

Outline: Nuclear mass

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web-page: <https://web-docs.gsi.de/~wolle/> and click on



1. chemical and nuclear reactions
2. binding energy
3. mass excess
4. characteristics of nuclear binding
5. penning trap measurement
6. Schottky mass spectroscopy

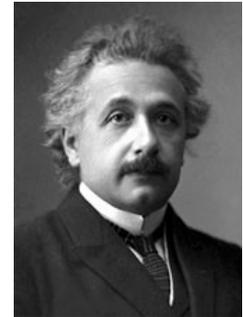
Nuclear masses

- ❖ In 1905 Albert Einstein following his derivation of the Special Theory of Relativity identifies relation between mass and energy of an object at rest:

$$E = mc^2$$

- ❖ The corresponding relation for moving object is

$$E = \frac{m_0 c^2}{\sqrt{1 + \left(\frac{v}{c}\right)^2}}$$

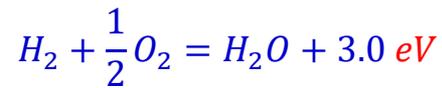


Albert Einstein
(1879 - 1955)

- ❖ **This discovery explains the energy powering nuclear decay.** The question of energy release in nuclear decay was a major scientific puzzle from the time of the discovery of natural radioactivity by Henry Becquerel (1896) until Einstein's postulate of mass-energy equivalence.

Chemical and nuclear reactions

Heat is evolved in the **chemical reaction** in which hydrogen and oxygen are combined to be water and generates 3.0 eV energy emission:

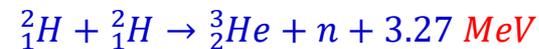


Such chemical reaction in which heat is evolved is called **exothermic reaction**.

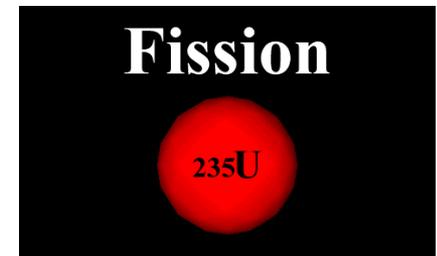
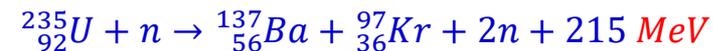
Another example is where one mol of carbon is oxidized to be carbon dioxide with producing 4.1 eV energy:



The **nuclear reaction** in which two deuterons bind with each other is an example of nuclear fusion. This exoergic reaction is written as



If a neutron is absorbed in the uranium-235 nucleus, it would split into two fragments of almost equal masses and evolves some number of neutrons and energy. One of the equations for the processes is written



Nuclear masses

	mass (kg)	mass (MeV/c ²)	mass (u)
electron	$9.109 \cdot 10^{-31}$	0.511	0.000549
proton	$1.673 \cdot 10^{-27}$	938.2794	1.007276
neutron	$1.675 \cdot 10^{-27}$	939.5728	1.008665
¹ H atom	$1.674 \cdot 10^{-27}$	938.7904	1.007825



nuclear and atomic masses are expressed in **ATOMIC MASS UNITS (u)**

definition: 1/12 of mass of neutral ¹²C → M(¹²C) = 12 u

1 u = $1.6605 \cdot 10^{-27}$ kg or 931.494 MeV/c² (E = mc²)

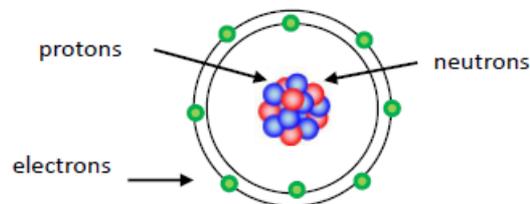
Expect: $M\left({}_Z^A X_N\right) = Z \cdot m_p + N \cdot m_n + Z \cdot m_e$

Find: $M\left({}_Z^A X_N\right) < Z \cdot m_p + N \cdot m_n + Z \cdot m_e$

$$\Delta m \cdot c^2 = BE$$

mass defect

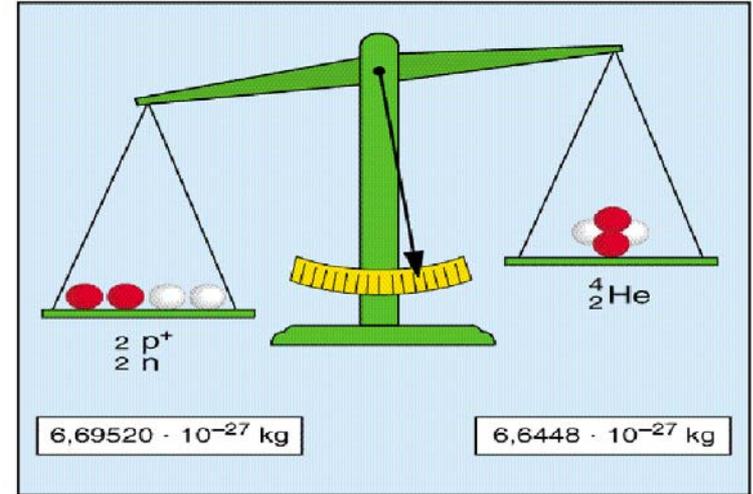
We typically use **ATOMIC** and **not NUCLEAR** masses → mass of electrons also included



Nuclear masses and binding energy

	mass (kg)	mass (MeV/c ²)	mass (u)
electron	$9.109 \cdot 10^{-31}$	0.511	0.000549
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$$1 \text{ u} = 1.6605 \cdot 10^{-27} \text{ kg} \text{ or } 931.494 \text{ MeV}/c^2$$

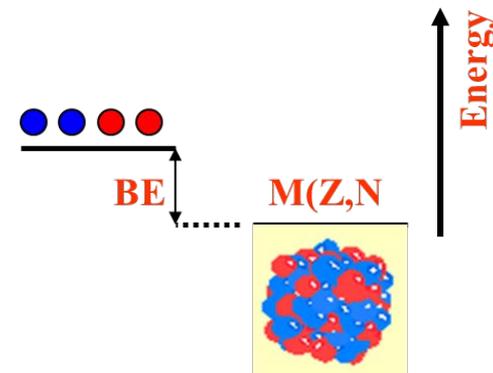


A bound system has a lower potential energy than its constituents !

