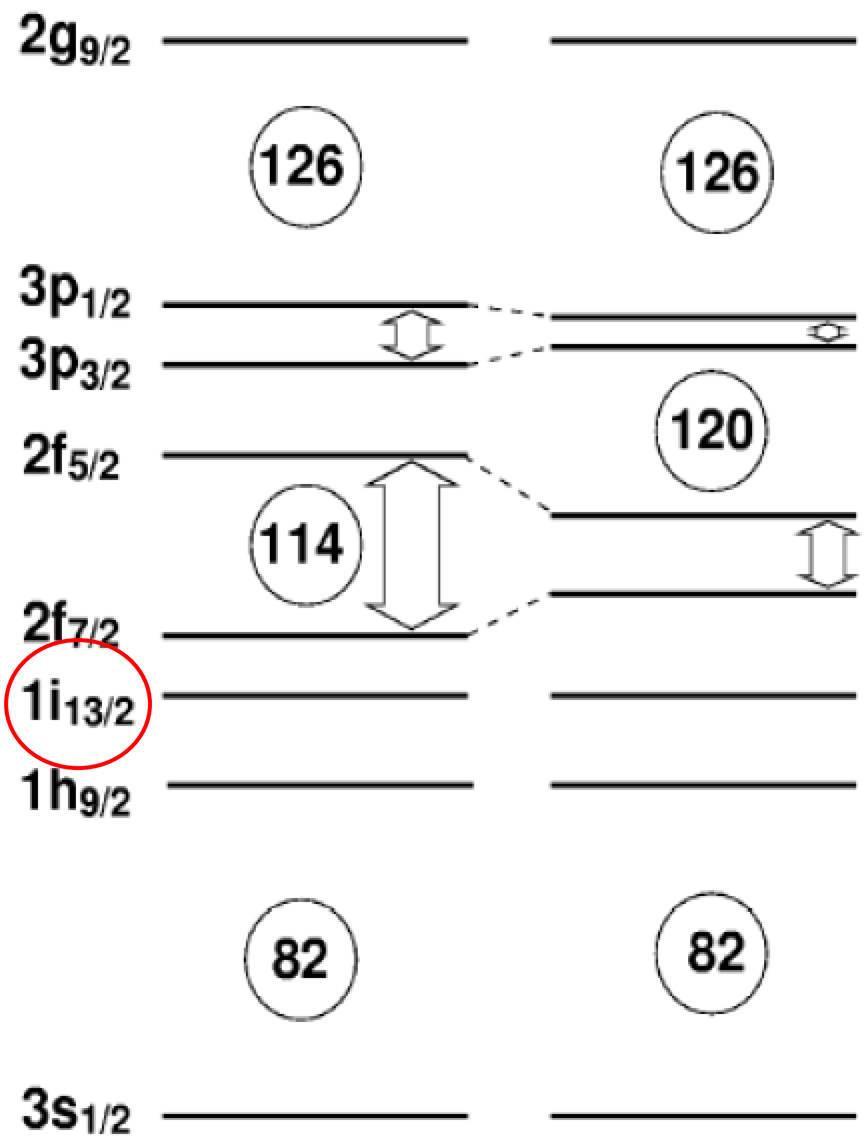
New challenges in nuclear structure new magic numbers

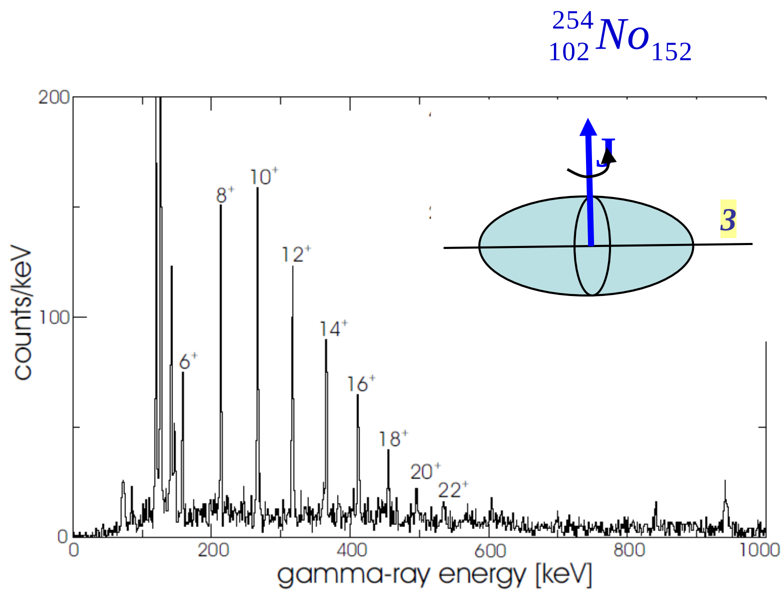


Spectroscopy of transfermium nuclei (Z=100-103)

super–heavy elements







The ***deformation*** of the nucleus changes the order of the single particle states (***Nilsson model***)

Deformed shell model – Nilsson model

Nilsson-model

- *deformed oscillator potential*
- *axially symmetry around the z-axis*
  $\rightarrow$ *nuclei can rotate*

$\omega_x = \omega_y \equiv \omega_\perp$ $\qquad$ $\omega_x \cdot \omega_y \cdot \omega_z = \omega_0^3$

Hamiltonian

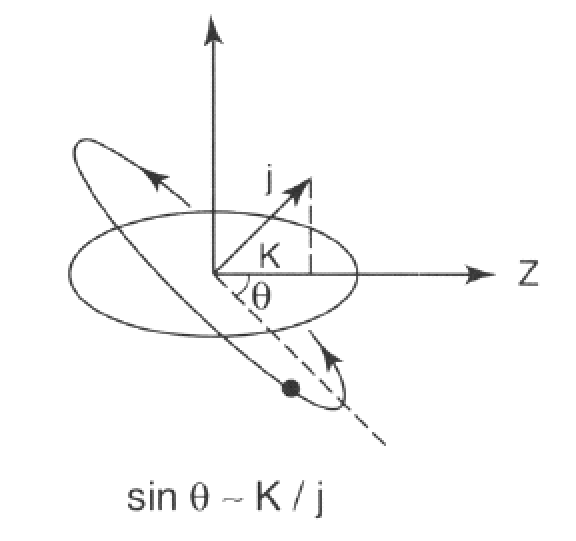
$$H = -\frac{\hbar^2}{2 \cdot m} \cdot \Delta + \frac{m}{2} \cdot [\omega_\perp^2 \cdot (x^2 + y^2) + \omega_z^2 \cdot z^2] + C \cdot \vec{L} \cdot \vec{S} + D \cdot \vec{L}^2$$

deformation parameter δ

$$\omega_\perp^2 = \omega_0^2 \cdot \left(1 + \frac{2}{3} \cdot \delta\right) \qquad \omega_z^2 = \omega_0^2 \cdot \left(1 - \frac{4}{3} \cdot \delta\right)$$

$$H = -\frac{\hbar^2}{2 \cdot m} \cdot \Delta + \frac{m}{2} \cdot \omega_0^2 \cdot r^2 + C \cdot \vec{L} \cdot \vec{S} + \mathrm{D} \cdot \vec{L}^2 - m \cdot \omega_0^2 \cdot r^2 \cdot \delta \cdot \frac{4}{3} \cdot \sqrt{\frac{5}{4 \cdot \pi}} \cdot Y_{20}(\theta, \phi)$$

shell model with H.O. potential $\qquad$ H_{def}



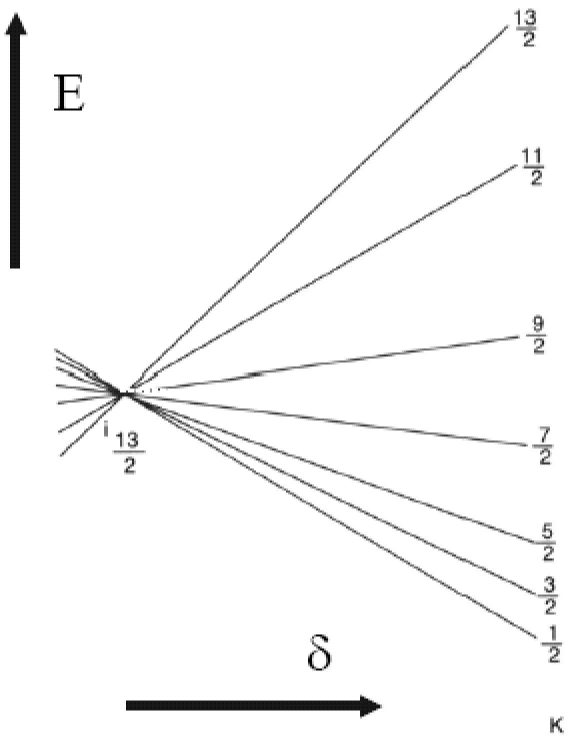
- *separation of laboratory system and body-fixed (intrinsic) system*
- ***K = projection of the single-particle angular momentum onto the symmetry axis***
- *Rotation perpendicular to the symmetry axis does not change the K-quantum number*

Deformed shell model – Nilsson model

Nilsson-model

- *deformed oscillator potential*
- *axially symmetry around the z-axis*

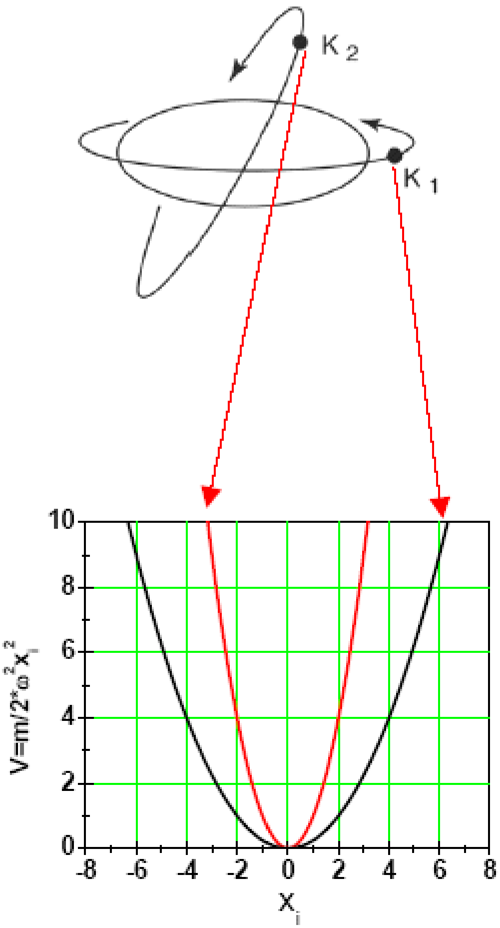
  → *nuclei can rotate*



Intruder

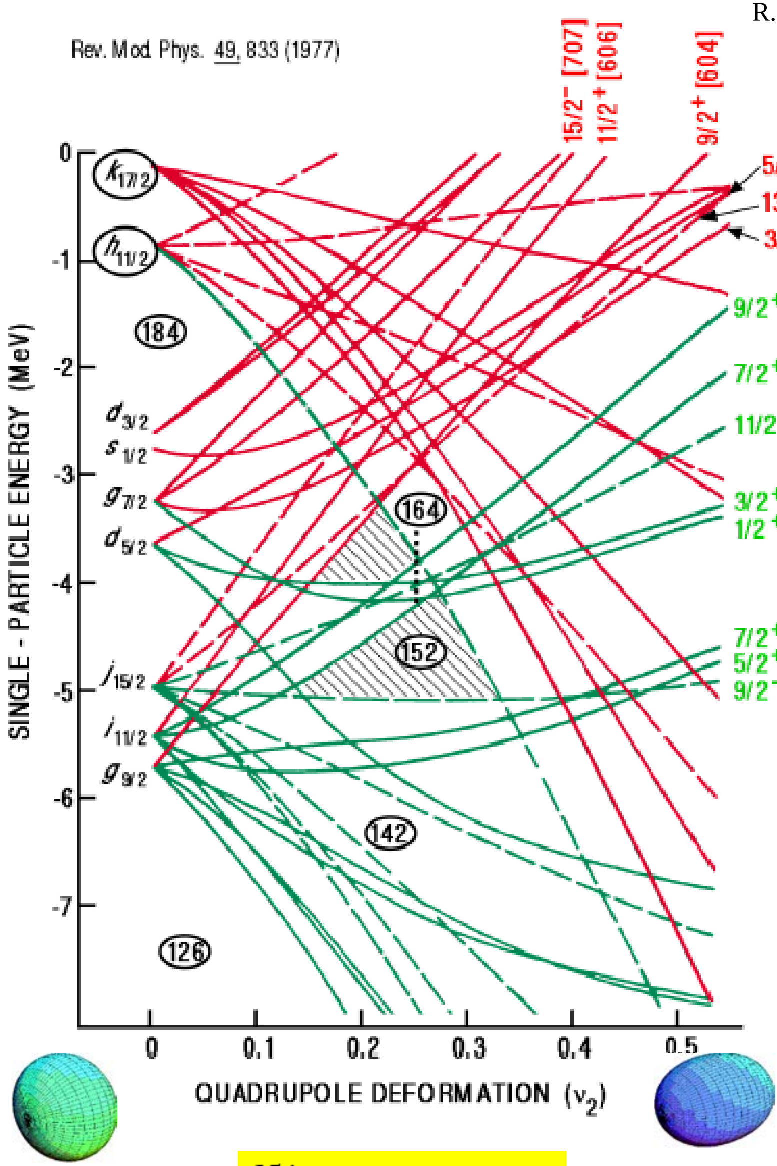
Orbitals are lifted or lowered, that orbitals from other shells with opposite parity are crossed

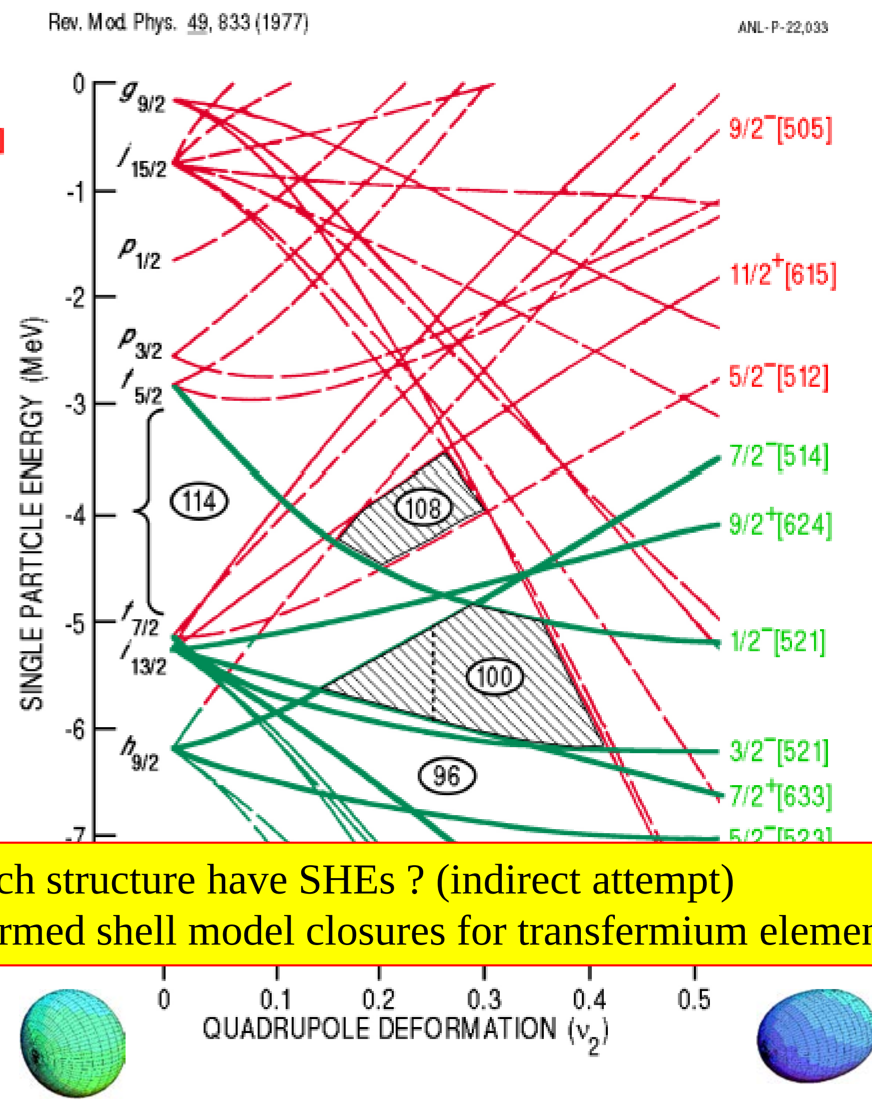
Orbital 1 is closer to the center of gravity than orbital 2. The energy of orbital 1 is lowest. (*attention: negative sign in Hamiltonian*)



Single particle orbitals

R. Chasman et al. Rev. Mod. Phys. 49 (1977), 833





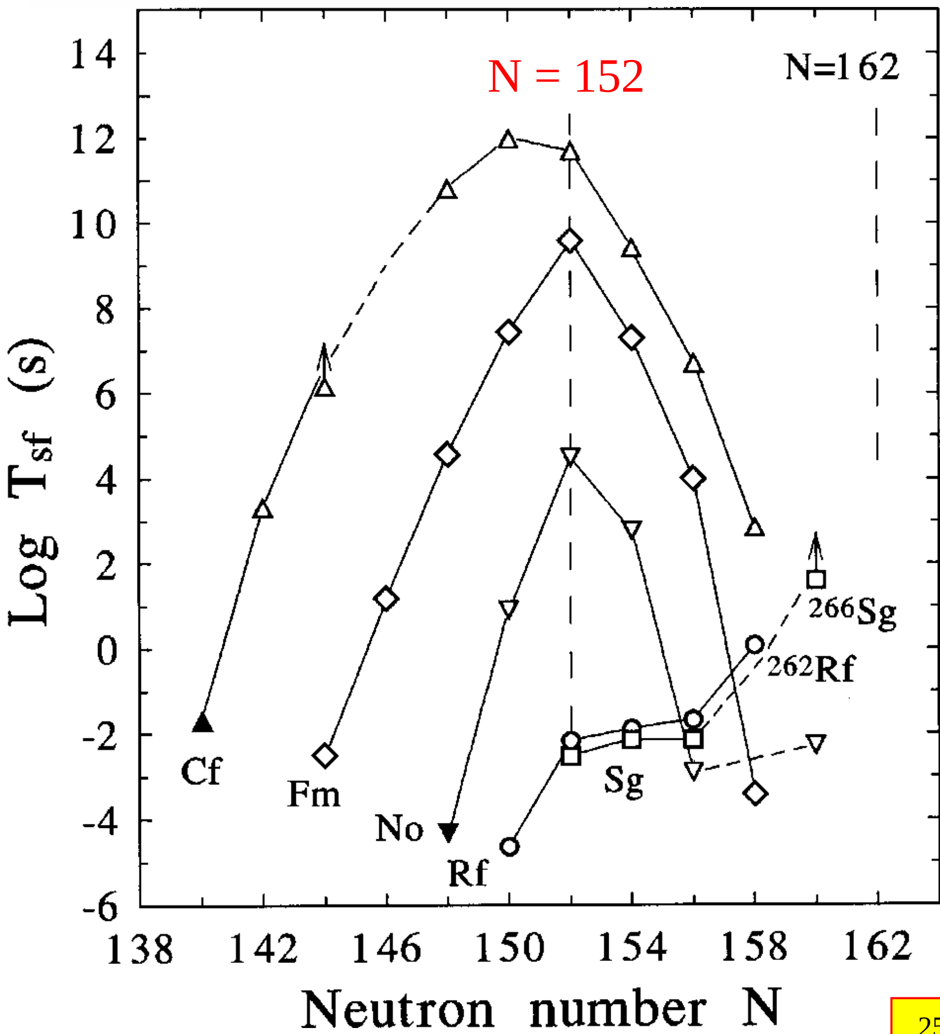
Which structure have SHEs ? (indirect attempt)
deformed shell model closures for transfermium elements

