Outline: Liquid drop model

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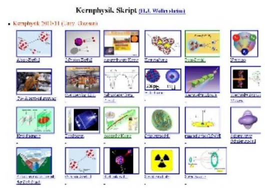
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. binding energy
2. volume, surface, Coulomb, asymmetry, pairing energy
3. line of stability
4. deviation from liquid drop model
5. neutron separation energy

The nucleus and its structure

Presently no complete theory to fully describe structure and behavior of nuclei based solely on knowledge of force between nucleons (although tremendous progress for A < 12 in the past few years!)

⇩

use MODELS:

- simplifying assumptions
- give reasonable account of observed properties
- make predictions

Liquid-Drop Model

nucleus regarded as collection of neutrons and protons forming a droplet of incompressible fluid

→ good description of overall trend of binding energy per nucleon

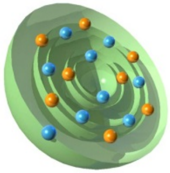
→ fails to account for magic numbers or give any prediction for J^π

SHELL Model

neutrons and protons arranged in stable quantum states in common potential well

→ accounts for ground state properties (e.g. J^π) and magic numbers

→ does not predict many of the observed nuclear excited states

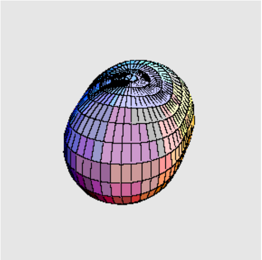


COLLECTIVE Model

neutrons and protons show collective motions give rise to vibrational and rotational states

→ accounts for properties of non-spherical nuclei

→ fails to reproduce other features



What have we learned about the nucleus so far?

1) The nuclear density is roughly constant for all nuclei

2) Nuclei are positively charged, and the nuclear charge density is also roughly constant

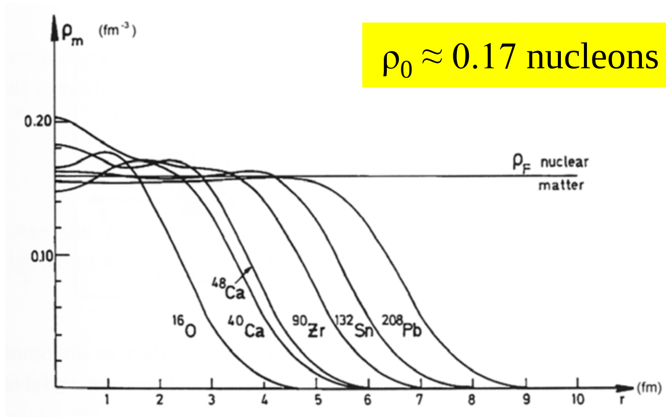
3) The strong force is attractive only at short range...

4) AND is repulsive at very short range (i.e. nuclear matter is highly incompressible)

These observations are remarkable, and have been performed with very simple concepts so far. We are now at the level of understanding where we can begin to *theoretically model* the nucleus in an attempt to predict our observations.

A charged drop of incompressible liquid

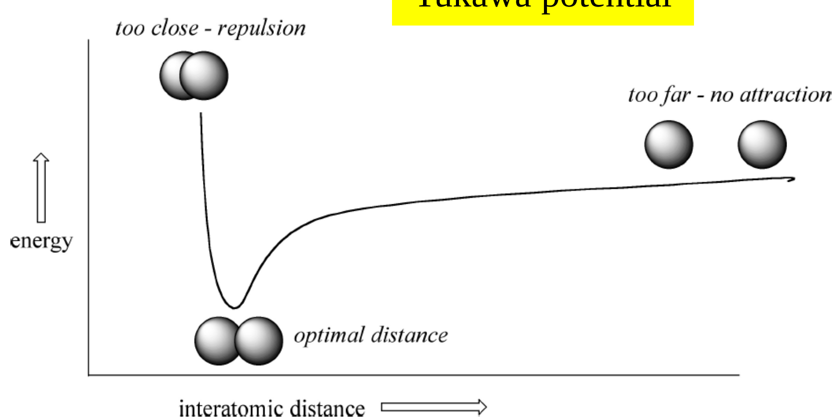
The scattering experiments we saw previously suggested that nuclei have approximately constant density. We were then able to calculate the nuclear radius assuming a uniform sphere. A drop of uniform liquid has the same property.