Binding energy:

$$BE = \{ [Z \cdot m_H + N \cdot m_n] - M({}_Z^A X_N) \} \cdot c^2 > 0$$

$$BE ({}^4_2\text{He}) = [2 \cdot 1.007825 + 2 \cdot 1.008665] - 4.002603 = 0.030377 \text{ u} \quad (= 28.296 \text{ MeV})$$



Binding energy and mass excess

	mass (kg)	mass (MeV/c ²)	mass (u)
electron	$9.109 \cdot 10^{-31}$	0.511	0.000549
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$$1 \text{ u} = 1.6605 \cdot 10^{-27} \text{ kg} \quad \text{or} \quad 931.494 \text{ MeV}/c^2$$

$$\text{Binding energy: } BE = \{ [Z \cdot m_H + N \cdot m_n] - M({}_Z^A X_N) \} \cdot c^2$$

$$\text{Mass excess: } \Delta M = M(A, Z) - A \cdot M(u)$$

Example:

$$\text{mass of } {}^{56}\text{Fe: } M(56,26) = 55.934942 \text{ [u]}$$

$$\begin{aligned} \text{mass excess: } \Delta M &= M(A,Z) - A \cdot M(u) = 55.934942 - 56 = -0.06506 \text{ [u]} \\ &= -0.06506 \cdot 931.494 = -60.601 \text{ [MeV}/c^2] \end{aligned}$$

$$\begin{aligned} \text{binding energy: } BE(Z,A) &= [Z \cdot M({}^1\text{H}) + N \cdot M_n - M(Z,A)] \cdot c^2 \\ &= [26 \cdot 1.007825 + 30 \cdot 1.008665 - 55.934942] \cdot 931.494 = 492.25 \text{ [MeV]} \end{aligned}$$

Binding energy

	mass (kg)	mass (MeV/c ²)	mass (u)
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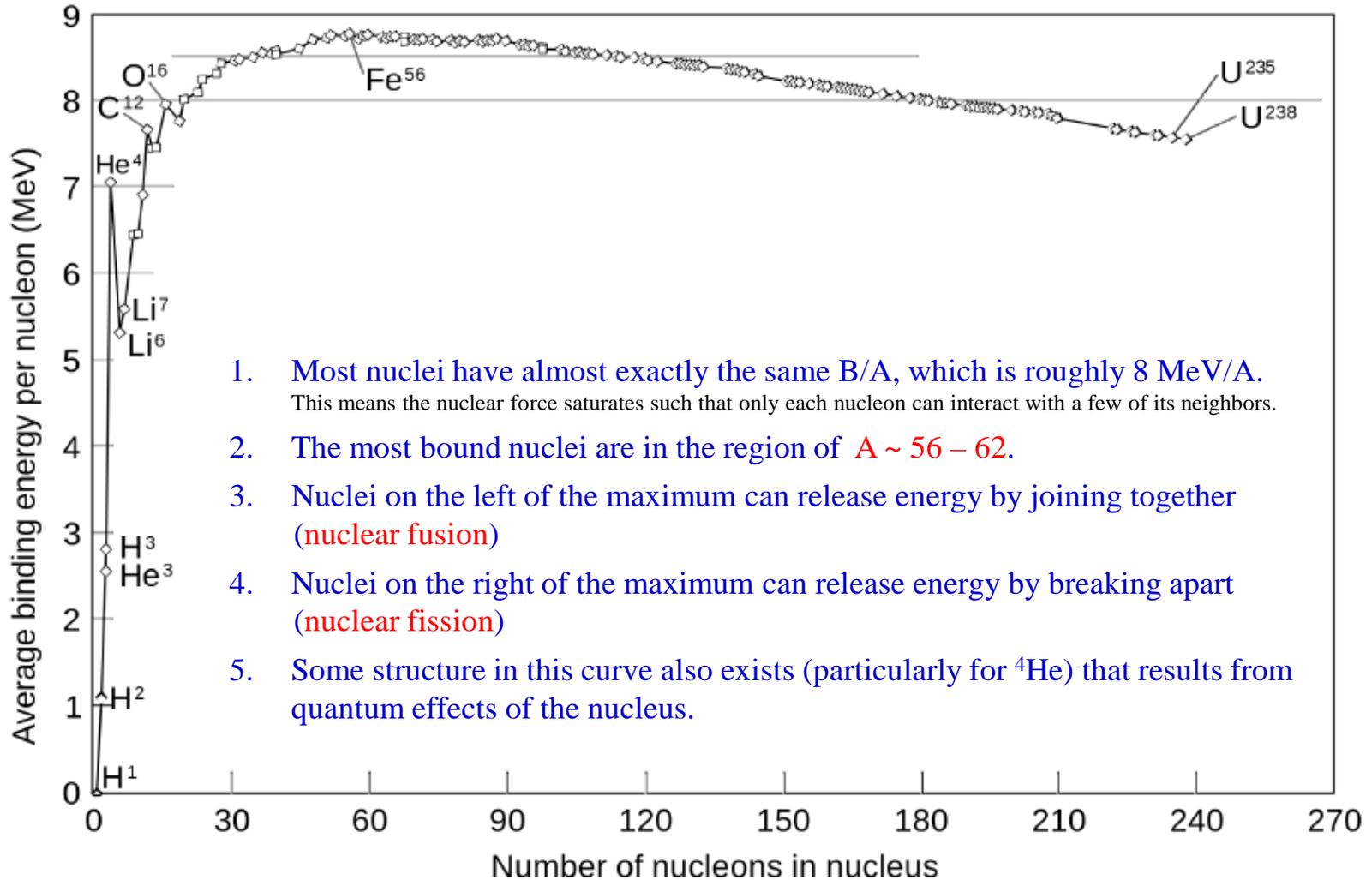
1 u = $1.6605 \cdot 10^{-27}$ kg or 931.494 MeV/c²