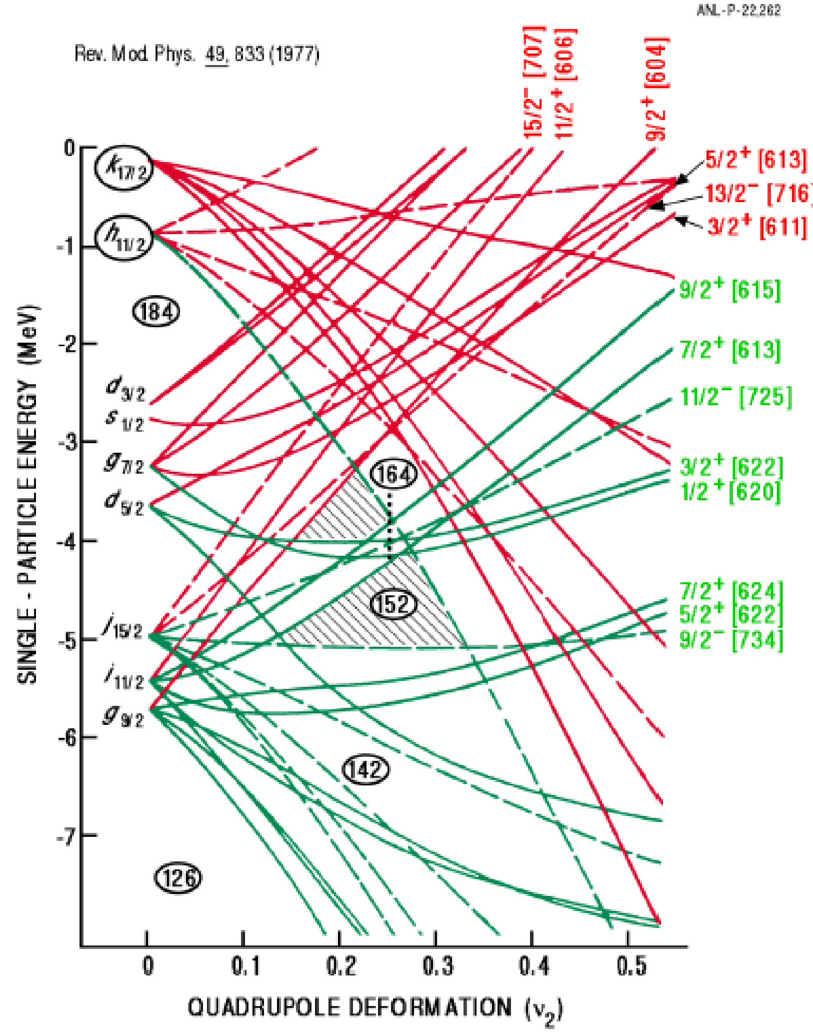
oblate

$^{254}_{102}No_{152}$ $\beta_2 \sim 0.28$

prolate

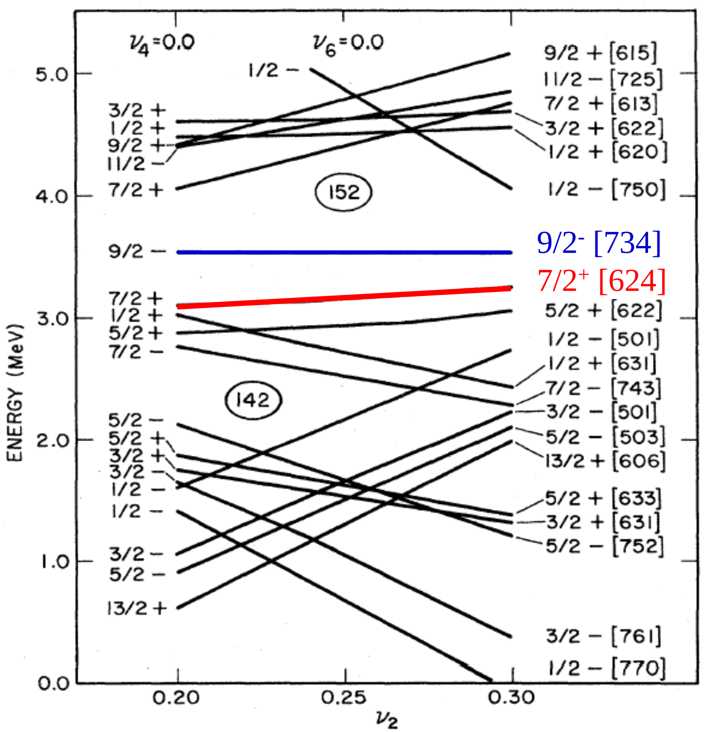
Stability of heavy elements – Nilsson energy levels

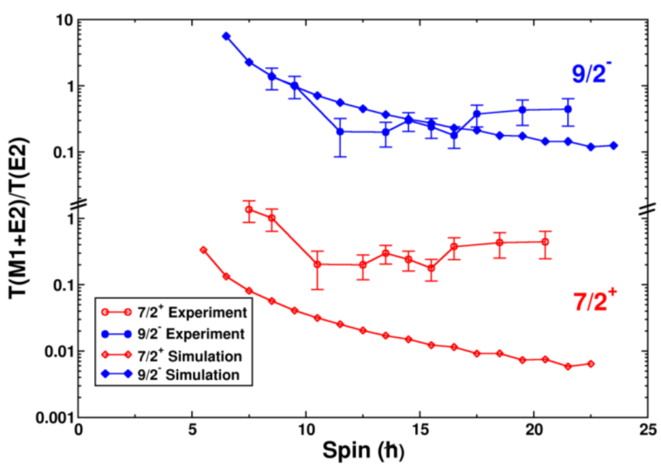




^{254}No (Z=102), ^{252}Fm (Z=100) and ^{250}Cf (Z=98)
with N=152
seem to be more stable than their neighbors

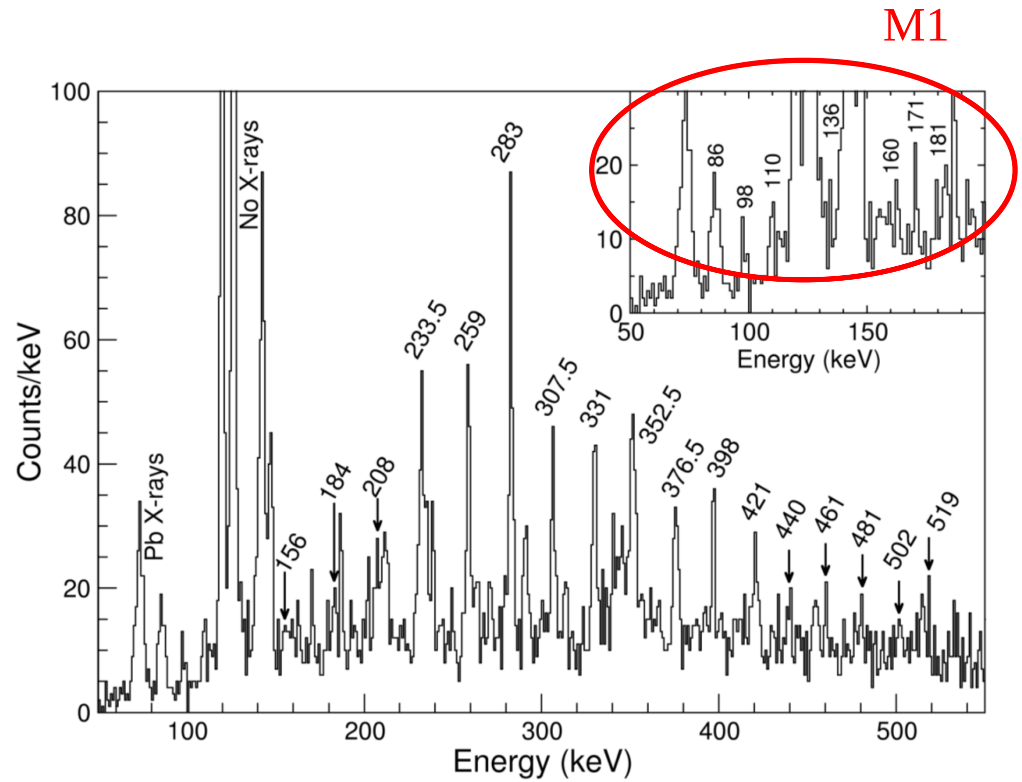
Level scheme of ^{253}No (151 neutrons)





- ground state 9/2-
- excited state 7/2+

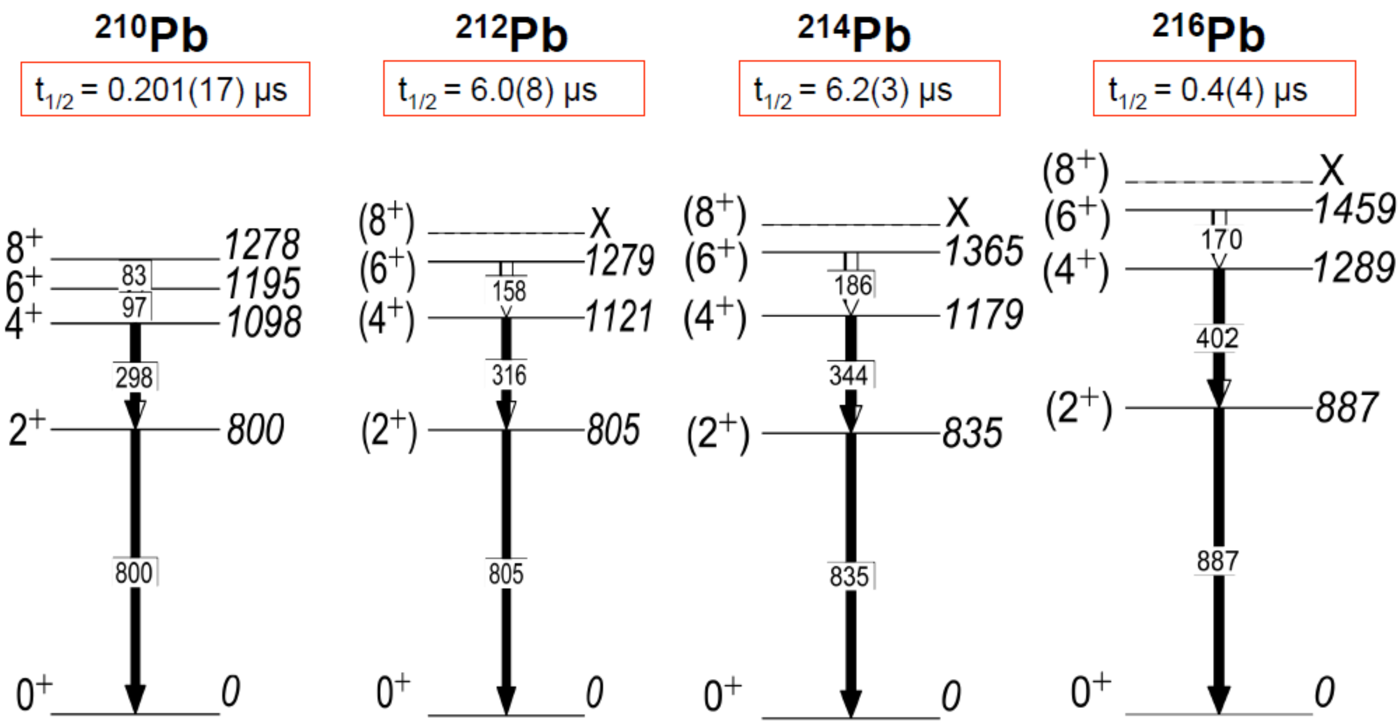
- rotational band observed at Gammasphere & JUROGAM



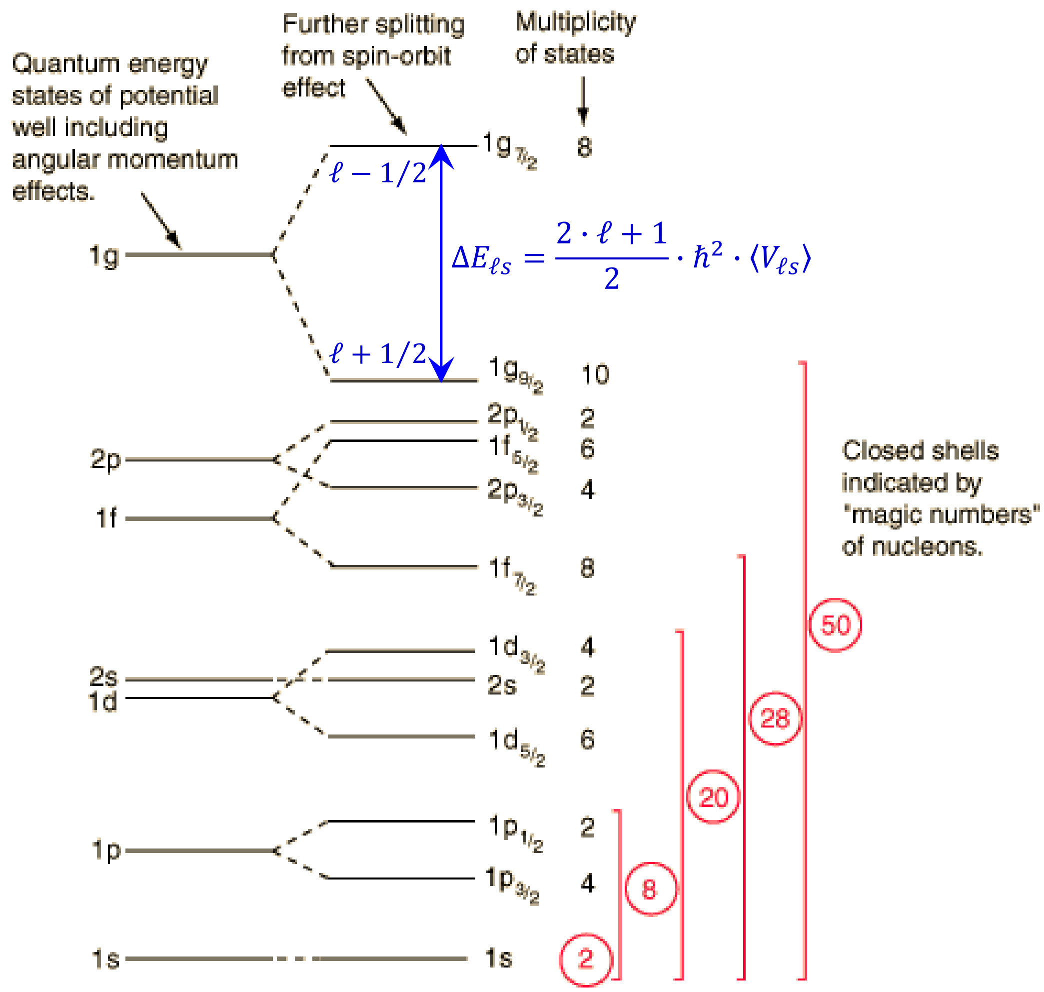
Level schemes in neutron-rich Pb isotopes

Pb205	Pb206	Pb207	Pb208	Pb209	Pb210	Pb211	Pb212	Pb213	Pb214	Pb215	Pb216	Pb217	Pb218
1.53E+7 y				3.253 h	22.3 y	36.1 m	10.64 h	10.2 m	26.8 m	36 s			
5/2-	0+	1/2-	0+	9/2+	0+	9/2+	0+	(9/2+)	0+	(5/2+)			
EC	24.1	22.1	52.4	β-	β-,α	β-	β-	β-	β-	β-			

$g_{9/2}$



The magic numbers near stable nuclei



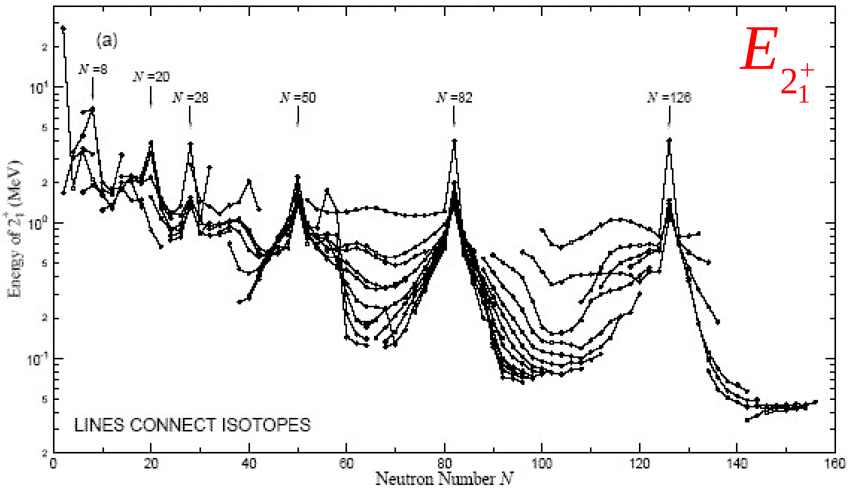
Maria Goeppert-Mayer (1906-1972)
Hans Jensen (1907-1973)

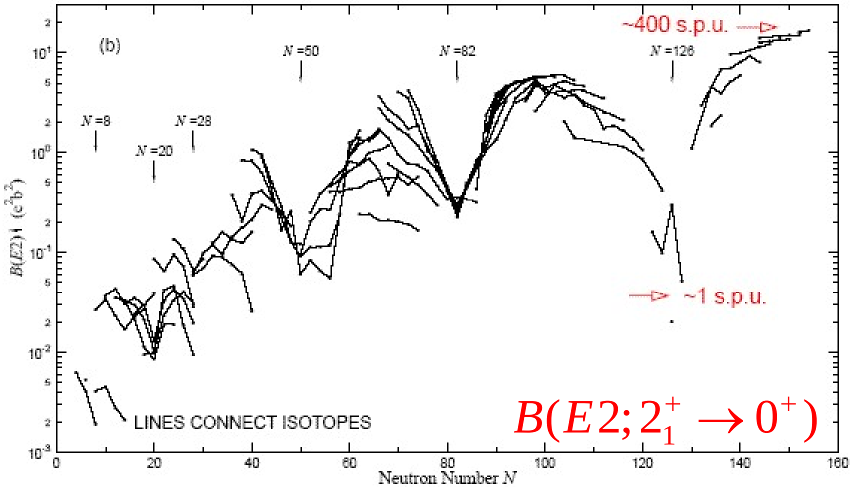
magic numbers with constant shell closures are not so robust, as we thought.