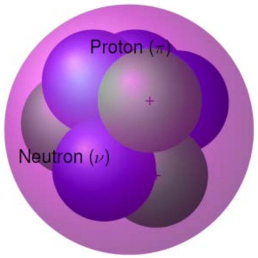
$\rho_0 \approx 0.17$ nucleons / fm^3

The nuclear force is short-range, but does not allow for compression of nuclear matter. Molecules in a liquid drop have the same basic properties.

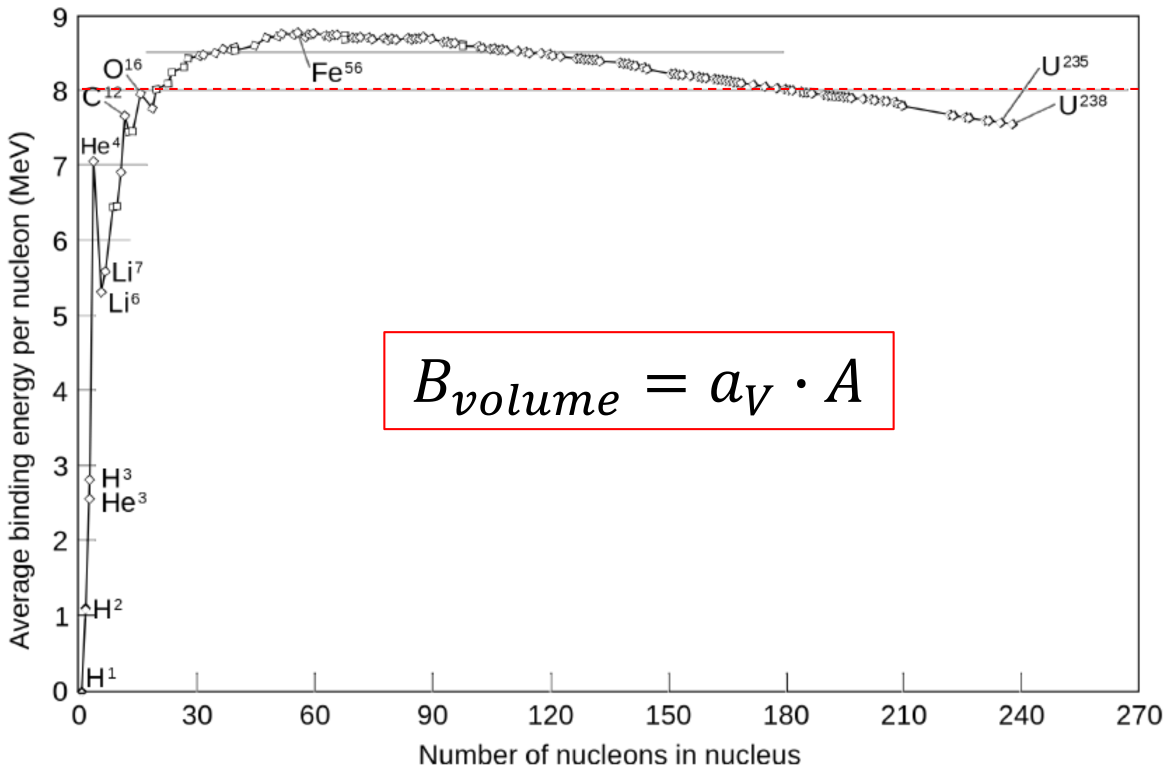
Yukawa potential



A charged drop of incompressible liquid

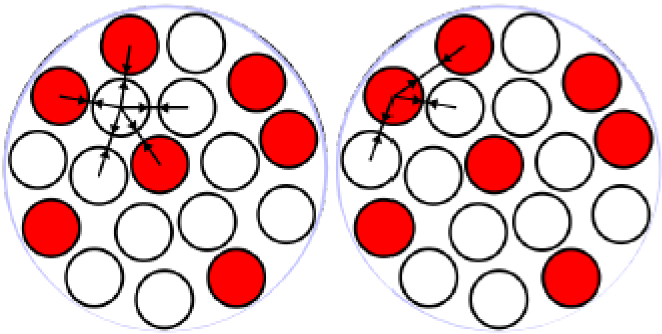


For the nucleus we assume a liquid drop with a uniform positive charge.