$$BE = \{ [Z \cdot m_H + N \cdot m_n] - M({}_Z^A X_N) \} \cdot c^2$$

element	mass of nucleons [u]	nuclear mass [u]	binding energy [MeV]	BE/A [MeV/u]
² H	2.01594	2.01355	2.23	1.12
⁴ He	4.03188	4.00151	28.29	7.07
⁷ Li	7.05649	7.01336	40.15	5.74
⁹ Be	9.07243	9.00999	58.13	6.46
⁵⁶ Fe	56.44913	55.92069	492.24	8.79
¹⁰⁷ Ag	107.86187	106.87934	915.23	8.55
¹²⁷ I	128.02684	126.87544	1072.53	8.45
²⁰⁶ Pb	207.67109	205.92952	1622.27	7.88
²¹⁰ Po	211.70297	209.93683	1645.16	7.73
²³⁸ U	239.93448	238.00037	1801.63	7.57

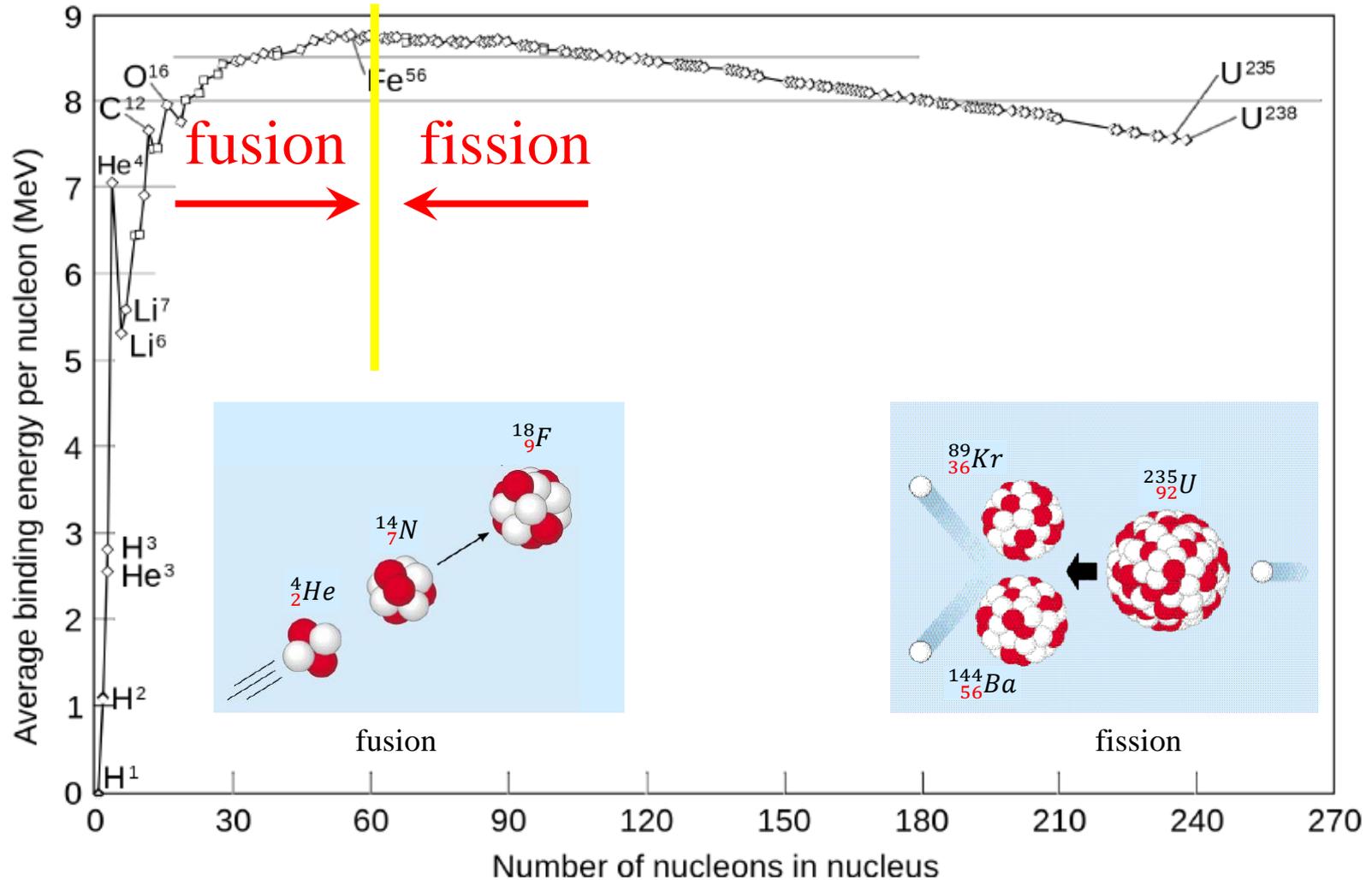
Characteristics of nuclear binding

Binding energy per nucleon (BE/A)