Nuclear shell structure

experimental hints for magic numbers

S. Raman et al., Atomic Data & Nuclear Data Tables 78, 1





nuclei with magic numbers

neutron / proton:

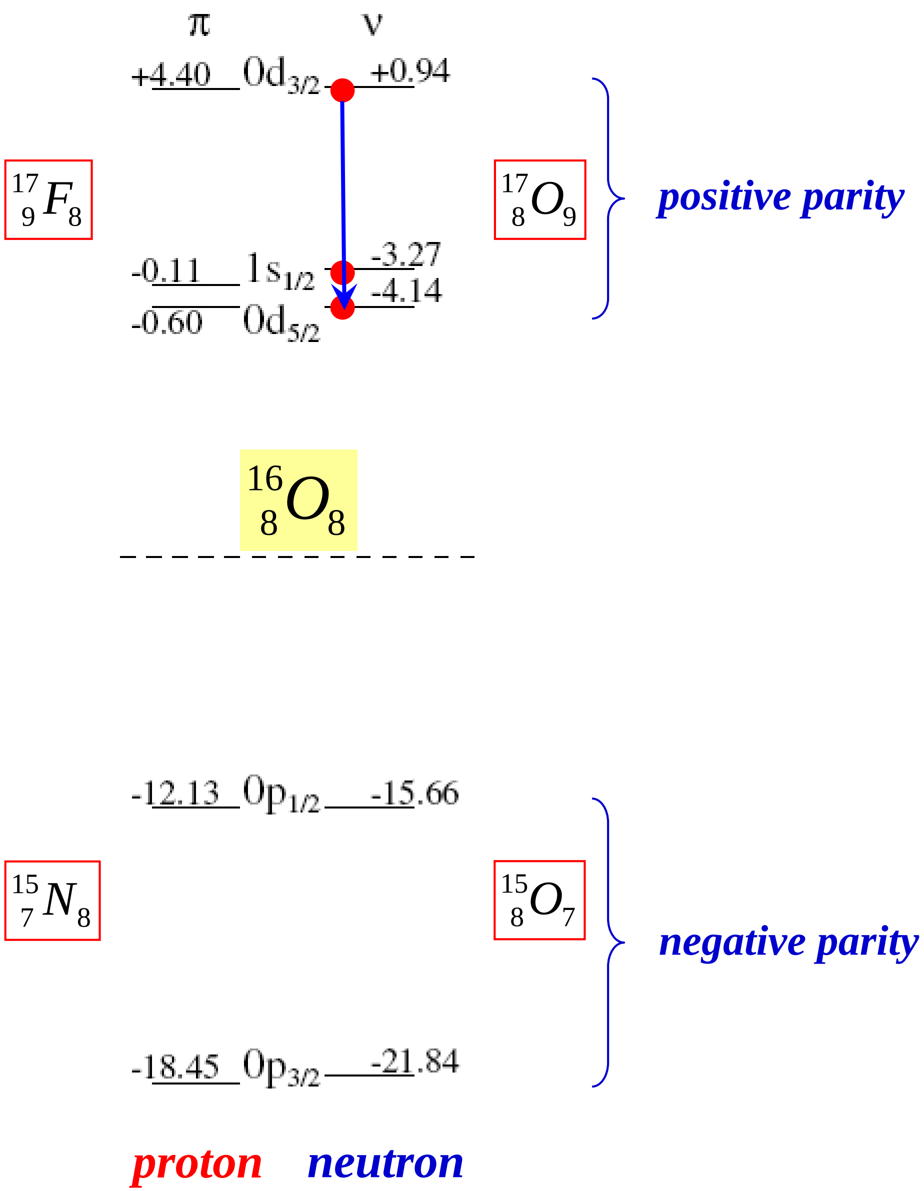
high energies for 2_1^+ states

small $B(E2; 2_1^+ \rightarrow 0^+)$ values

transition probabilities are measured in Weisskopf units (spu)

What happens far away from the valley of stability?

Extreme single-particle shell model



energies of shell closure:

$$BE({}^{17}_{9}F_7) - BE({}^{16}_{8}O_8) = E(0d_{5/2})$$

$$BE({}^{15}_{7}N_8) - BE({}^{16}_{8}O_8) = -E(0p_{1/2})$$

$$E(0d_{5/2}) - E(0p_{1/2}) = BE({}^{17}_{9}F_8) + BE({}^{15}_{7}N_8) - 2\cdot BE({}^{16}_{8}O_8)$$
$$= -11.526\,MeV$$

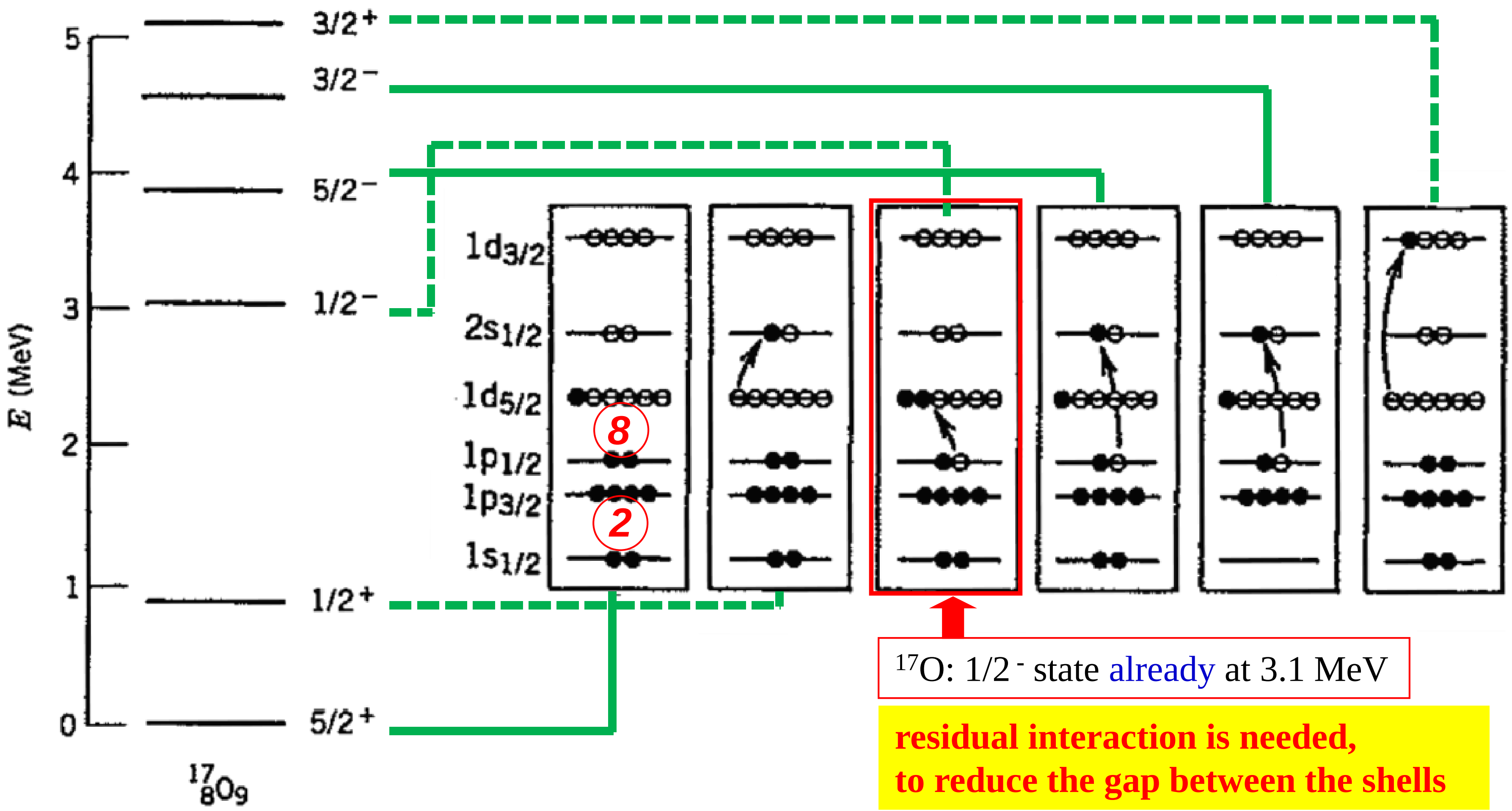
$$BE({}^{17}_{8}O_9) - BE({}^{16}_{8}O_8) = E(0d_{5/2})$$

$$BE({}^{15}_{8}O_7) - BE({}^{16}_{8}O_8) = -E(0p_{1/2})$$

$$E(0d_{5/2}) - E(0p_{1/2}) = BE({}^{17}_{8}O_9) + BE({}^{15}_{8}O_7) - 2\cdot BE({}^{16}_{8}O_8)$$
$$= -11.519\,MeV$$

good prediction of
spin
parity $\pi = (-1)^{\ell}$
magnetic moment

Single-particle energies



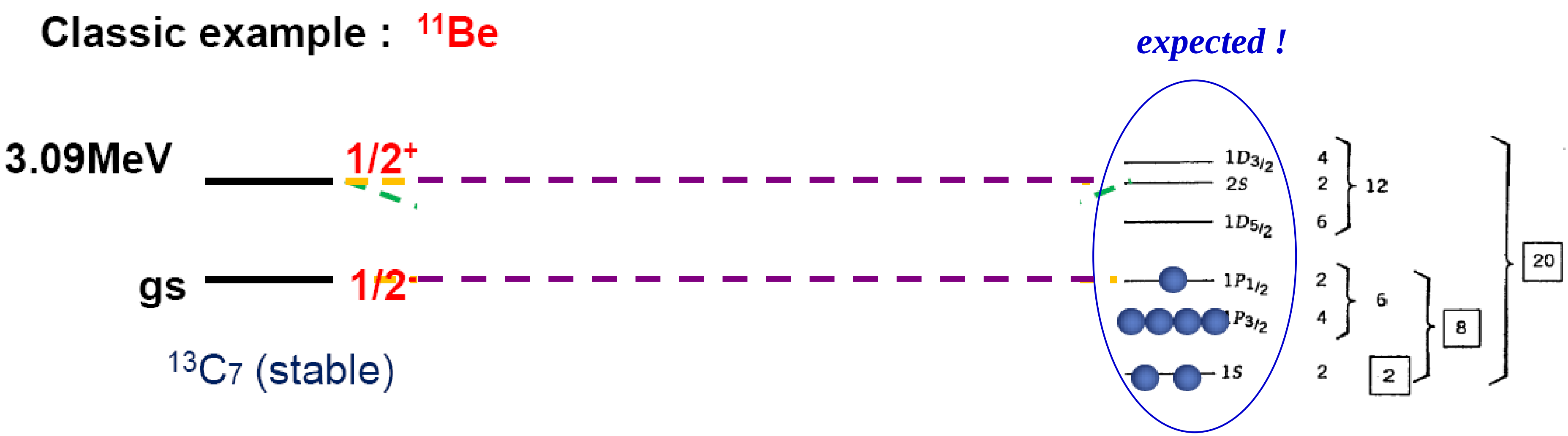
^{17}O: 1/2 $^-$ state already at 3.1 MeV

residual interaction is needed,
to reduce the gap between the shells

Single-particle states observed in odd-A nuclei (especially one nucleon + doubly magic nucleus as ^{4}He, ^{16}O, ^{40}Ca) are characterized by the single-particle energies in the shell model picture.

Classic example of anomaly

Several anomalies were observed in shell structures of exotic nuclei: proton-rich or neutron-rich



The $2s_{1/2}$ orbital (parity +) and the $1p_{1/2}$ orbital (parity -) are inverted ?? (***parity inversion***)

Formation of halos and the s-orbital

The s component in the ground state is essential for creating a halo structure.

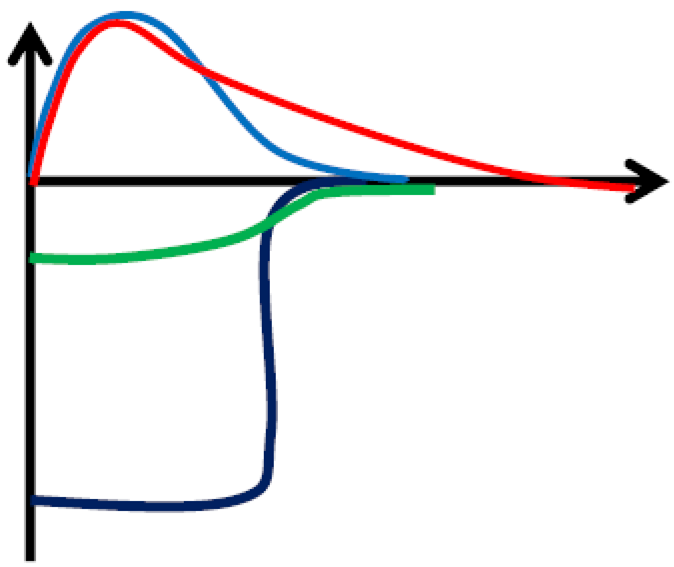
Schrödinger equation: $\left[-\frac{\hbar^2}{2\cdot\mu}\nabla^2 + V(r)\right]\Psi(r) = E\,\Psi(r)$ $\Psi(r) = u_{nl}(r)\cdot Y_{lm}(\vartheta,\varphi)$

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \left[\frac{2\cdot\mu}{\hbar^2}\left(E - V(r)\right) - \frac{\ell\cdot(\ell+1)}{r^2}\right]u(r)$$

centrifugal barrier ($\ell = 0$ for s-wave)

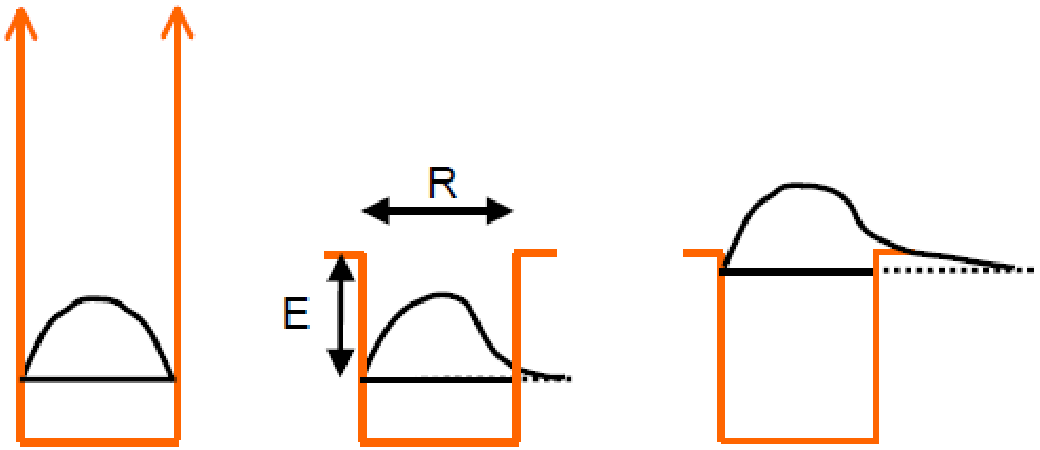
neutron-rich nuclei (^{11}Be, ^{11}Li)

→ instable: flat nuclear potential
→ the wave function is extended
→ for s-orbitals, the radial extension
is not blocked by the centrifugal barrier
(halo)



REMINDER

What can we expect at the neutron-dripline?



wave function outside of the potential

$$\Psi(r) \propto \frac{e^{-\kappa r}}{r}$$

$$\kappa^2 = \frac{2 \cdot \mu \cdot E}{\hbar^2} \approx 0.05 \cdot E(MeV) \quad [fm^{-2}]$$

$$\langle r^2 \rangle = \frac{\int r^4\, dr \left(e^{-\kappa \cdot r} / \kappa \cdot r\right)^2}{\int r^2\, dr \left(e^{-\kappa \cdot r} / \kappa \cdot r\right)^2}$$

The smaller the binding energy, the more extended is the wave function

$$\langle r^2 \rangle = \frac{1}{2 \cdot \kappa^2} \cdot (1 + \kappa \cdot R) \approx \frac{\hbar^2}{4 \cdot \mu \cdot S_n}$$

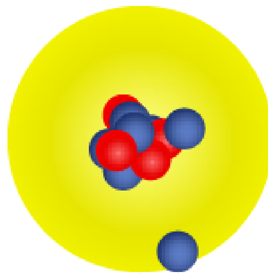
E	κ^2	κ	$1/\kappa \sim r$
7 MeV	0.35 fm^{-2}	0.6 fm^{-1}	1.7 fm
1 MeV	0.05 fm^{-2}	0.2 fm^{-1}	4.5 fm
0.1 MeV	0.005 fm^{-2}	0.07 fm^{-1}	14 fm

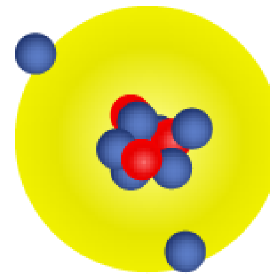
Halo nuclei

Anomalies of the shell structure was first observed in

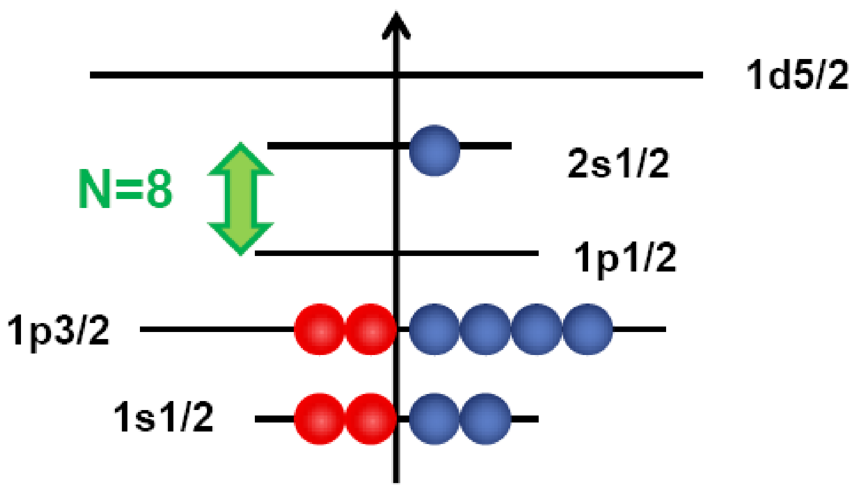
^{11}**Be** (Z=4, N=7) and ^{11}**Li** (Z=3, N=8) ,

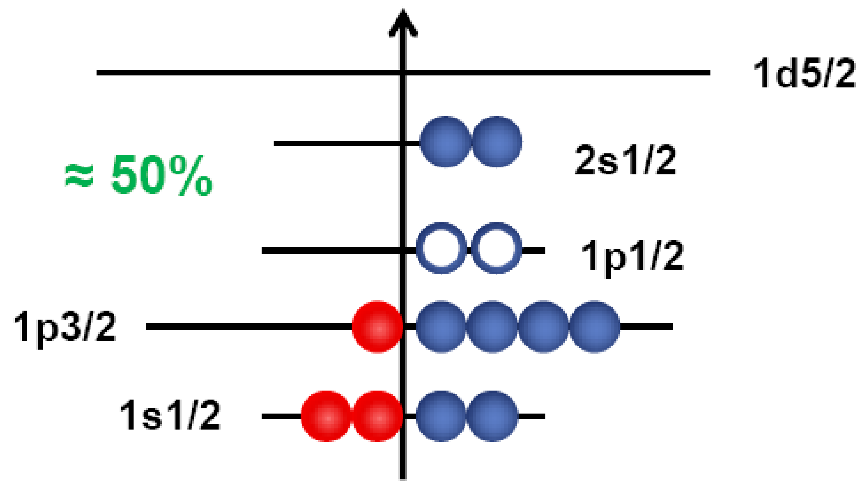
the most famous one-neutron halo and two-neutron halo-nuclei.