$$B_{volume} = a_V \cdot A$$

Liquid drop model



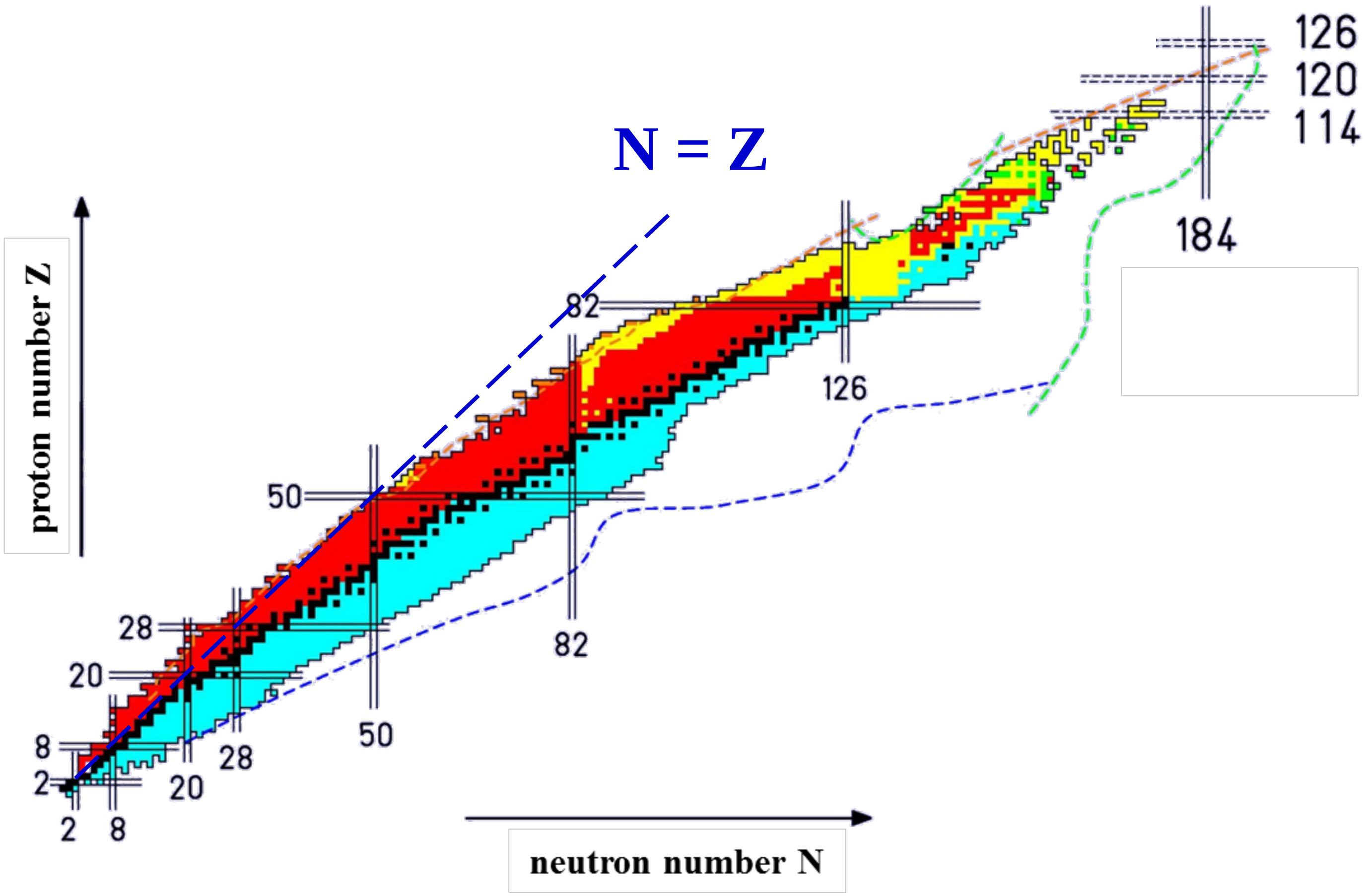
First, we need to account for the fact that the nucleons on the surface have less neighbours, and do not exhibit the same binding as those in the interior (volume)....