Characteristics of nuclear binding

Binding energy per nucleon (BE/A)



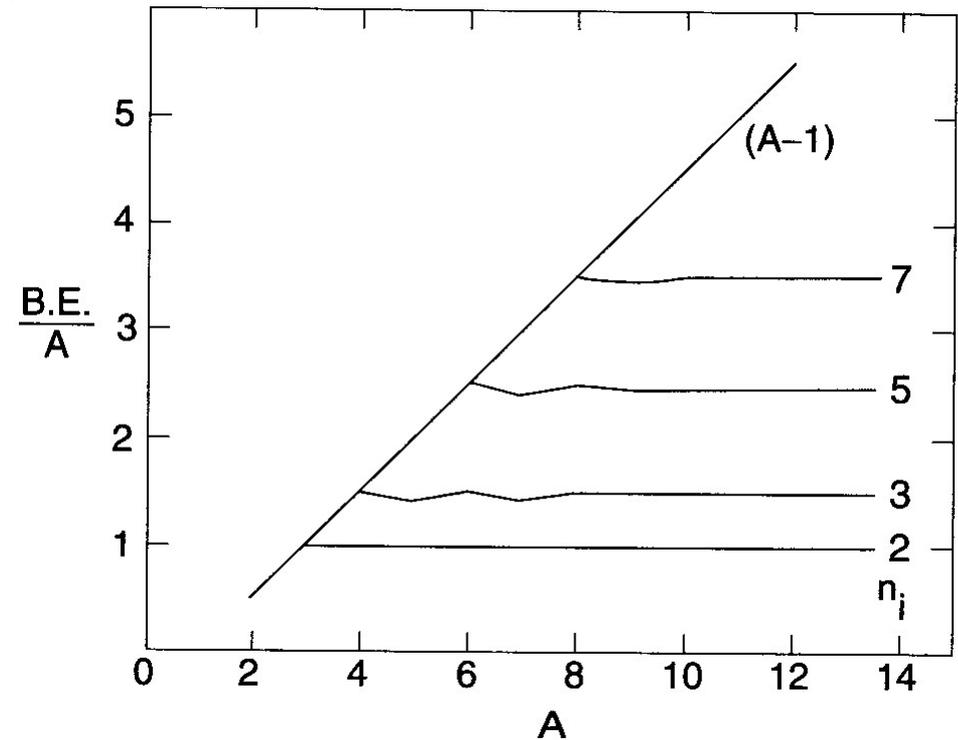
Long vs short range interaction

Long range force: $B \propto \frac{A(A-1)}{2}$ \hookrightarrow $B/A \propto A$

Short range force: saturation

NUMBER OF INTERACTIONS (n_i) PER NUCLEON

A	2	3	5	(A-1)
3	 1.0			1.0
4	 1.0	 1.5		1.5
5	 1.0	 1.4	 2.0	2.0
6	 1.0	 1.5	 2.5	2.5
8	 1.0	 1.5	 2.5	3.5 ⋮ $(A-1)/2$



Mass spectroscopy

Mass of nuclei are measured by passing ion beams through magnetic and electric field (2 steps)

- 1) select a constant velocity with a velocity (Wien) filter

\mathbf{E} and \mathbf{B}_1 are at right angles

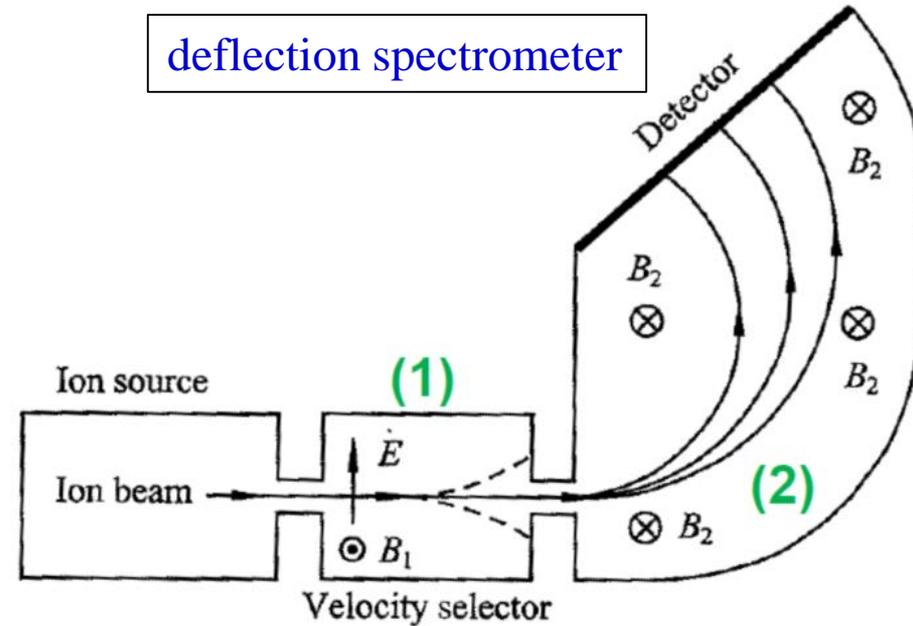
$$q \cdot E = q \cdot v \cdot B_1$$

$$v = E / B_1$$

- 2) Measure a trajectory of ions in a uniform magnetic \mathbf{B}_2 field

$$\frac{m \cdot v^2}{r} = q \cdot v \cdot B_2$$

If $B_1 = B_2 = B$ $\frac{q}{m} = \frac{v}{B \cdot r} = \frac{E}{B^2 \cdot r}$



Penning trap measurement

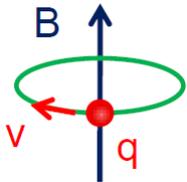
The most precise measurements come from storage devices that confine ions in three dimensions by means of well-controlled electromagnetic fields.