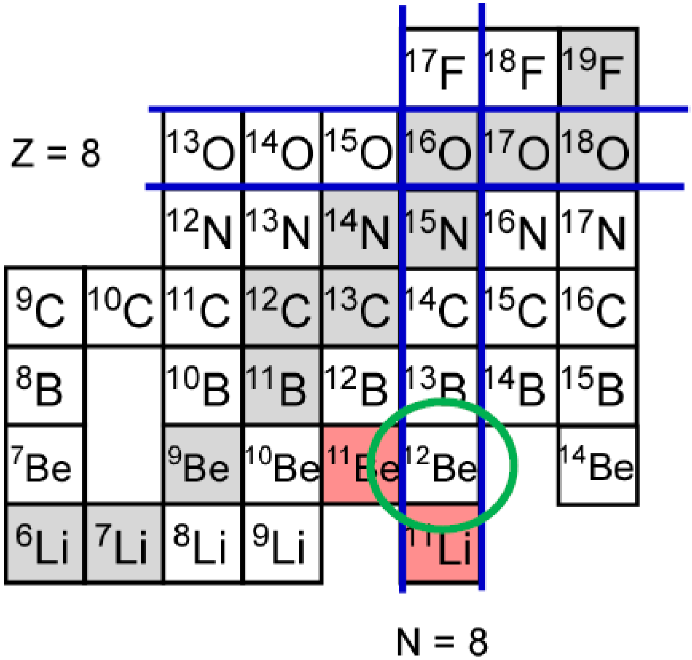


ground-state spin (J^{π}) $1/2^{+}$ $3/2^{-}$





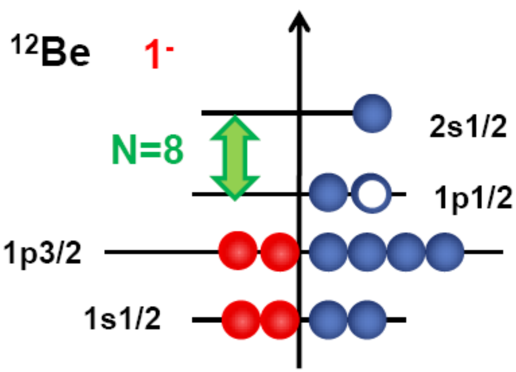
Change of the magic number near N=8: ^{12}Be

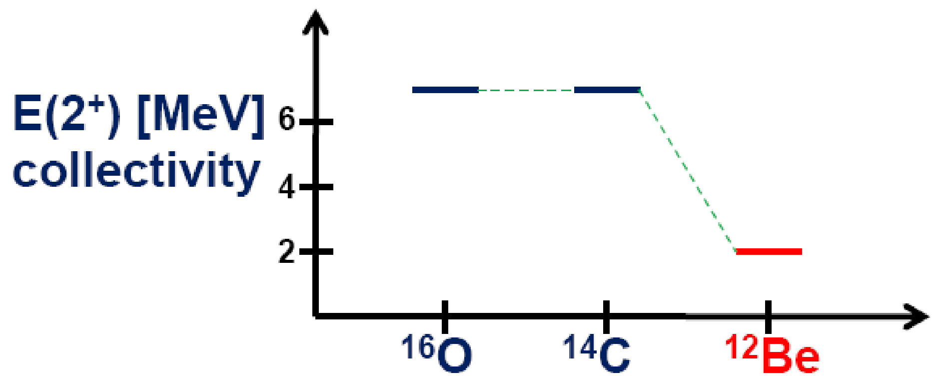


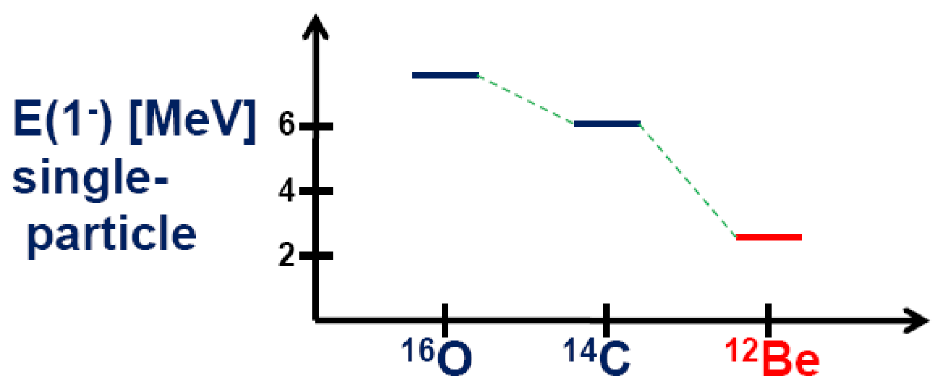
Is the magic number changed only in halo nuclei ?

No! It holds also for ^{12}Be.

This observation indicates a universal evolution of the shell structure.





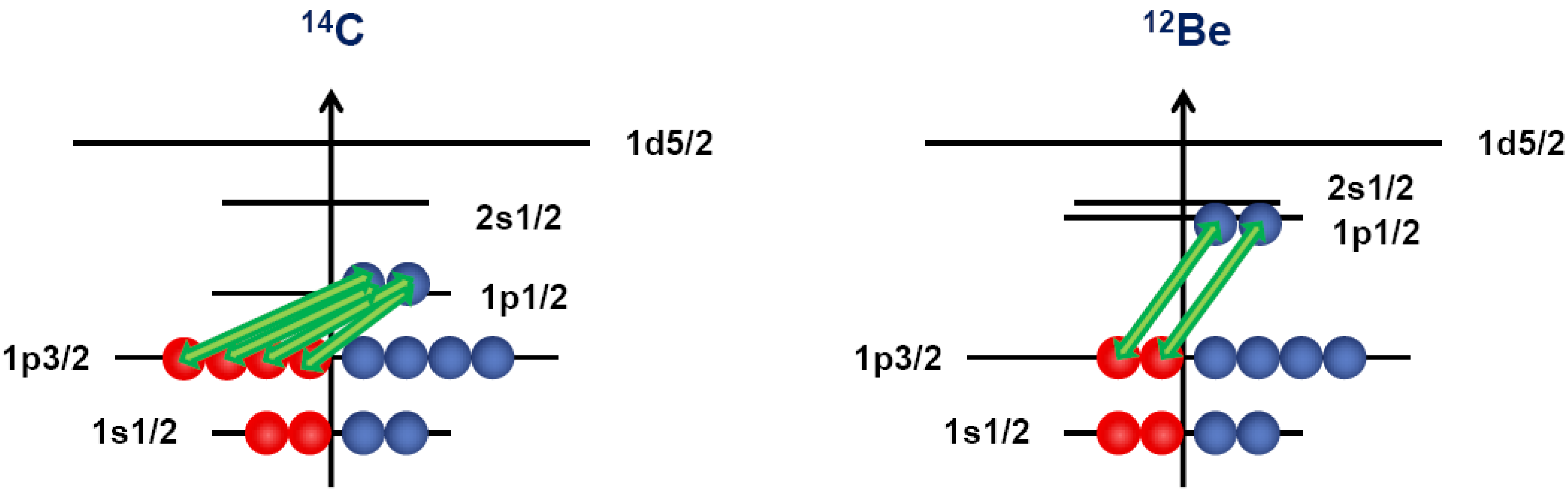


Simplified picture of monopole effects of the tensor force

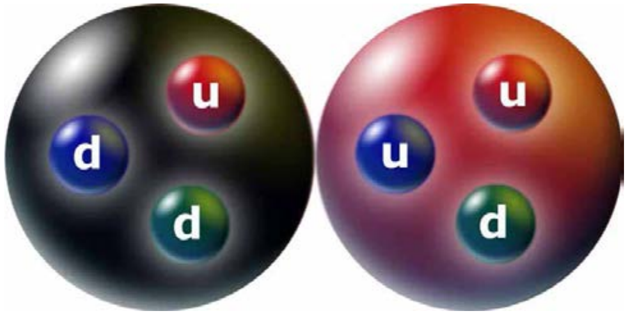
nucleon-nucleon residual interaction

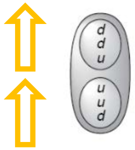
The specific proton-neutron interaction (**monopole term of the tensor-force**) can change the single-particle order, depending on the proton-neutron ratio of the nucleus.

The strong attractive p-n force between $J_>$ and $J_<$ orbitals ($\begin{cases} >: \ \ell + 1/2 \\ <: \ \ell - 1/2 \end{cases}$ for example, $\pi\ p_{3/2}$ and $\nu\ p_{1/2}$)



The deuteron



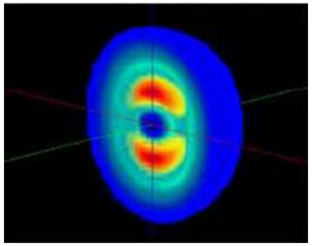
mass (MeV/c^2)	1875.61	
charge (e)	1	
I^π	1^+	 n p
binding energy (MeV)	2.2245	
magnetic moment (μ_N)	0.8574	0.8798 $\mu_N = \mu_S\,({}^{2}_{1}H)$ → the deuteron can not be a pure ***s*** state! ~ 96% s and 4% d.
quadrupole moment (b)	0.0029	not spherical consistent with s/d-ratio = 96/4

The deuteron is an ideal candidate for tests of our basic understanding of nuclear physics

Deuteron: quadrupole moment

- The measured ***nuclear spin*** of the deuteron is ***J = 1***
- The ***parity*** of the deuteron is ***positive*****,** only even ***angular momenta ℓ = 0*** and ***ℓ = 2*** .
- The ***magnetic moment*** of the deuteron yields to $\mu = 0.8574 \cdot \mu_K$ The angular momentum has to **4%** the value of ***ℓ = 2***
- The deuteron is ***not spherical***.

  It has an experimentally determined ***quadrupole moment*** of **Q = 0.00282 eb.**



A free neutron and a free proton have no electric quadrupole moment.

The deuteron can only possess a quadrupole moment because of its angular momentum of ***ℓ = 2***.

$$Q_{zz} = \int \rho(\vec{r}) \cdot r^2 \cdot (3 \cdot cos^2\theta - 1) d\tau$$

A pure $\ell = 0$ wave function has a vanishing quadrupole moment, because of its rotational symmetry.

The nuclear force is spin dependent !

The nuclear forces must raise a torsional moment, that depend on the radius r and the angle θ .

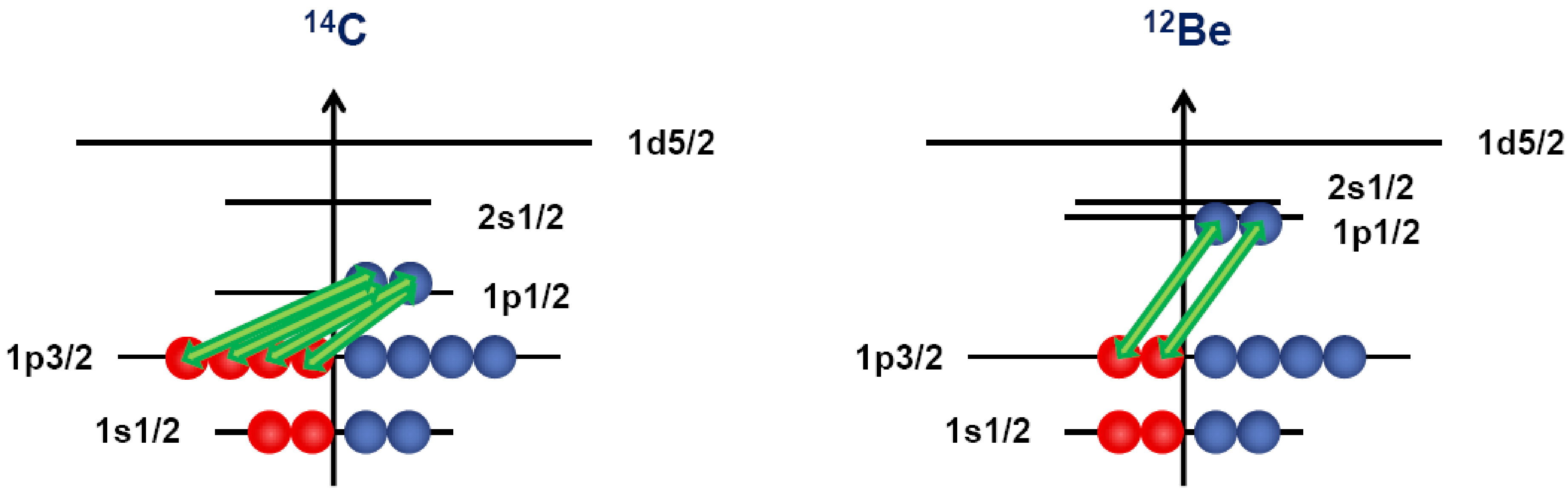
If the nuclear force depends on r and θ , then there is a non-central force component a Tensor force

Simplified picture of monopole effects of the tensor force

nucleon-nucleon residual interaction

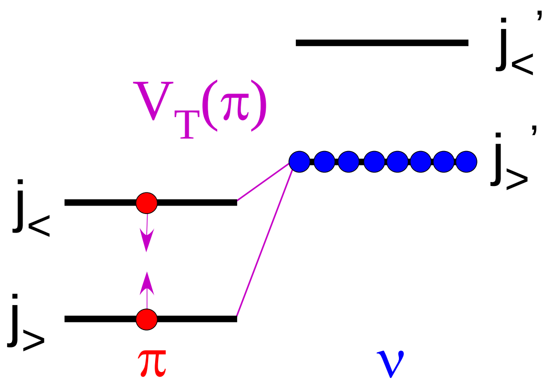
The example shows the proton configuration ($0p_{3/2}$) of $^{14}C_8$. The more protons are in $0p_{3/2}$ orbital, the more the $0p_{1/2}$ neutron orbital will be attracted and the shell closure at N=8 develops.

For $^{12}Be_8$ the proton orbital $0p_{3/2}$ will be emptied, the interaction is weaker and the neutron orbital $0p_{1/2}$ will be lifted.

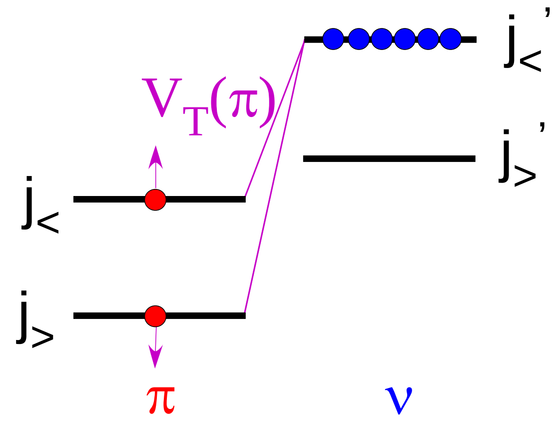


T. Otsuka et al., Phys. Rev. Lett. 95, 232502 (2005), Phys. Rev. Lett. 97, 162501 (2006)

The effect of the tensor force on the ℓs-coupling



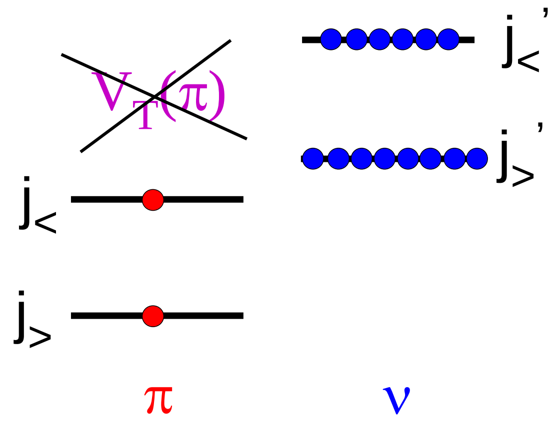
V_T reduces ls-splitting.



V_T enhances ls-splitting.

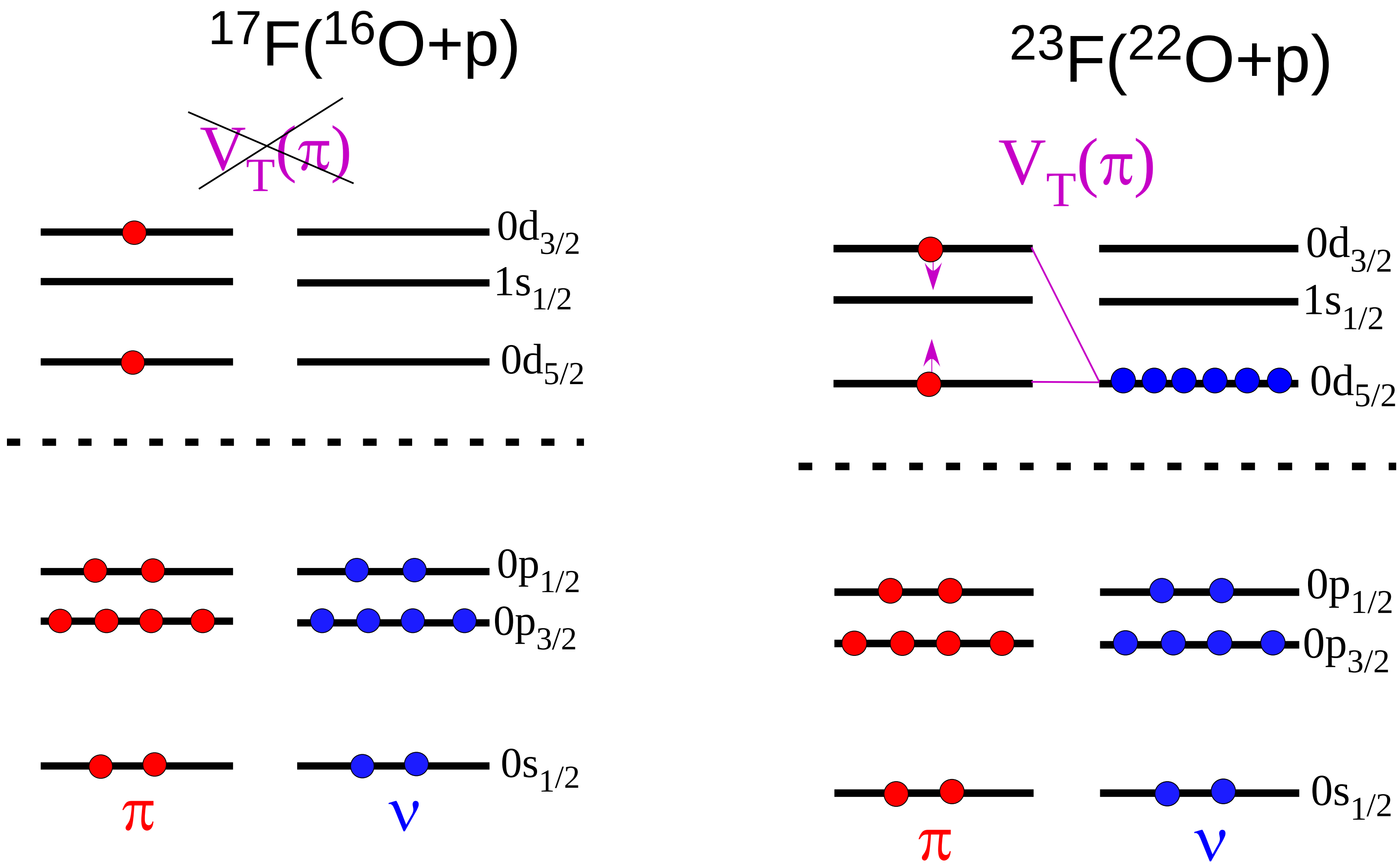
1. $j_<$ - $j_<$' or $j_<$ - $j_<$': repulsion
2. $j_<$ - $j_>$': attraction
3. If both $j_<$' and $j_>$' orbits are fully occupied the tensor force does not act.

cf. Bouyssy et al. PRC **36** (1987) 380
Otsuka et al. PRL **95** (2005) 232502



V_T does not act.