$$B_{surface} = -a_S \cdot A^{2/3}$$

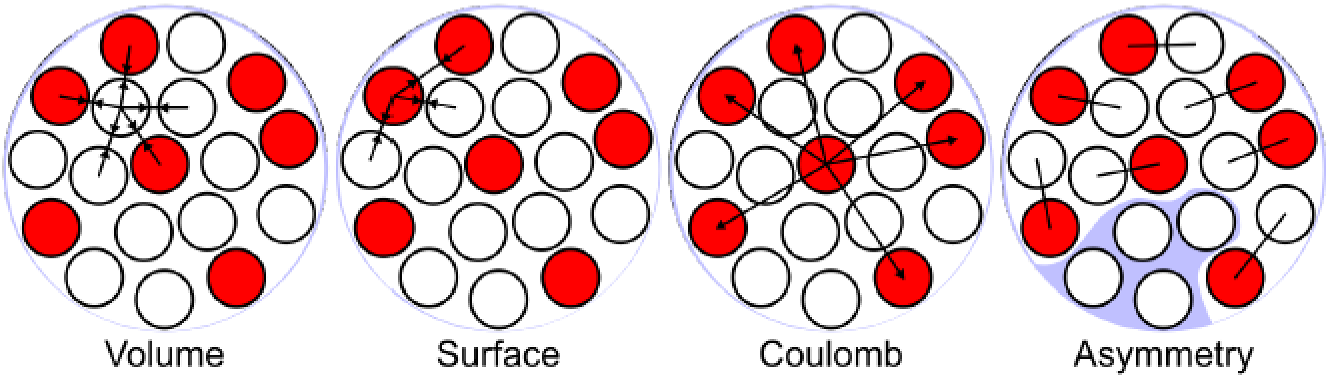
Protons in the nucleus repel each other due to their mutual positive charge, this reduces the binding energy further....