Examples: storage rings, Penning trap, Paul trap

In a Penning trap,

- 1) Ions are confined in 2D with a strong magnetic field B
- 2) A weak electrostatic field is used to trap ions in 3D (along the Z-axis)

1) Cyclotron oscillations (ω_c)

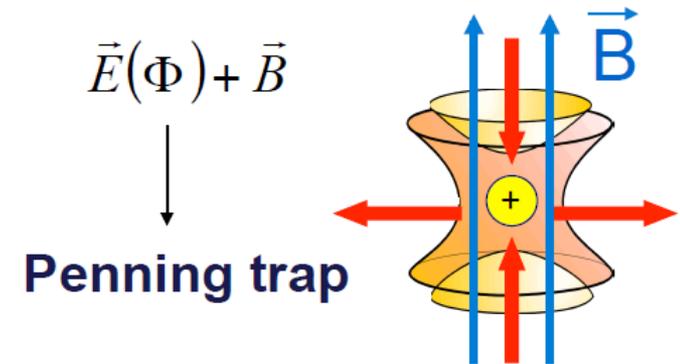
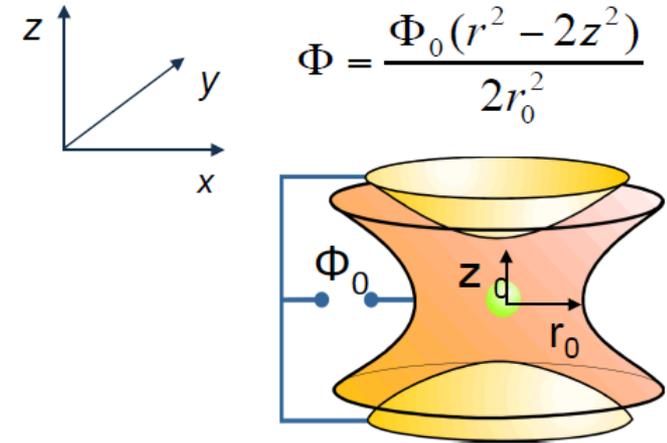


$$\frac{m \cdot v^2}{r} = q \cdot v \cdot B$$

$$v = \frac{q \cdot B \cdot r}{m}$$

$$t = \frac{2\pi \cdot r}{v} = \frac{2\pi \cdot m}{q \cdot B}$$

$$\omega_c = 2\pi \cdot f = \frac{2\pi}{t} = \frac{qB}{m}$$



Penning trap measurement

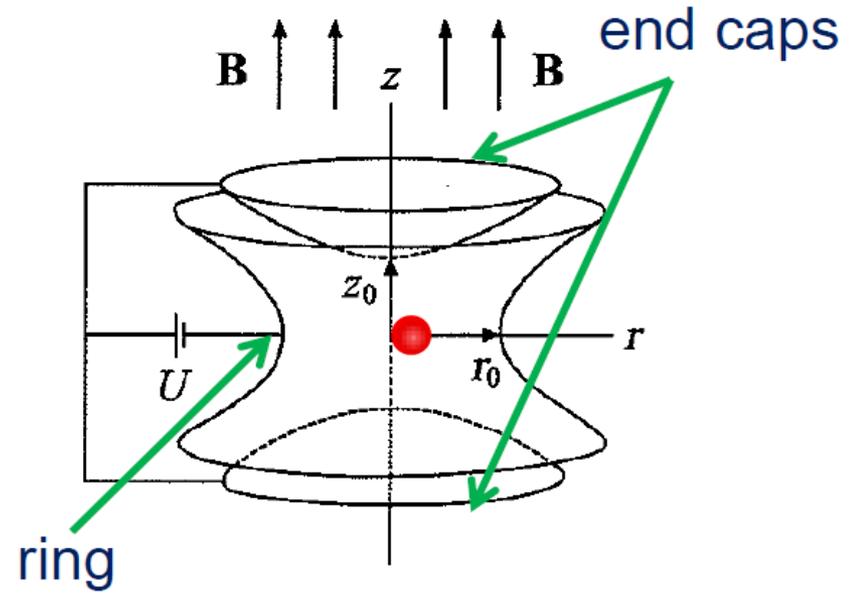
- 2) A weak axially symmetric electrostatic potential is superimposed to produce a saddle point at the center.