The effect of the tensor force on the ℓs-coupling



The tensor force does not act

The tensor force reduces the ℓs-splitting

Michimasa et al.
(from NPA **787** (2007) 569)

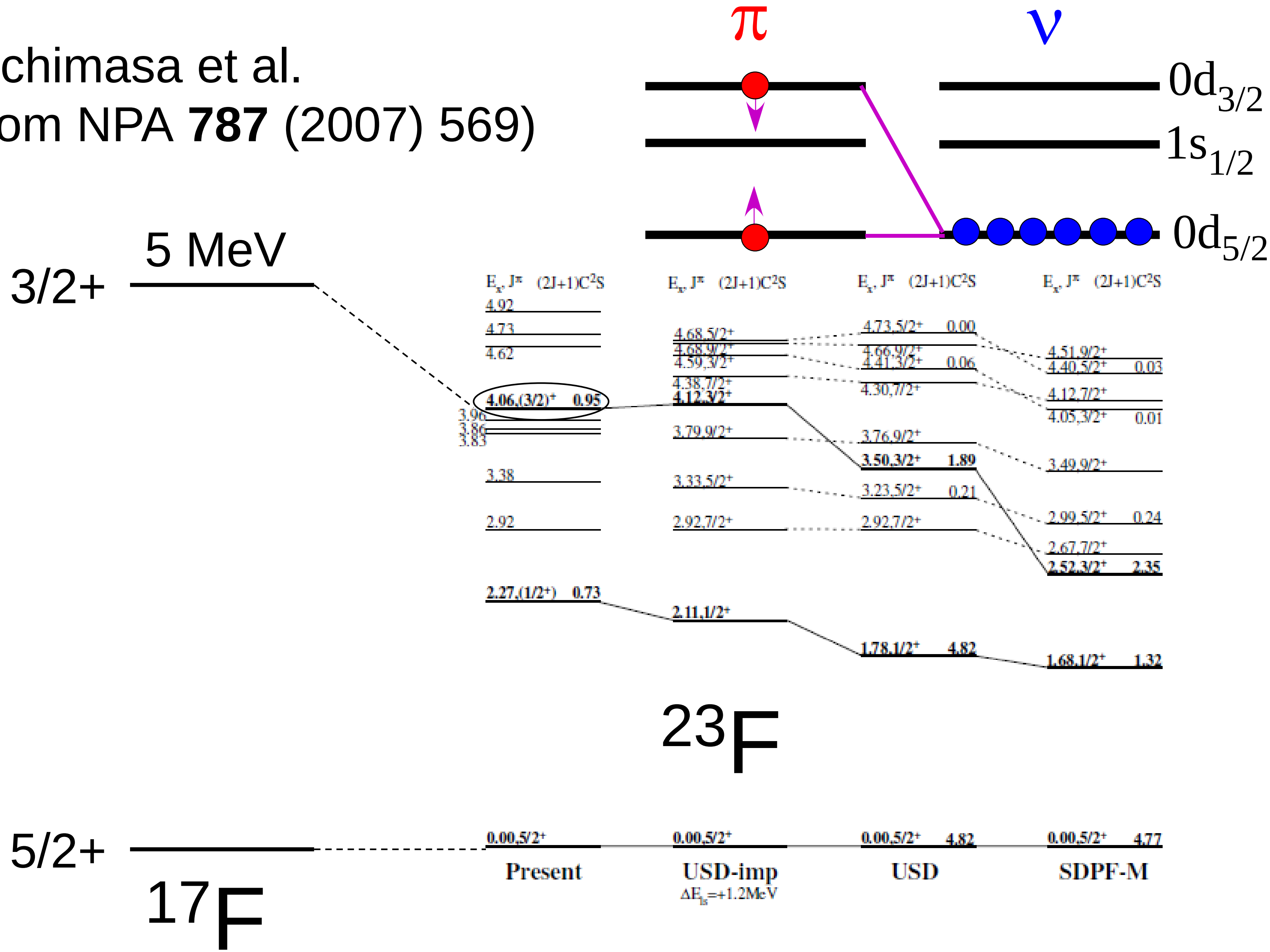


Figure 3. Level schemes in ^{23}F, together with the shell model calculations using USD [9] and SDPF-M [10] effective interactions. The level scheme labeled by USD-imp was obtained based on the USD by widening the single-particle energy gap between $\pi d_{5/2}$ and $\pi d_{3/2}$ by +1.2 MeV.

Bohr & Mottelson vol. 1

β-DECAY SCHEMES OF VERY NEUTRON-RICH SODIUM ISOTOPES AND THEIR DESCENDANTS

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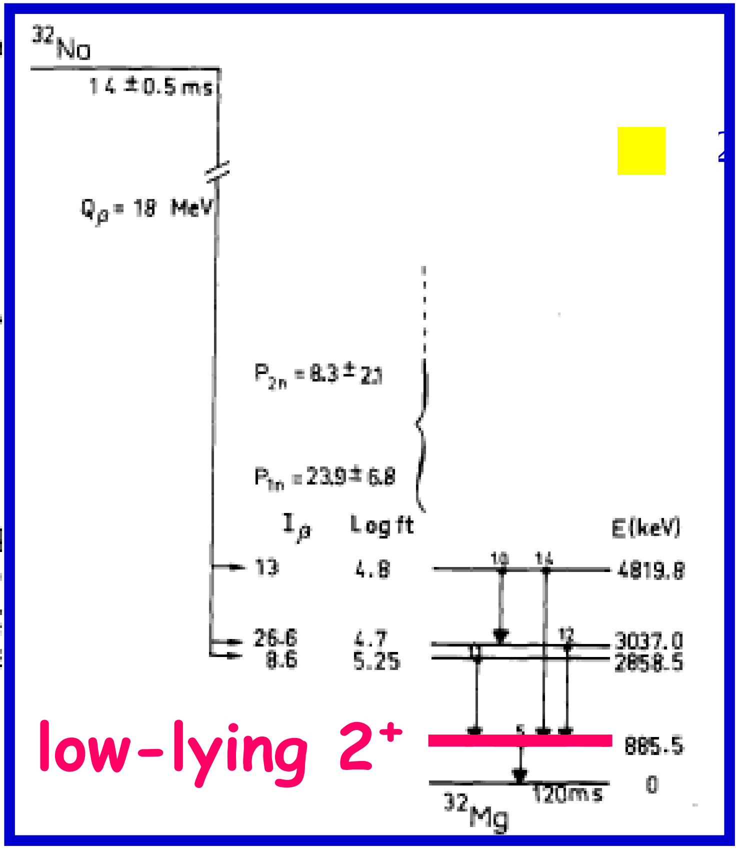
Abstract: The γ-activities from the β-decay of Na isotopes up
fragmentation and analysed through mass-spectrometry techniq
from their Mg descendants. The I_γ intensities, the β-delayed o
and the I_β intensities are measured. Decay schemes are prop
location of the first 2^+ level of ^{32}Mg, the occurrence of a nucle



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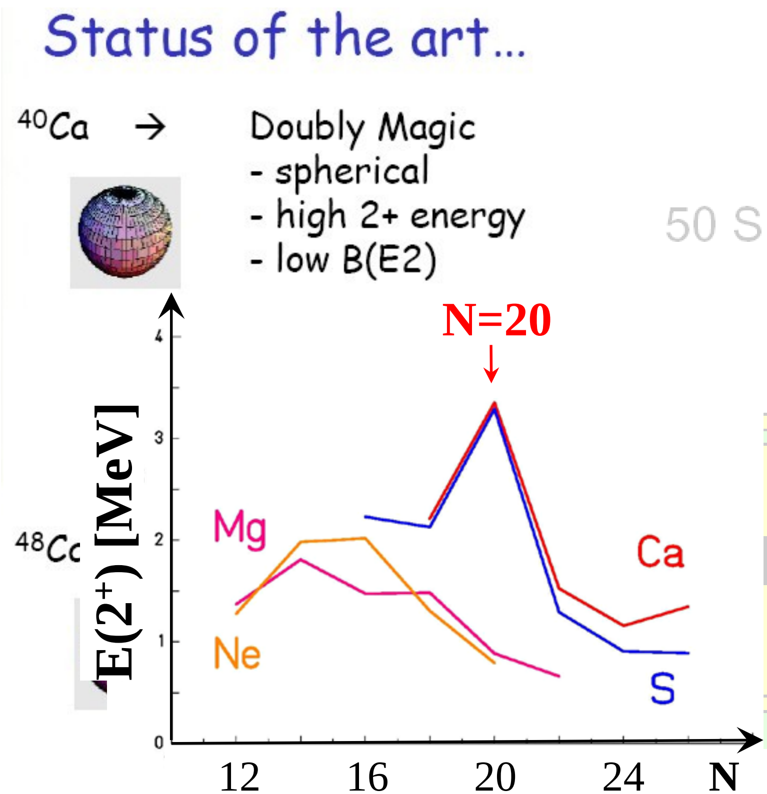
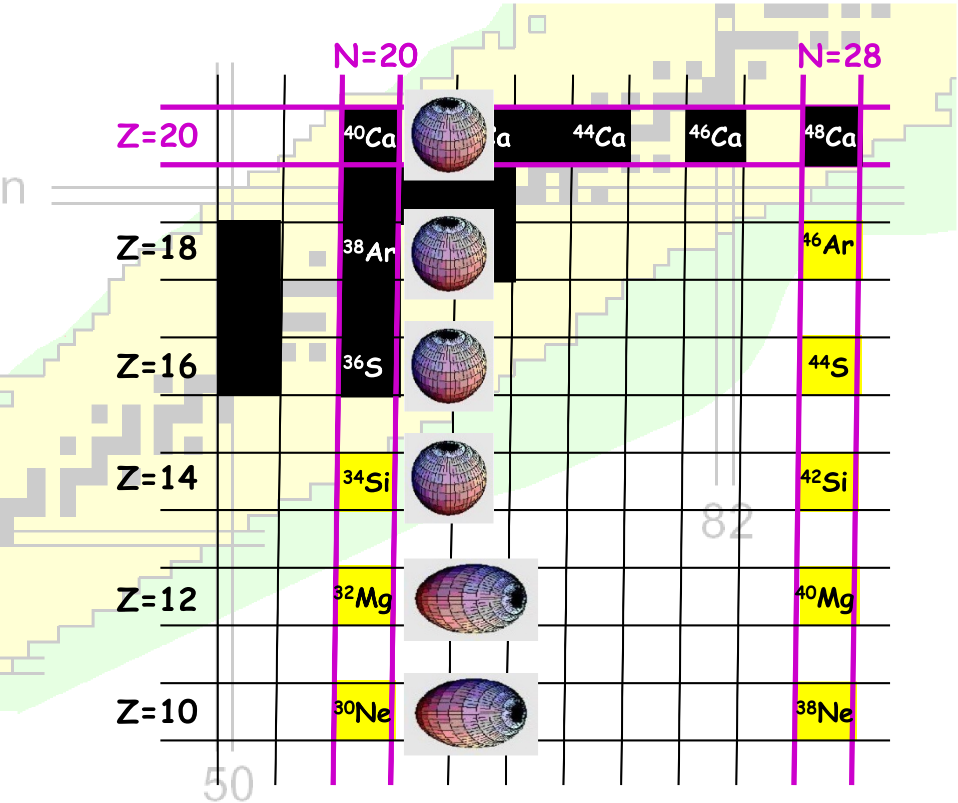




Evidence of the nuclear shell model:

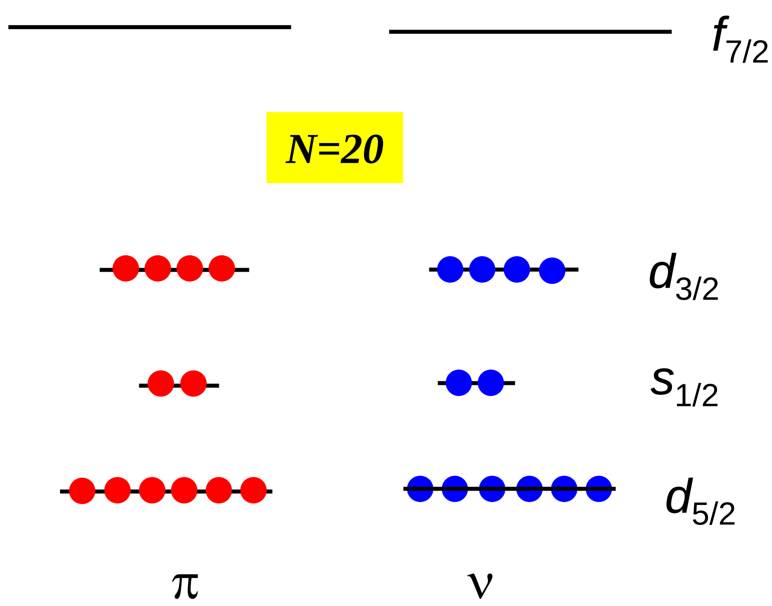
high energies of the 2_1^+ states

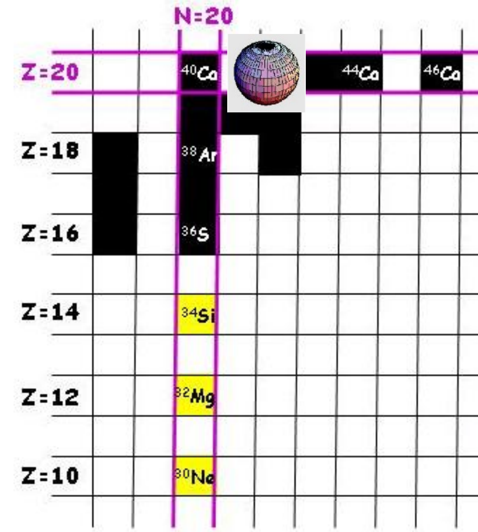
for nuclei with magic numbers

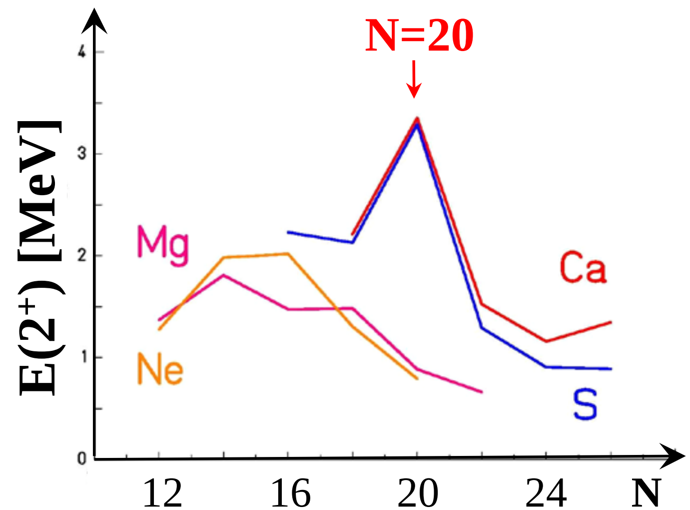
Monopole-interaction of the tensor force

Experimental evidence of the magic number **N=20**

$^{40}_{20}Ca_{20}$



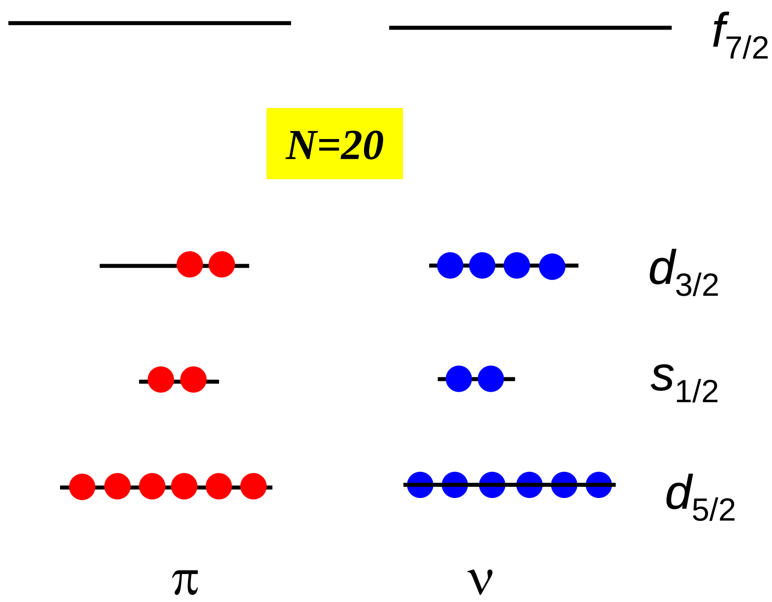


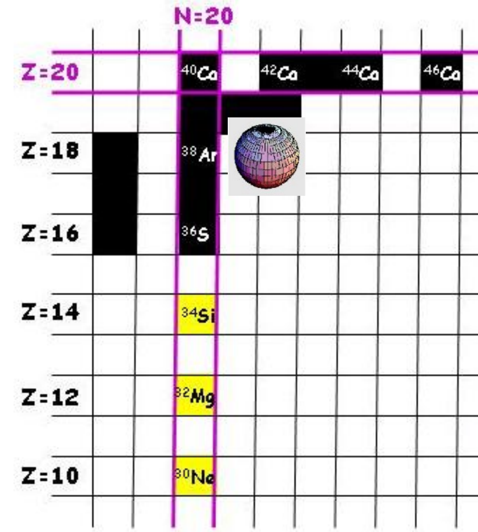


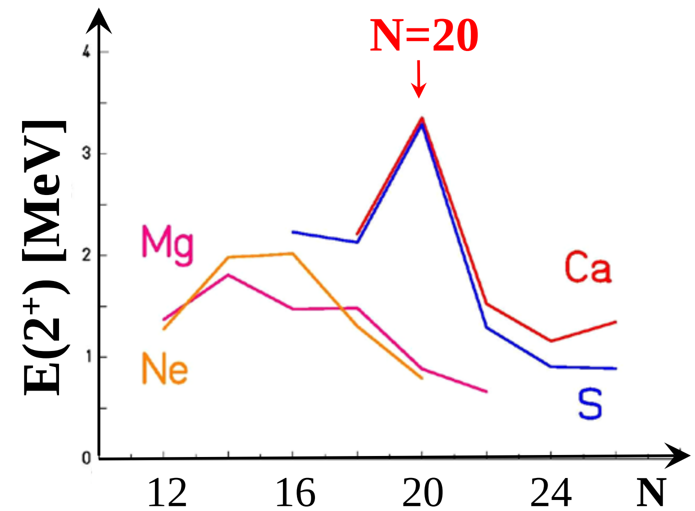
Monopole-interaction of the tensor force