$$B_{Coulomb} = -a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}}$$

Liquid drop model asymmetry between proton and neutrons



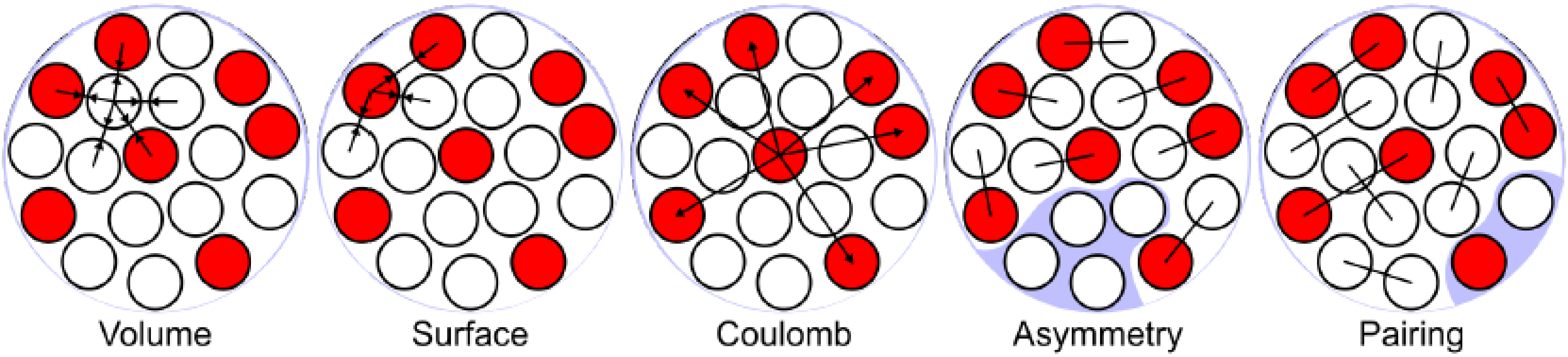
Liquid drop model



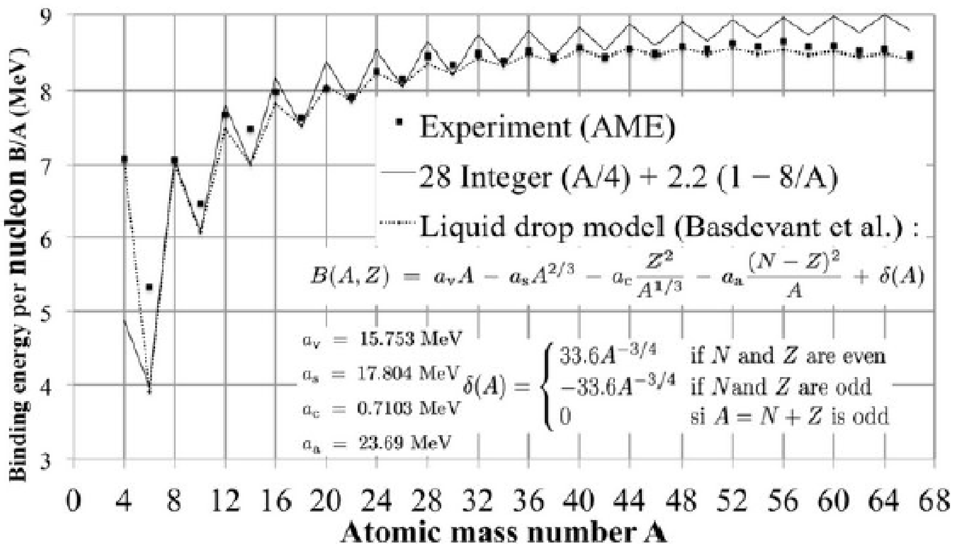
For light nuclei, N ~ Z (for heavy nuclei N is only slightly larger than Z). Where the Coulomb term would always favour Z = 0 for any A, we must account for the fact that nuclei are quantum objects (specifically that nucleons are fermions), and must obey the Pauli exclusion principle....

$$B_{asymmetry} = -a_{asym} \cdot \frac{(N-Z)^2}{A}$$

Liquid drop model



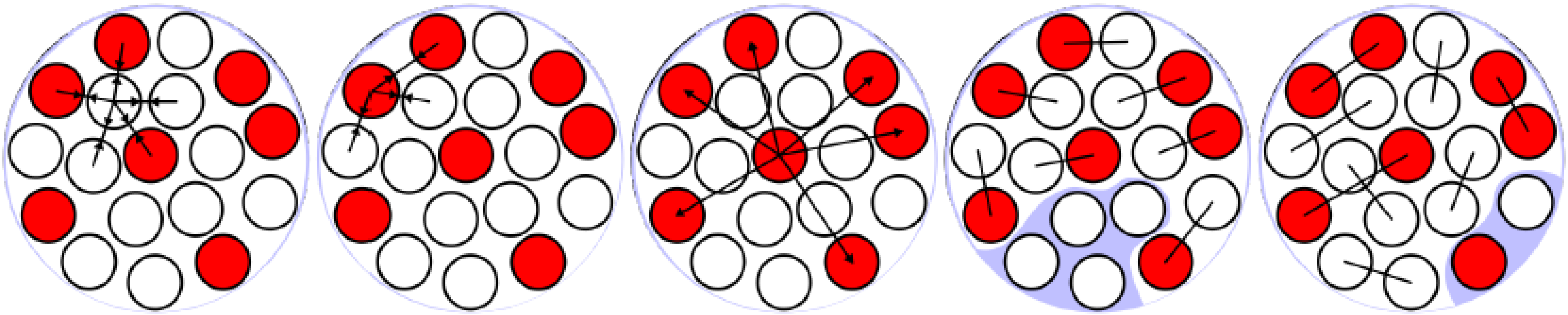
There is still one observation that can tell us something about the binding energy, and how nucleons interact with one another. How many nuclei with an even or odd number of protons and neutrons are stable?



nuclear pairing

$$B_{pair} = \begin{cases} +\delta & for\ even-even\ nuclei \\ 0 & for\ odd-even\ nuclei \\ -\delta & for\ odd-odd\ nuclei \end{cases}$$