This requires the **quadrupole potential**:

$$\Phi(z, r) = \frac{U}{4d^2} \cdot (2z^2 - r^2)$$

$$d = \frac{1}{2} (2z_0^2 - r_0^2)^{1/2}$$

(so that U is the potential difference between the endcap and the ring electrodes)



Penning trap measurement

Solving the equations of motions result in three independent motional modes with frequencies;

$$\omega_z = \sqrt{q \cdot U / m \cdot d^2} \quad (\text{axial motion})$$

$$\omega_+ = \omega_c / 2 + \sqrt{\omega_c^2 / 4 - \omega_z^2 / 2} \quad (\text{cyclotron motion})$$

$$\omega_- = \omega_c / 2 - \sqrt{\omega_c^2 / 4 - \omega_z^2 / 2} \quad (\text{magnetron motion})$$

The condition on the magnetic field,

$$\omega_c^2 / 4 - \omega_z^2 / 2 > 0 \quad B^2 > 2 \cdot m \cdot U / q \cdot d^2$$

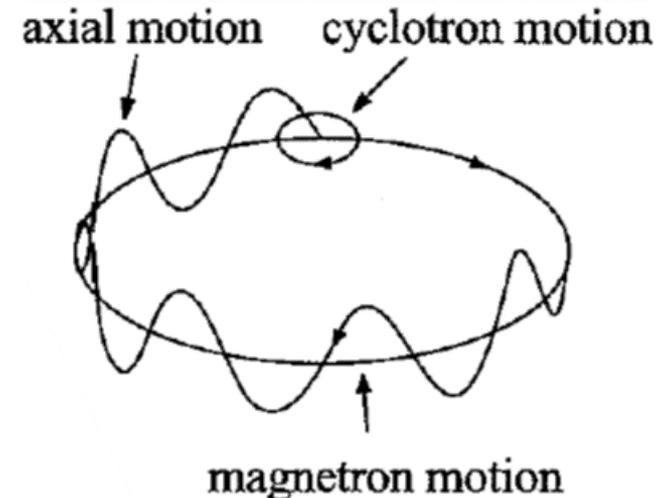
$$\omega_{\pm} = \omega_c / 2 \cdot \left[1 \pm \left(1 - 2 \cdot \omega_z^2 / \omega_c^2 \right)^{1/2} \right]$$

$$= \omega_c / 2 \cdot \left[1 \pm \left(1 - \omega_z^2 / \omega_c^2 \right) \right]$$

$$\omega_- = \omega_z^2 / 2\omega_c = U / 2 \cdot B \cdot d^2$$

$$\omega_+ = \omega_c - U / 2 \cdot B \cdot d^2$$

$$\rightarrow \omega_c = \omega_+ + \omega_- \quad \omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2 \quad (\omega_+ \cdot \omega_- = \omega_z^2 / 2)$$

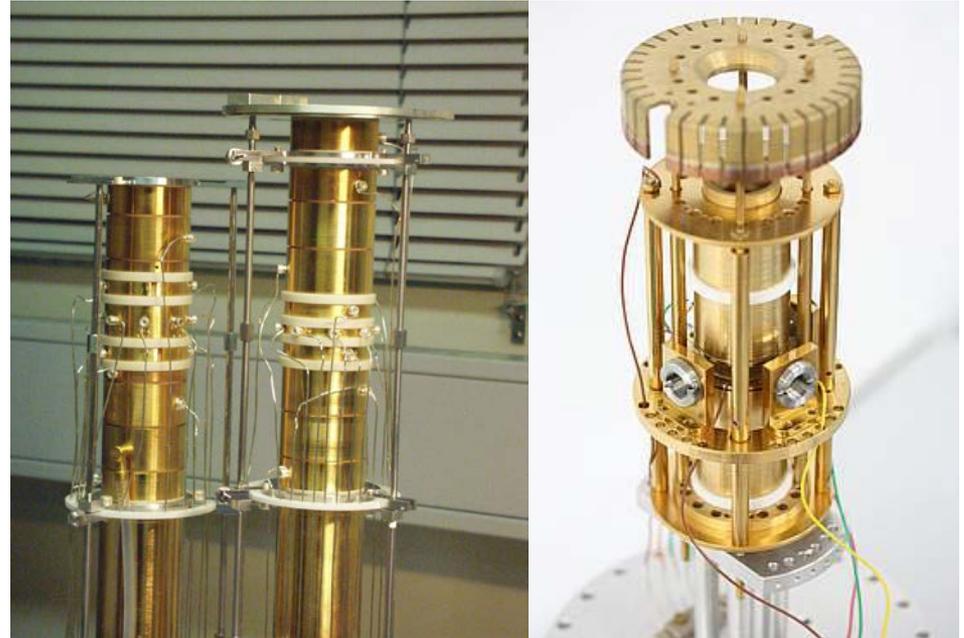
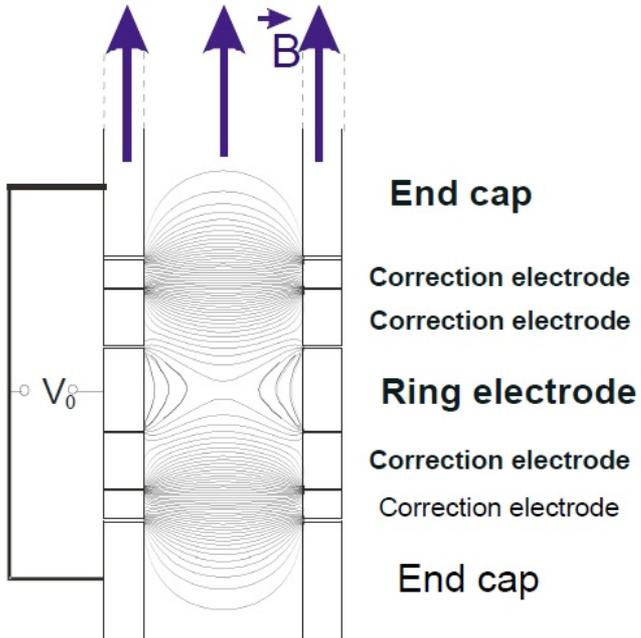