Experimental evidence of the magic number **N=20**

$^{38}_{18}Ar_{20}$



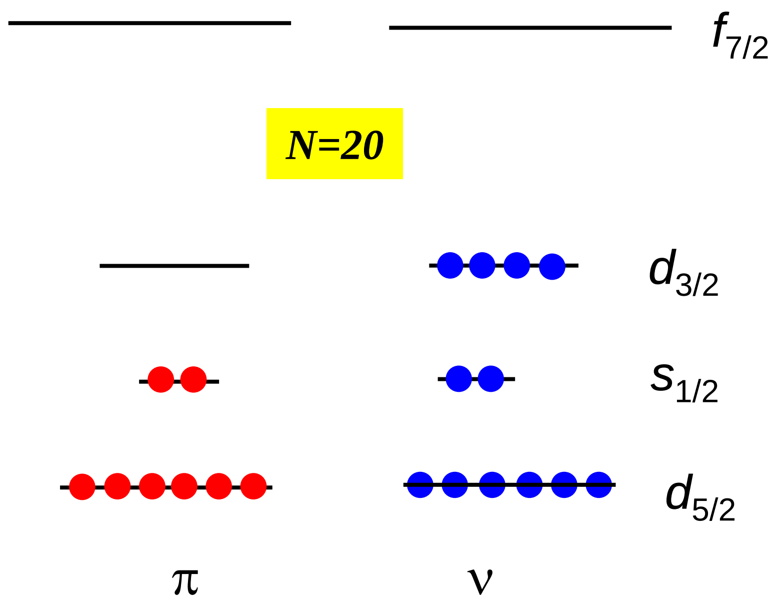


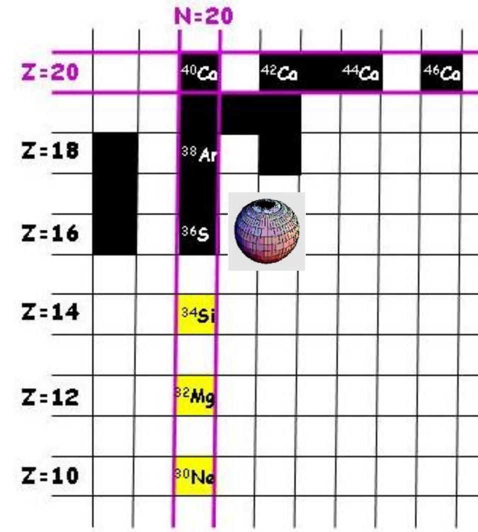


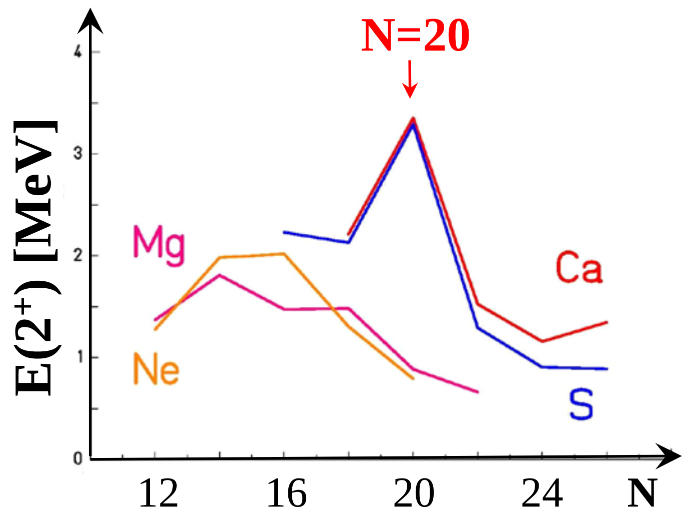
Monopole-interaction of the tensor force

Experimental evidence of the magic number **N=20**

$^{36}_{16}S_{20}$



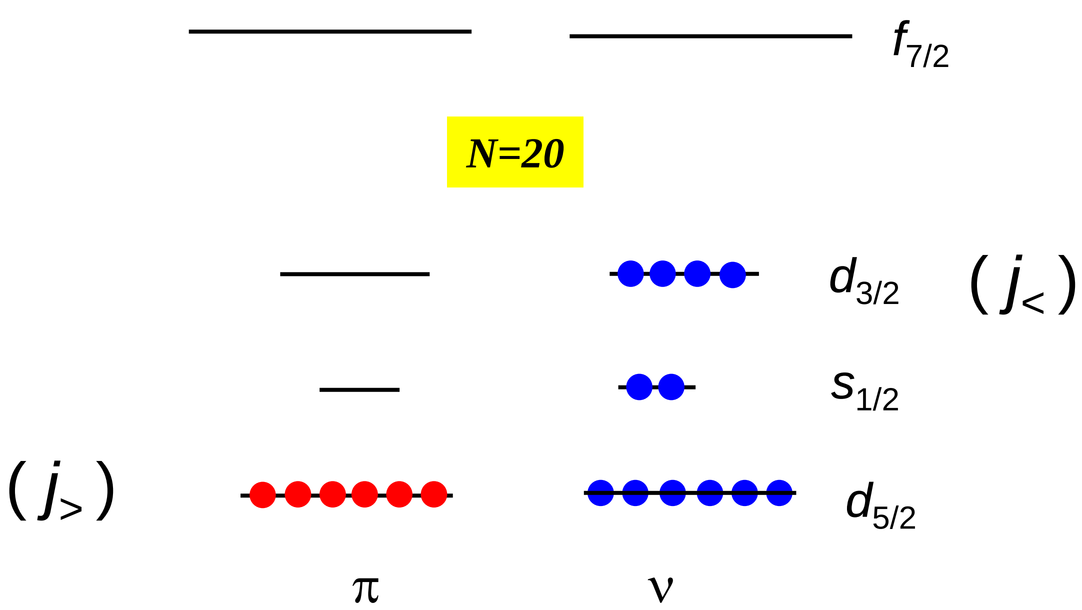


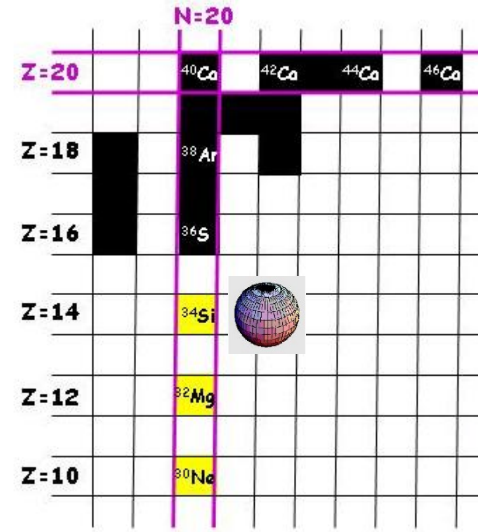


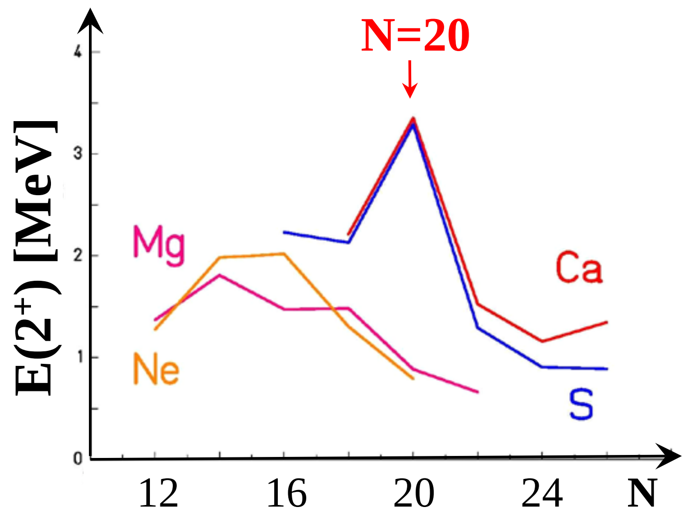
Monopole-interaction of the tensor force

Experimental evidence of the magic number ***N=20***

$^{34}_{14}Si_{20}$



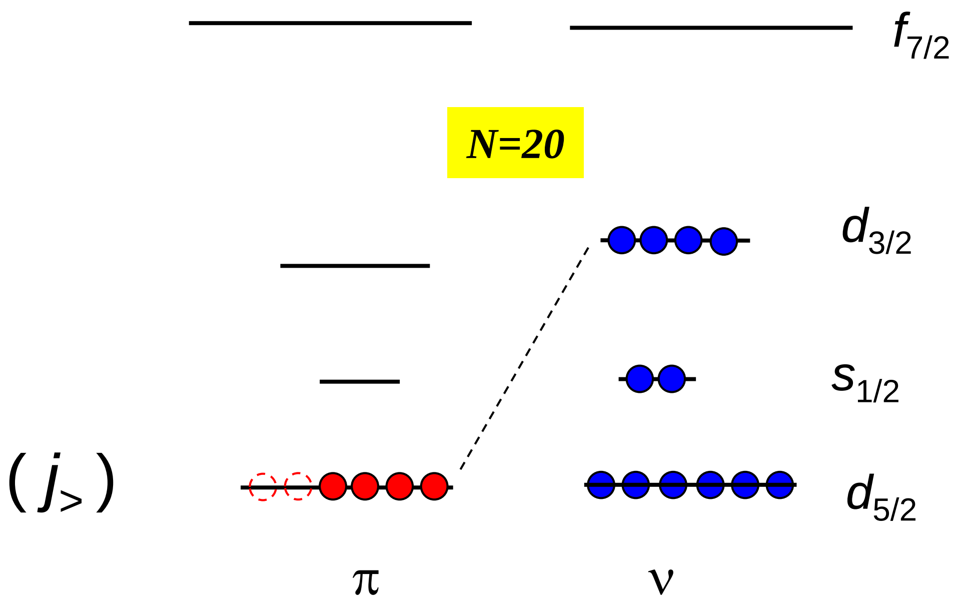


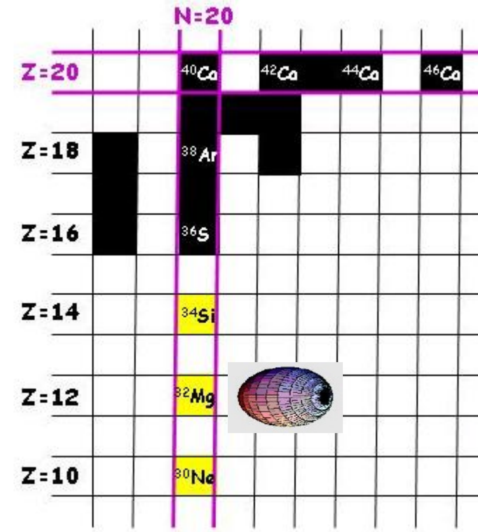


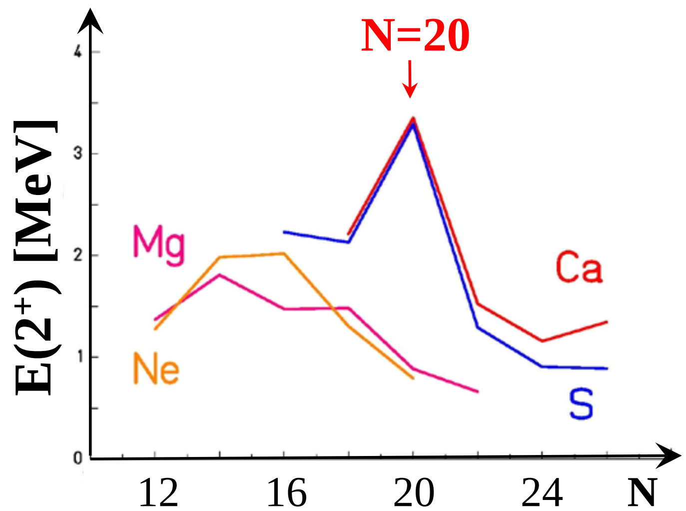
Monopole-interaction of the tensor force

Experimental evidence of the magic number **N=20**

$^{32}_{12}Mg_{20}$



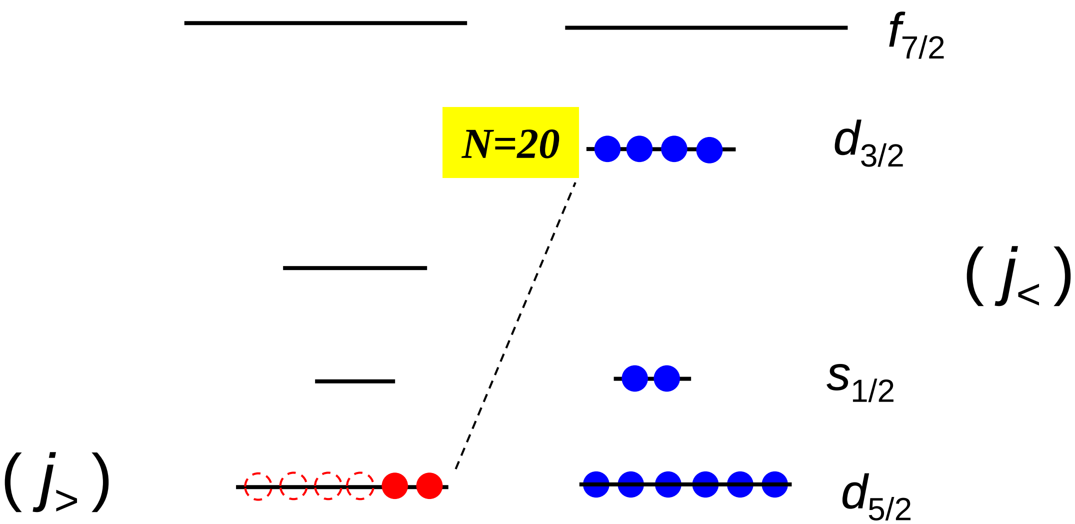


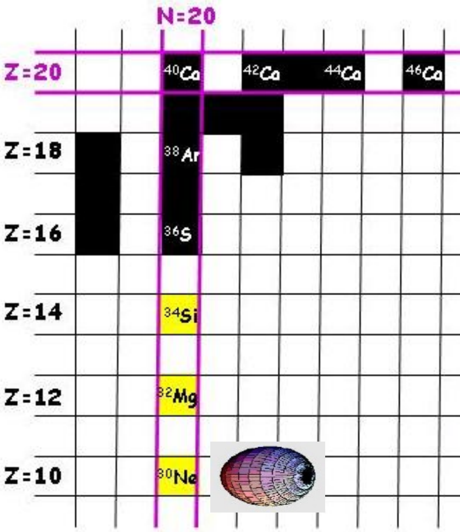


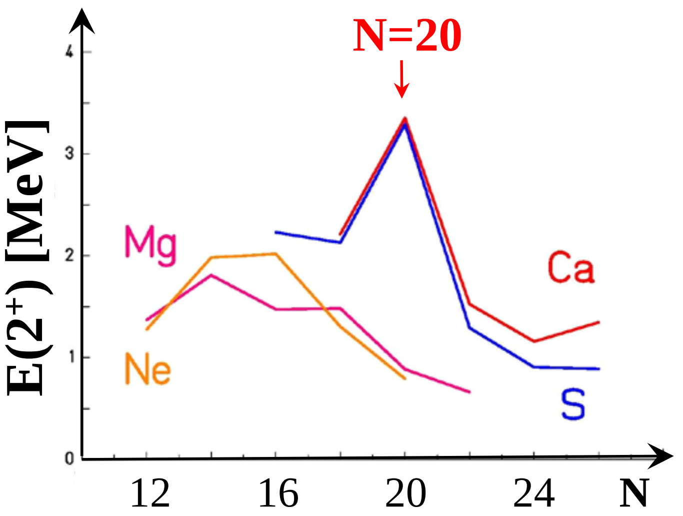
Monopole-interaction of the tensor force

Experimental evidence of the magic number **N=20**

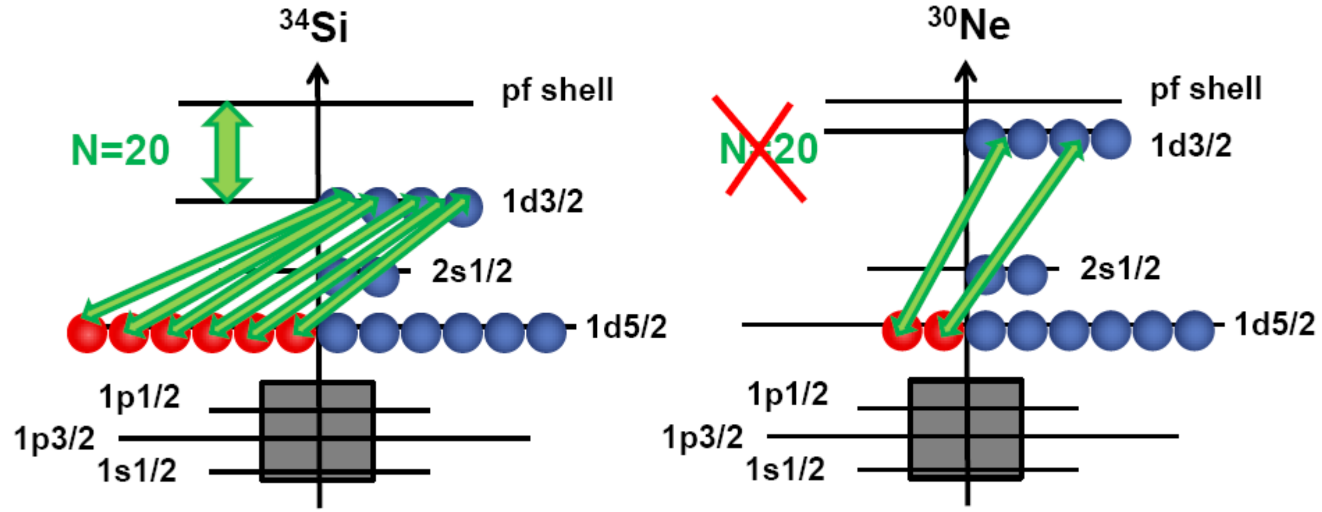
$^{30}_{10}Ne_{20}$

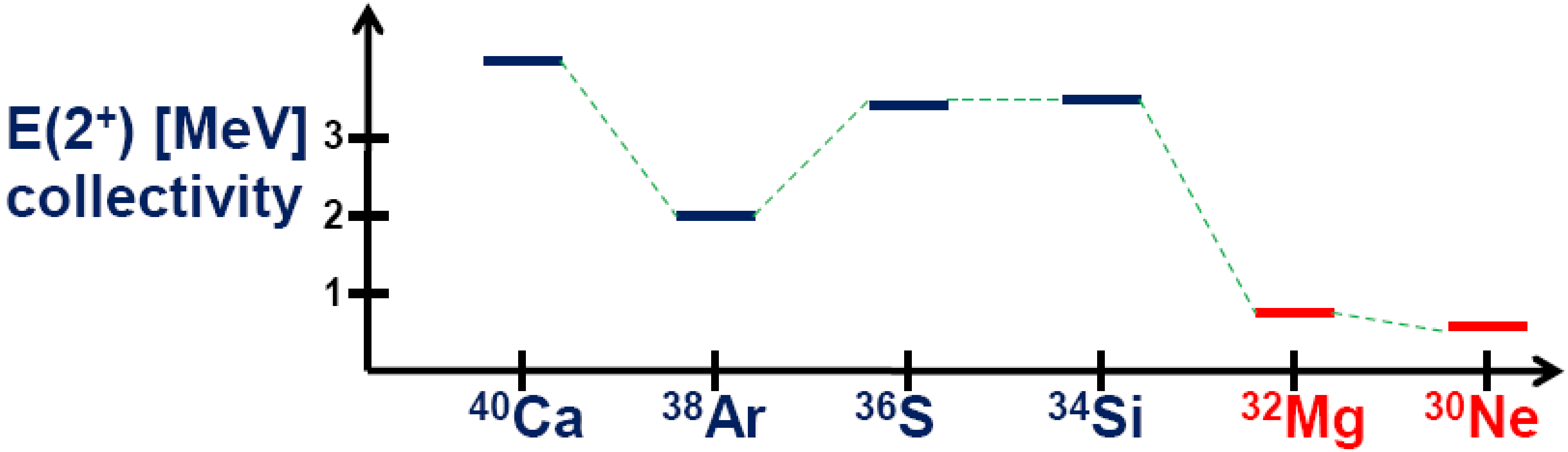




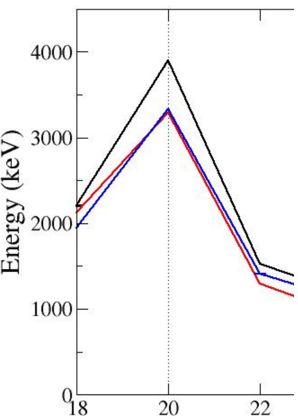


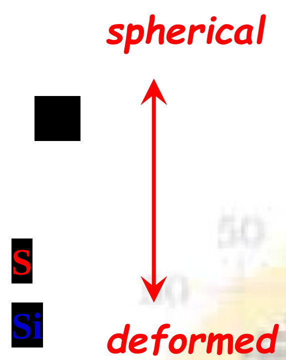
The shell structure is strongly influenced by the attractive p-n force between $J_>$ and $J_<$ orbitals ($\pi\ d_{5/2}$ and $\nu\ d_{3/2}$).

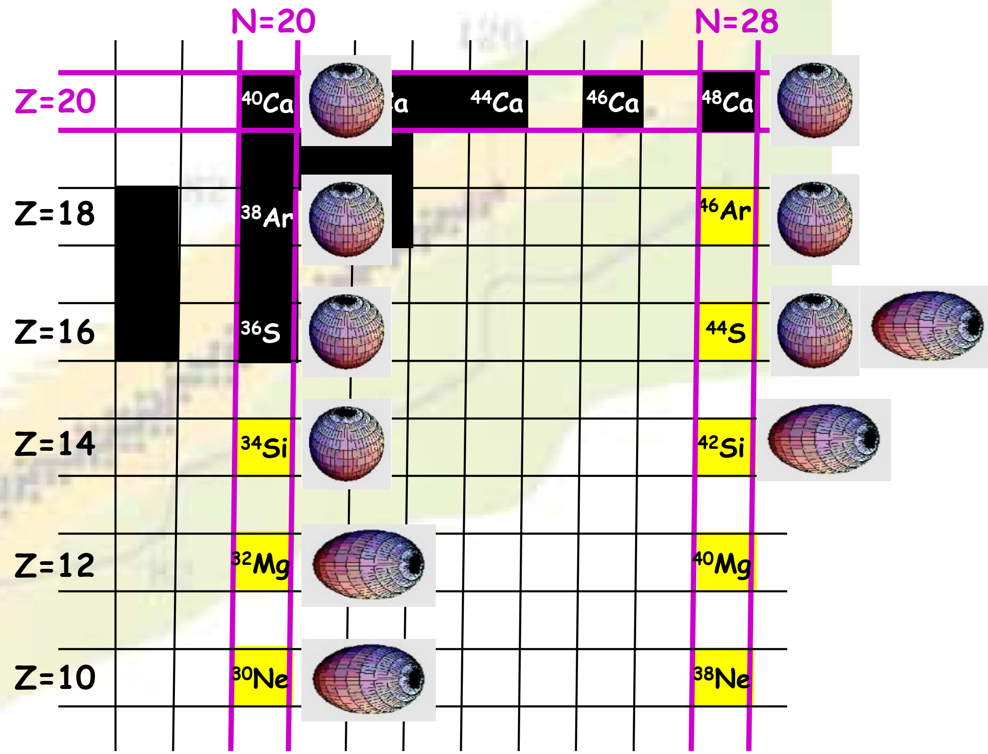




N=28







Evidence of the nuclear shell model:

high energies of the 2_1^+ state

for nuclei with magic number

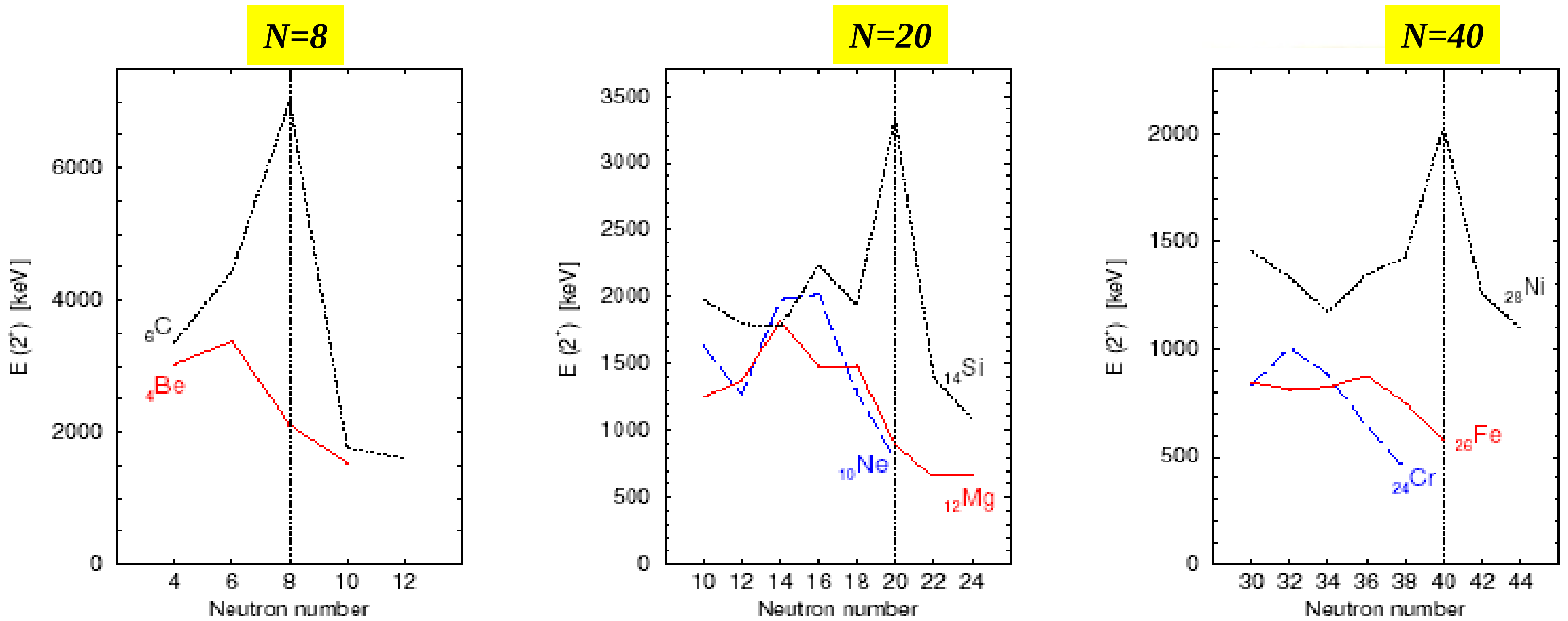
Nuclear field theory:

Nuclear many-particle problem will be solved relativistically
with the consequence: attractive scalar field (S-V)
repulsive vector field (S+V)

Relativistic quasi-particle random phase approximation

Nuclear shell structure

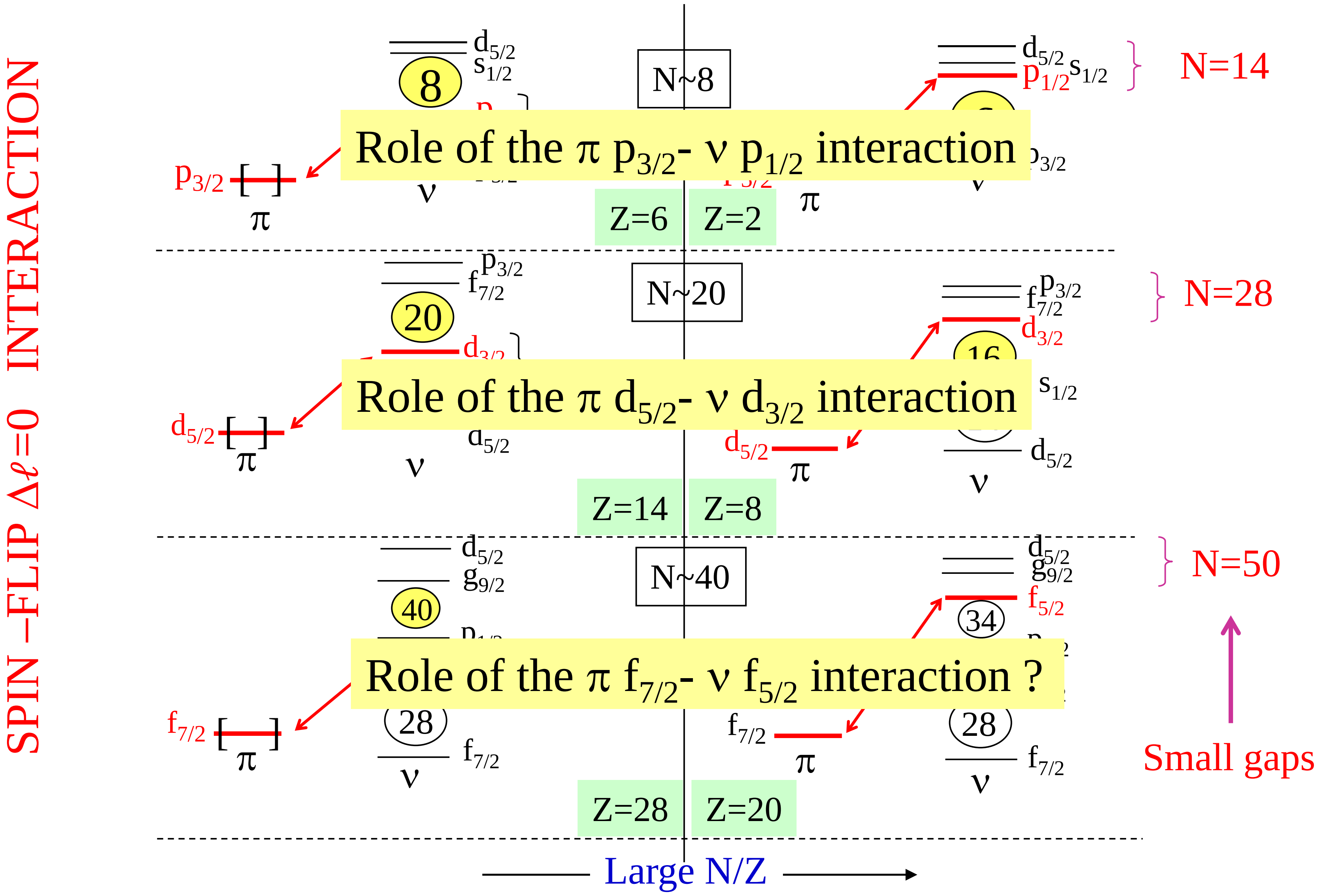
Large similarity between three numbers of the HO-shell model



O. S. , MG Porquet PPNP (2008)

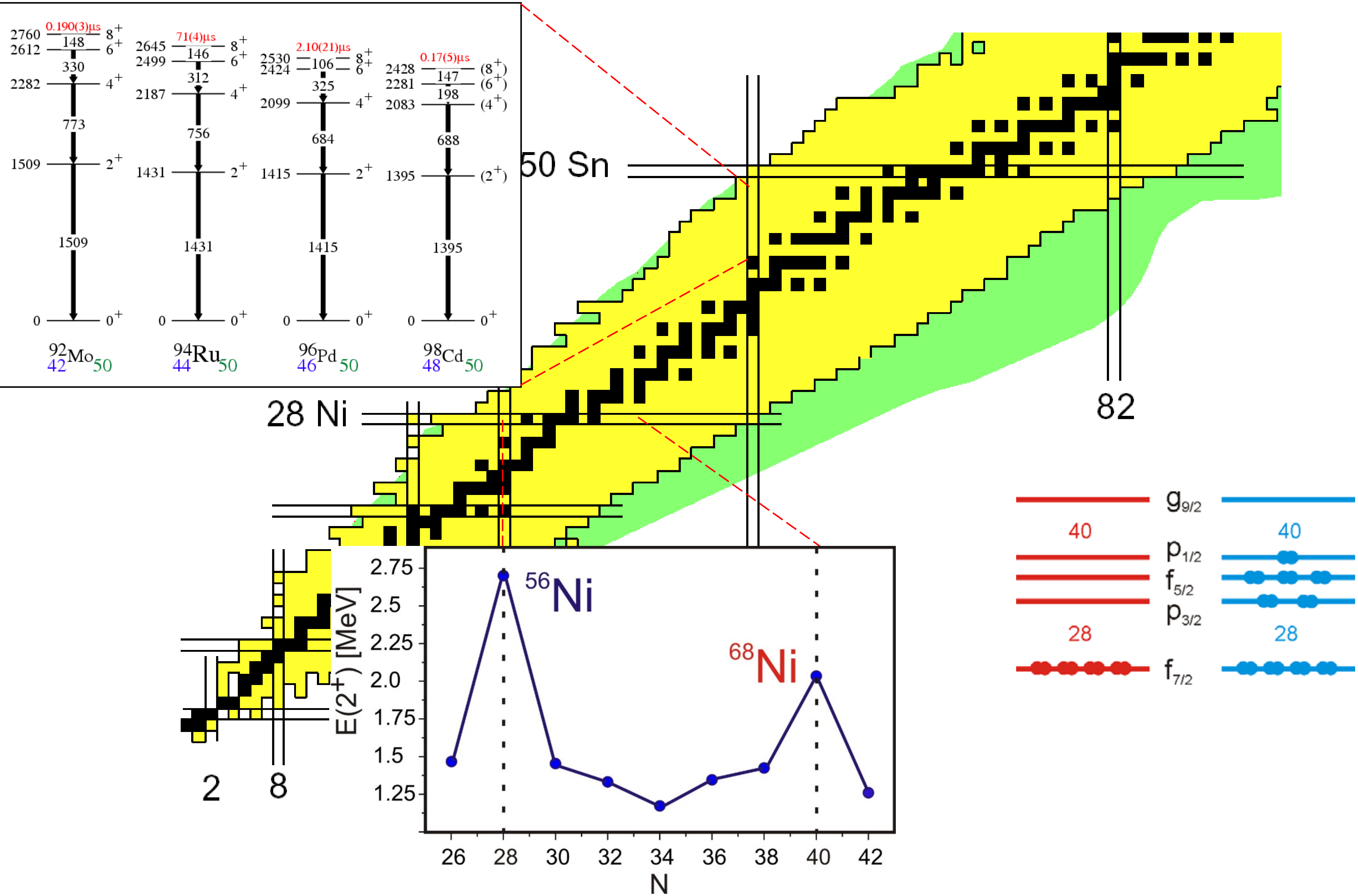
Same mechanism :

- small 2^+ energies for N=8, 20 and 40
- Inversion between normal and intruder states for N=40
- Search for a (super)deformed 0^+_2 state in ^{68}Ni
- Proof the extreme deformation of ^{64}Cr

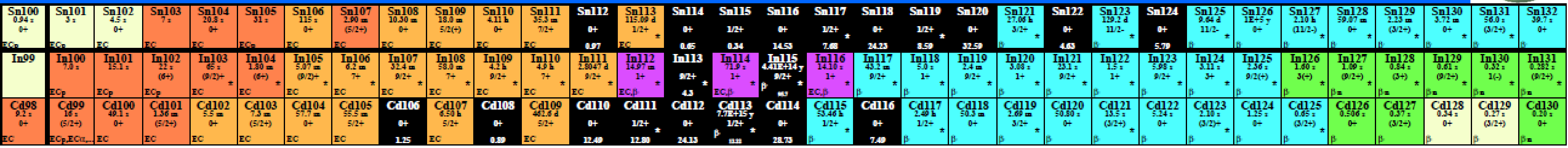


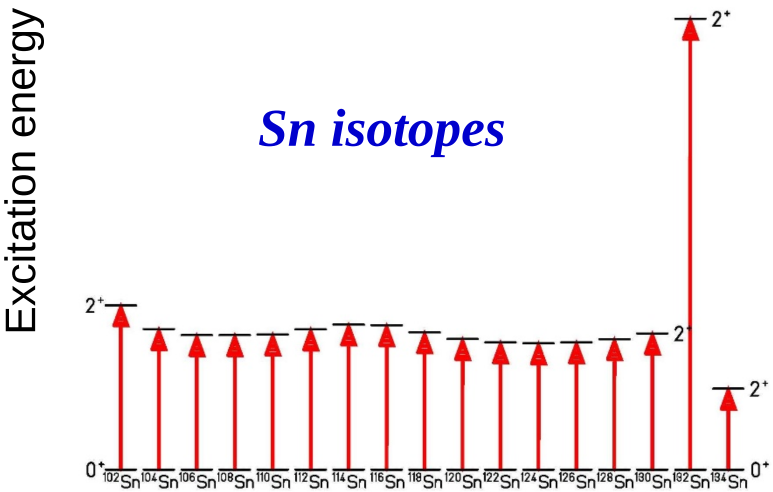
SPIN –FLIP Δℓ=0 INTERACTION
N~8
Role of the π $p_{3/2}$- ν $p_{1/2}$ interaction
Z=6
Z=2
N=14
N~20
Role of the π $d_{5/2}$- ν $d_{3/2}$ interaction
Z=14
Z=8
N=28
N~40
Role of the π $f_{7/2}$- ν $f_{5/2}$ interaction ?
Z=28
Z=20
N=50
Small gaps
Large N/Z