In the experiment one needs to measure the sum

$$\omega_+ + \omega_-$$

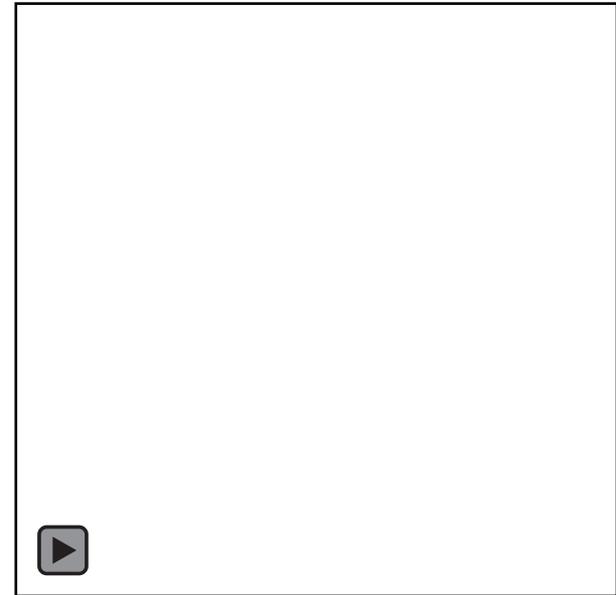
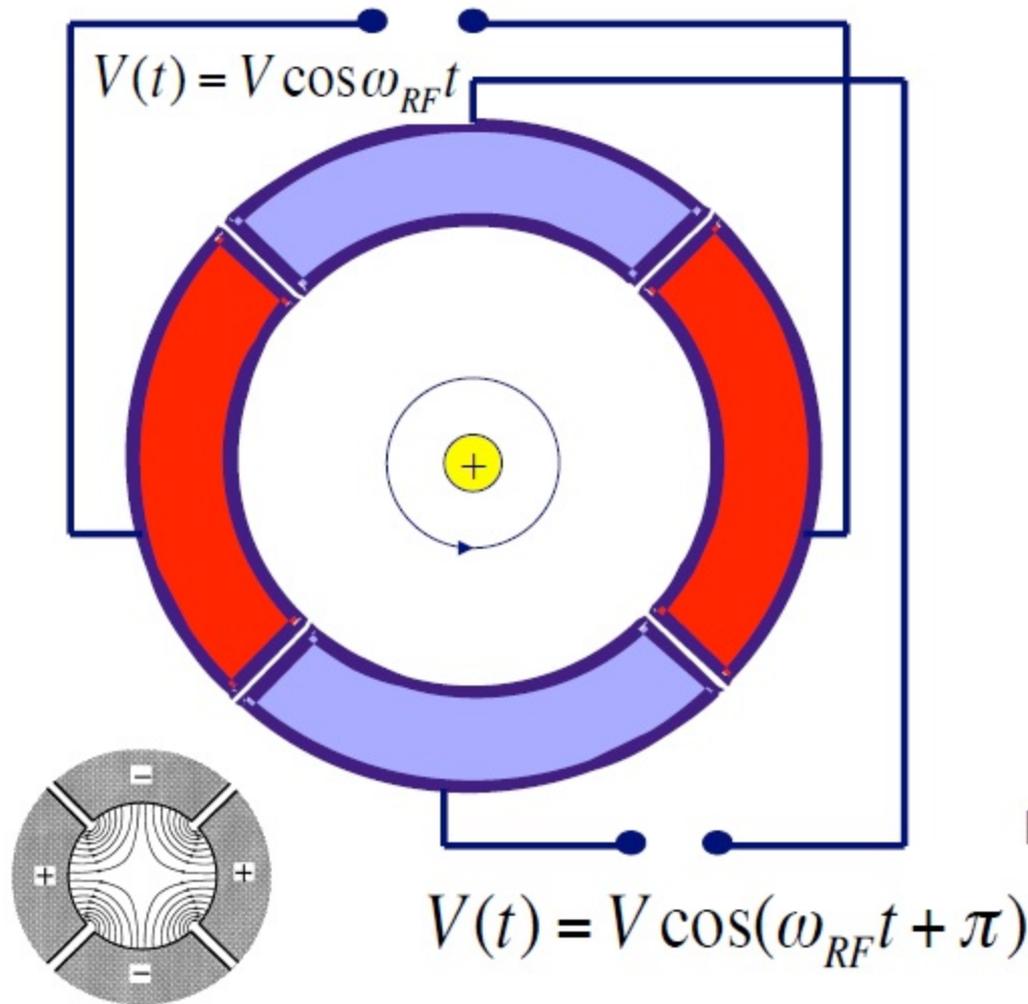
(not cyclotron frequency from ω_+ only)

Cylindrical Penning trap



GSI-Darmstadt

Ion manipulation

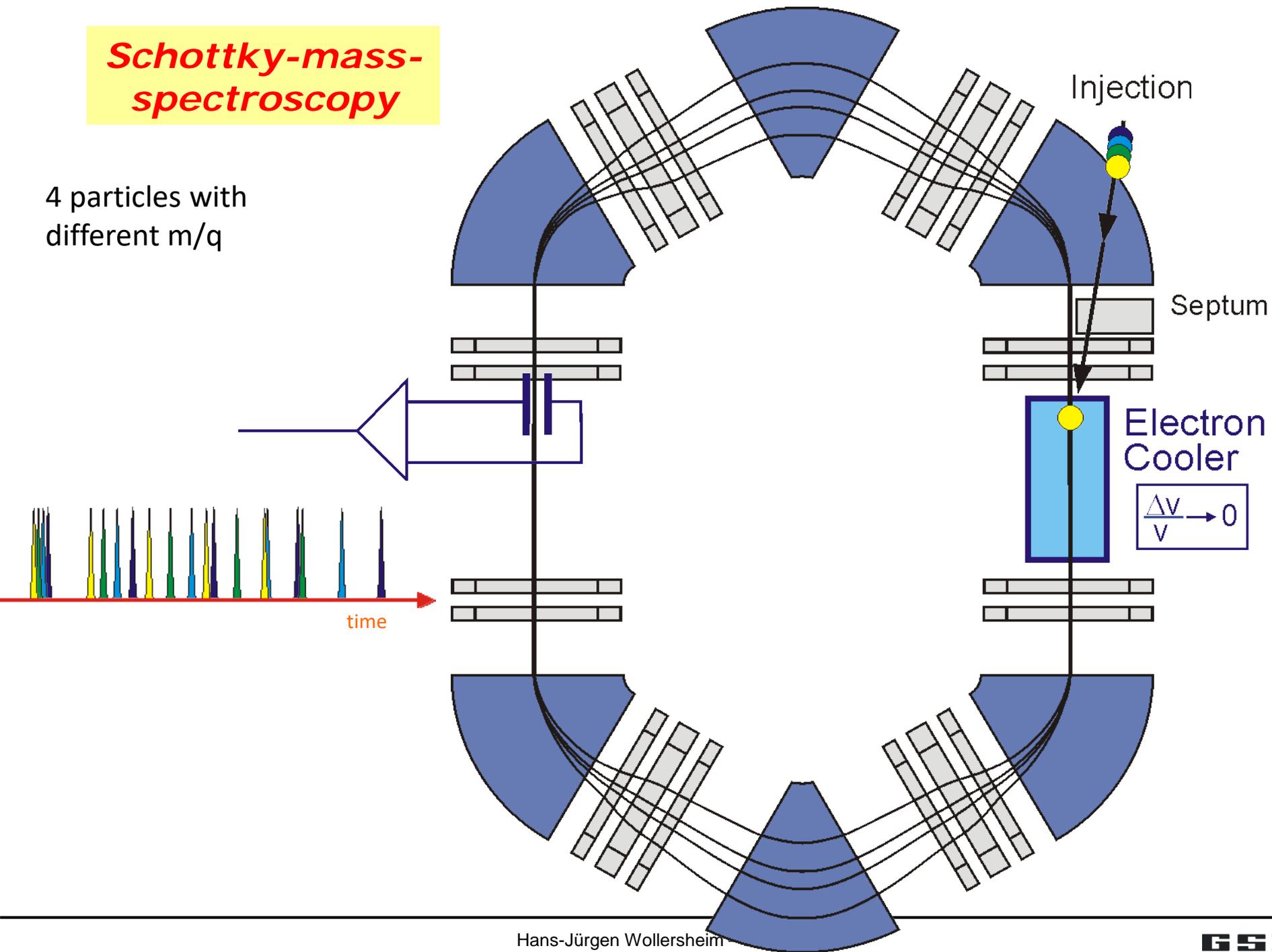


Energy exchange
between
eigenmotions

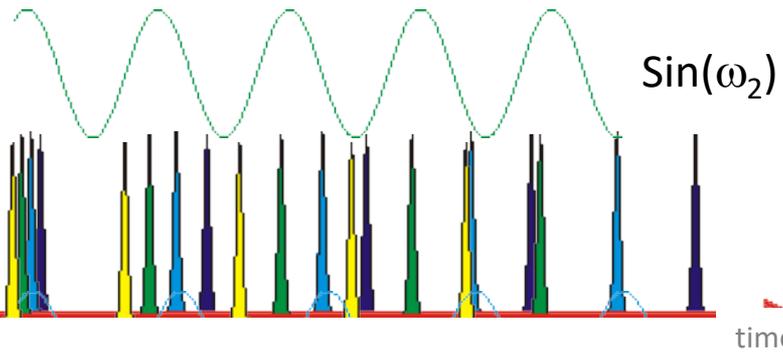
Magnetron to reduced-cyclotron
Reduced-cyclotron to axial

Schottky-mass-spectroscopy

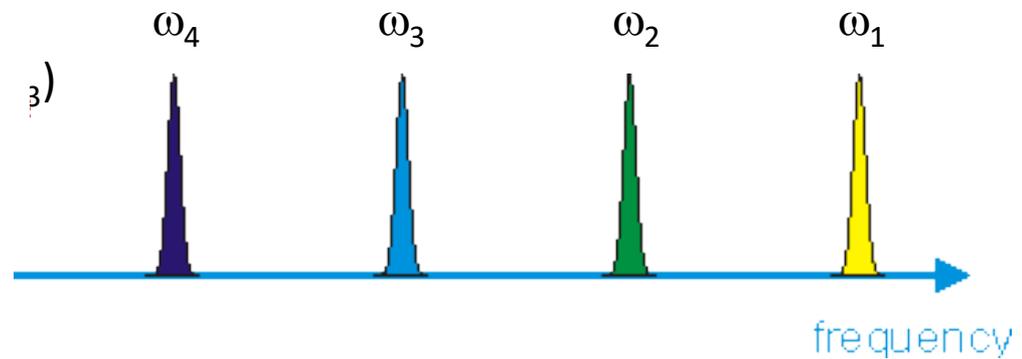
4 particles with different m/q



Schottky-mass-spectroscopy



fast Fourier transform



Small-band Schottky frequency spectra