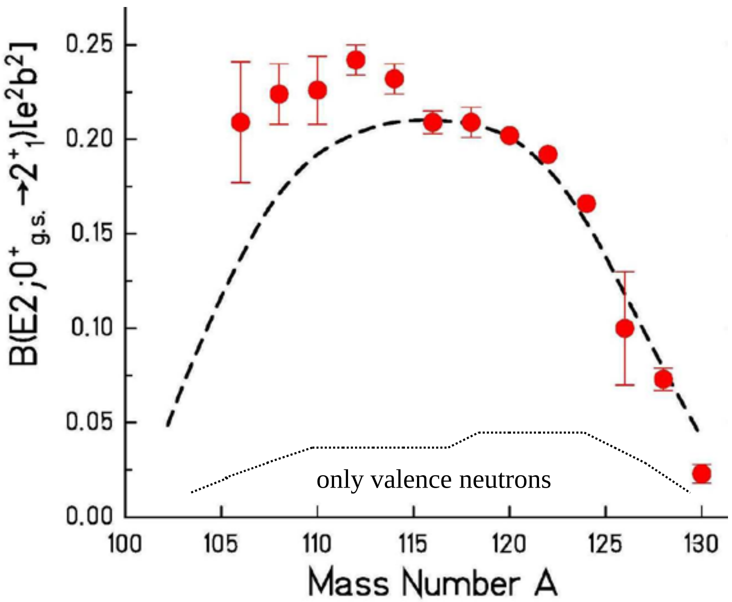
Nuclear shell structure

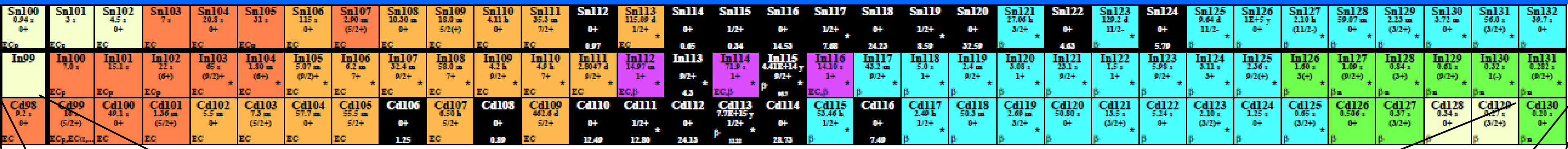


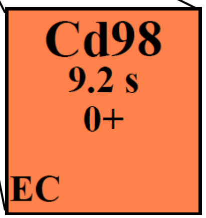
Signatures near closed shells

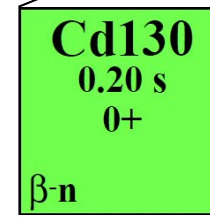




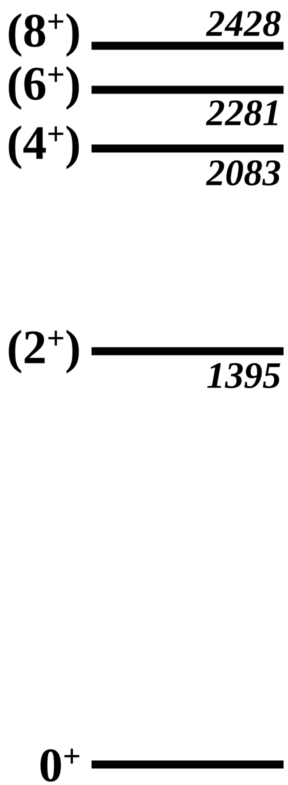


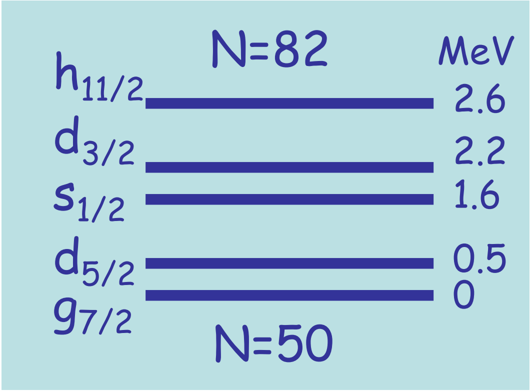




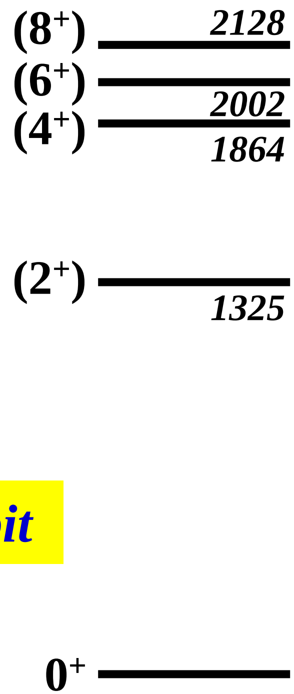


N=50
Z=48





participating neutron-orbitals



N=82
Z=48

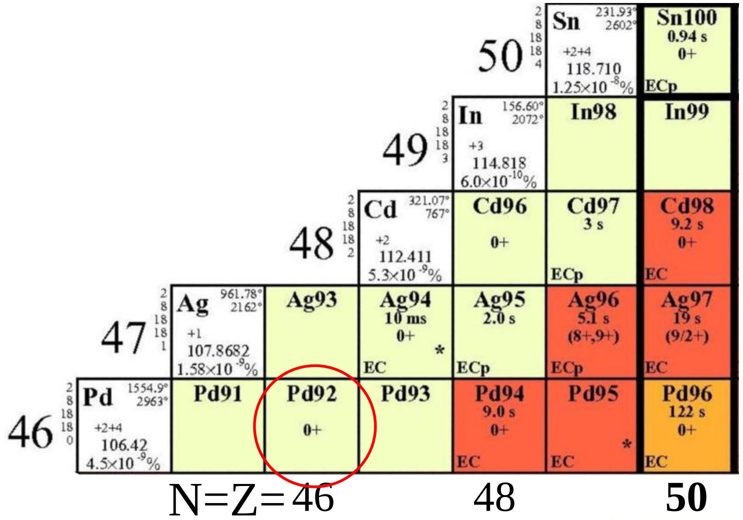
two proton holes in the $g_{9/2}$ orbit

No dramatic shell quenching!

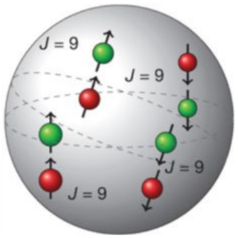
A. Blazhev et al., Phys. Rev. C69 (2004) 064304

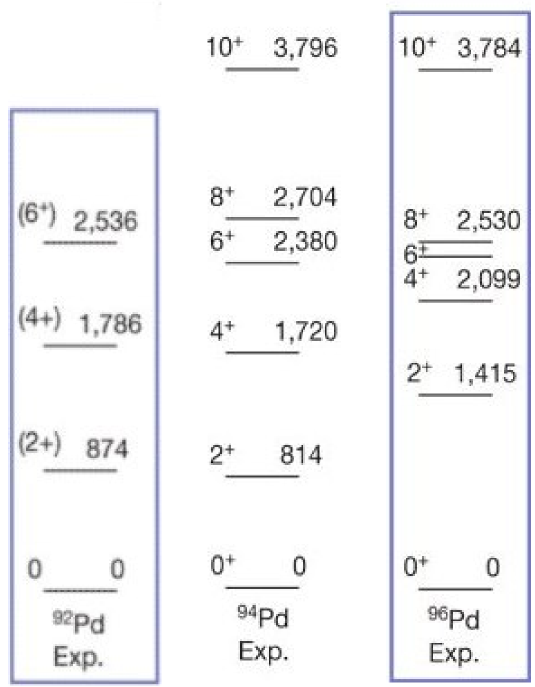
A. Jungclaus et al., Phys. Rev. Lett. 99 (2007), 132501

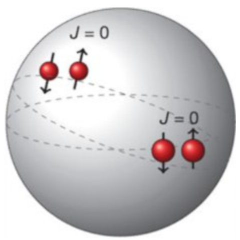
Isoscalar neutron-proton pairing in ^{92}Pd



four proton holes in $g_{9/2}$ orbit





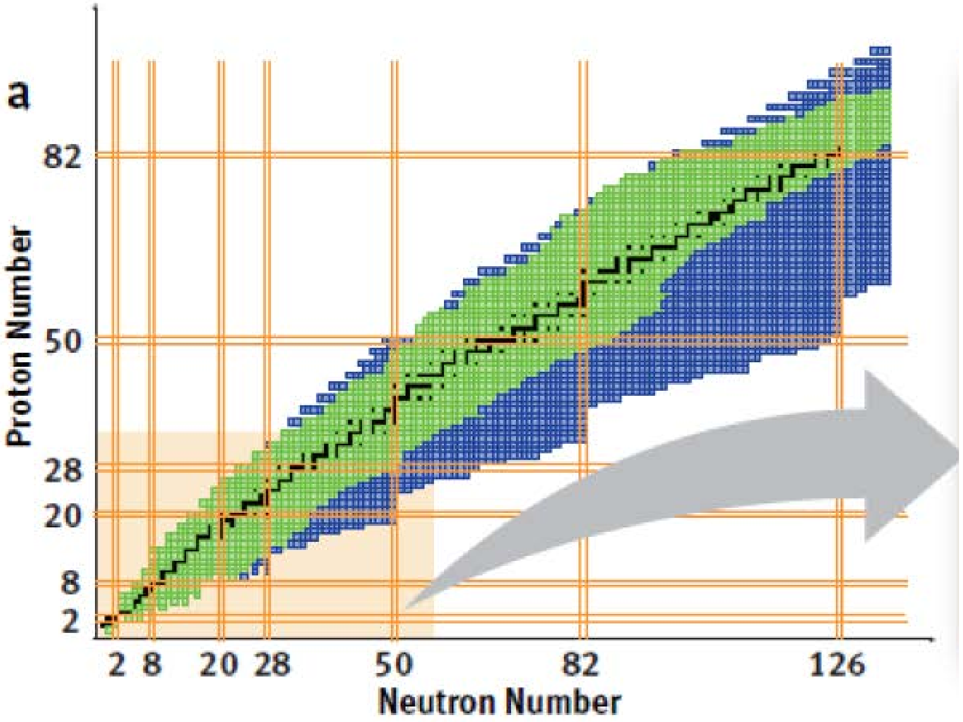


J_{max}=12

results reveal evidence for a spin-aligned, isoscalar neutron-proton coupling scheme and replaces seniority coupling

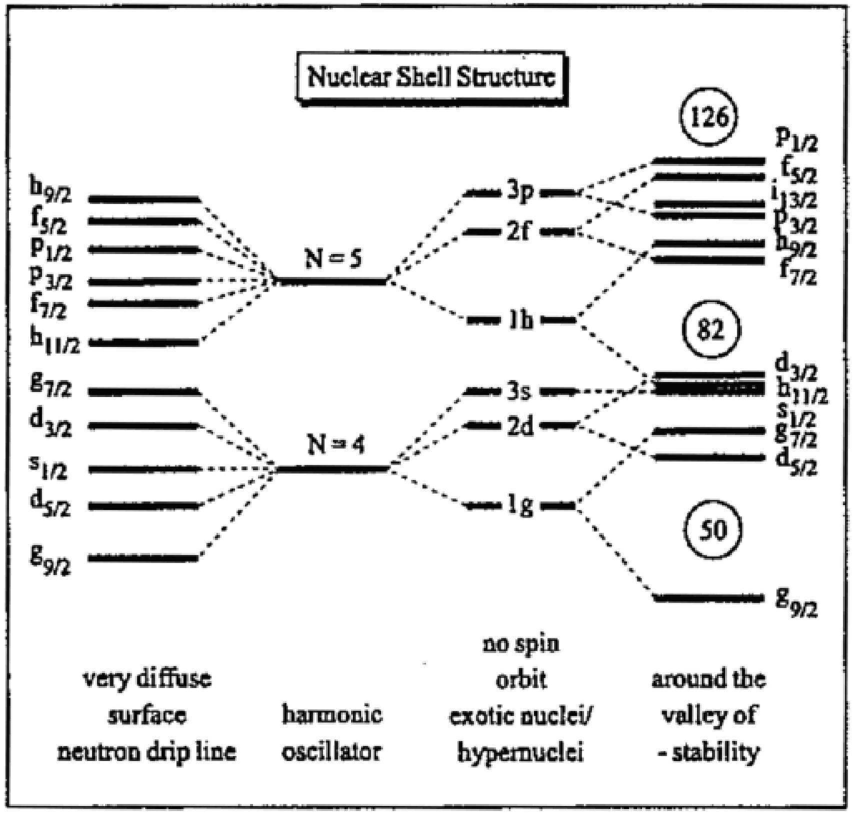
B. Cederwall et al., Nature 469 (2011), 68 *T.S. Brock et al., Phys. Rev. C82 (2010) 061309*

New magic numbers

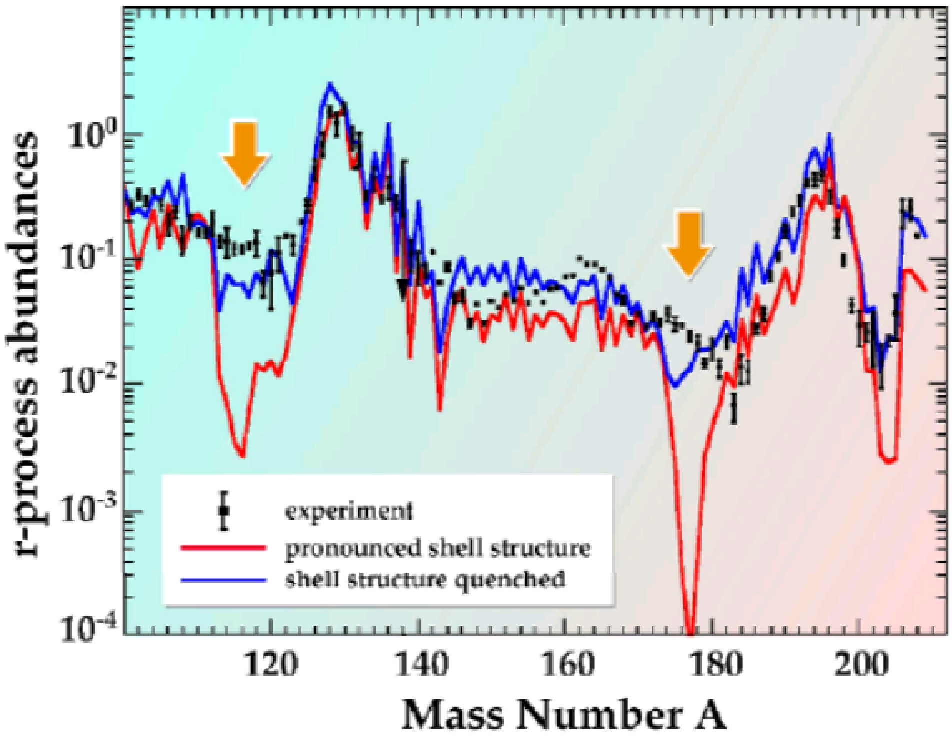


Future: Nuclear- and Astrophysics

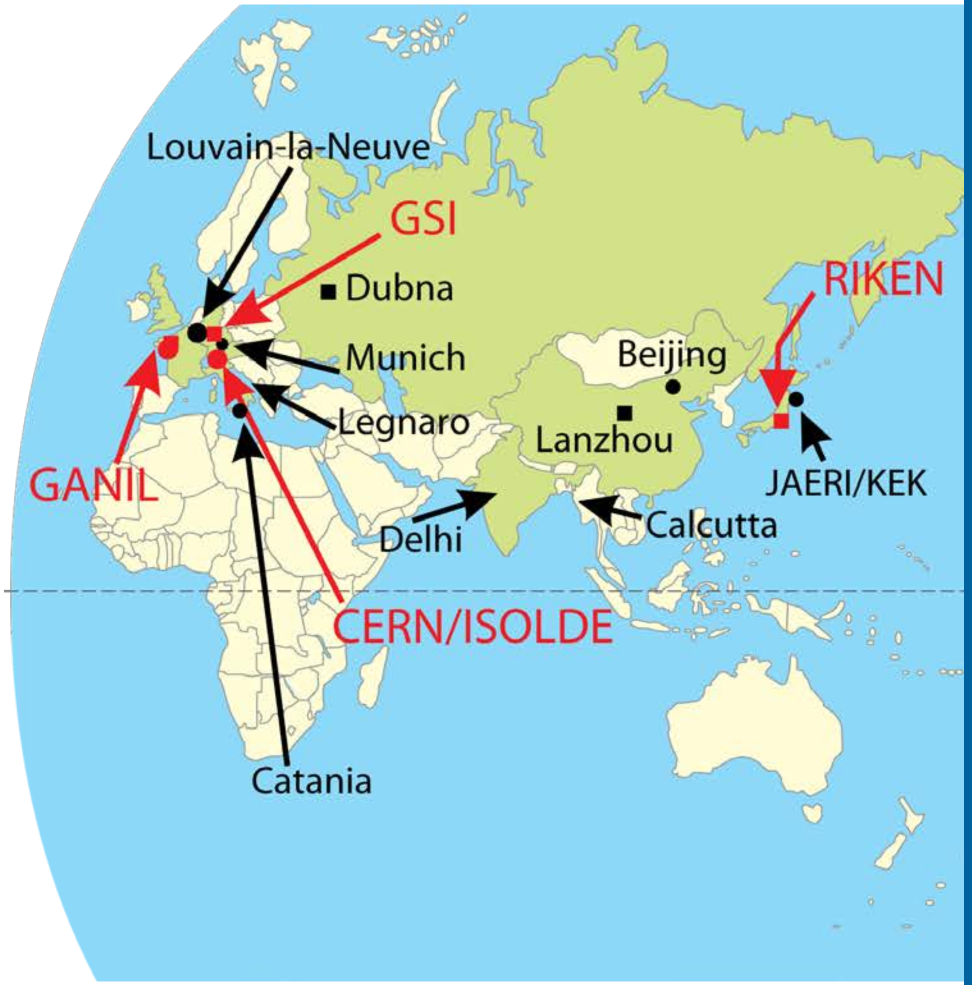
The shell evolution expected for medium-mass and heavy nuclei



The influence to r – process abundances



Rare Isotope Beam Capabilities Worldwide

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feature article

RISING: Gamma Spectroscopy Far from Stability

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