Bethe-Weizsäcker mass formula



The Semi-Empirical Mass Formula

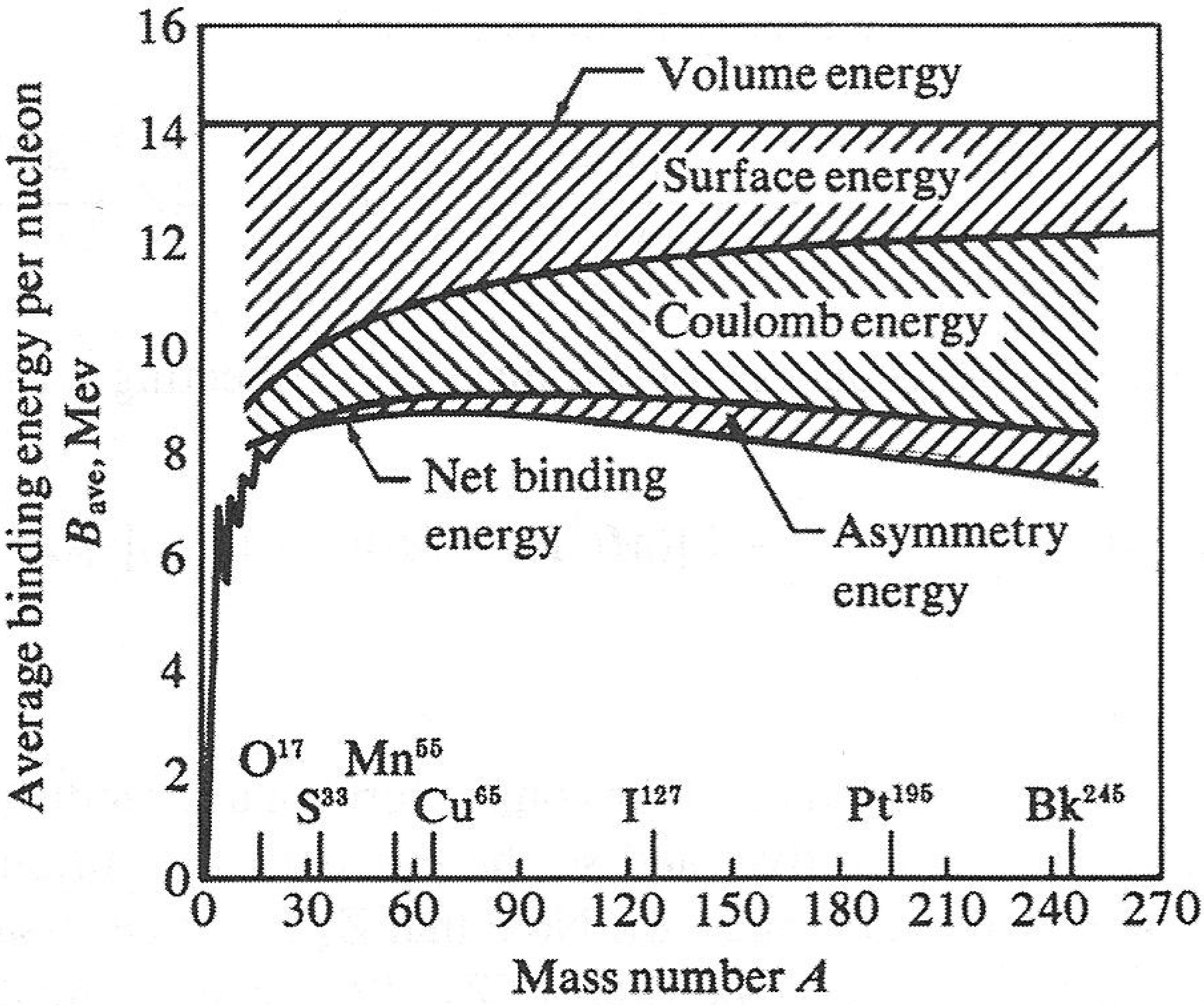
$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$

with

a_V	15.85 MeV
a_S	18.34 MeV
a_C	0.71 MeV
a_{asym}	23.21 MeV
a_{pair}	12 MeV

$$\delta = \begin{cases} +1 & for\ even - even\ nuclei \\ 0 & for\ odd - even\ nuclei \\ -1 & for\ odd - odd\ nuclei \end{cases}$$

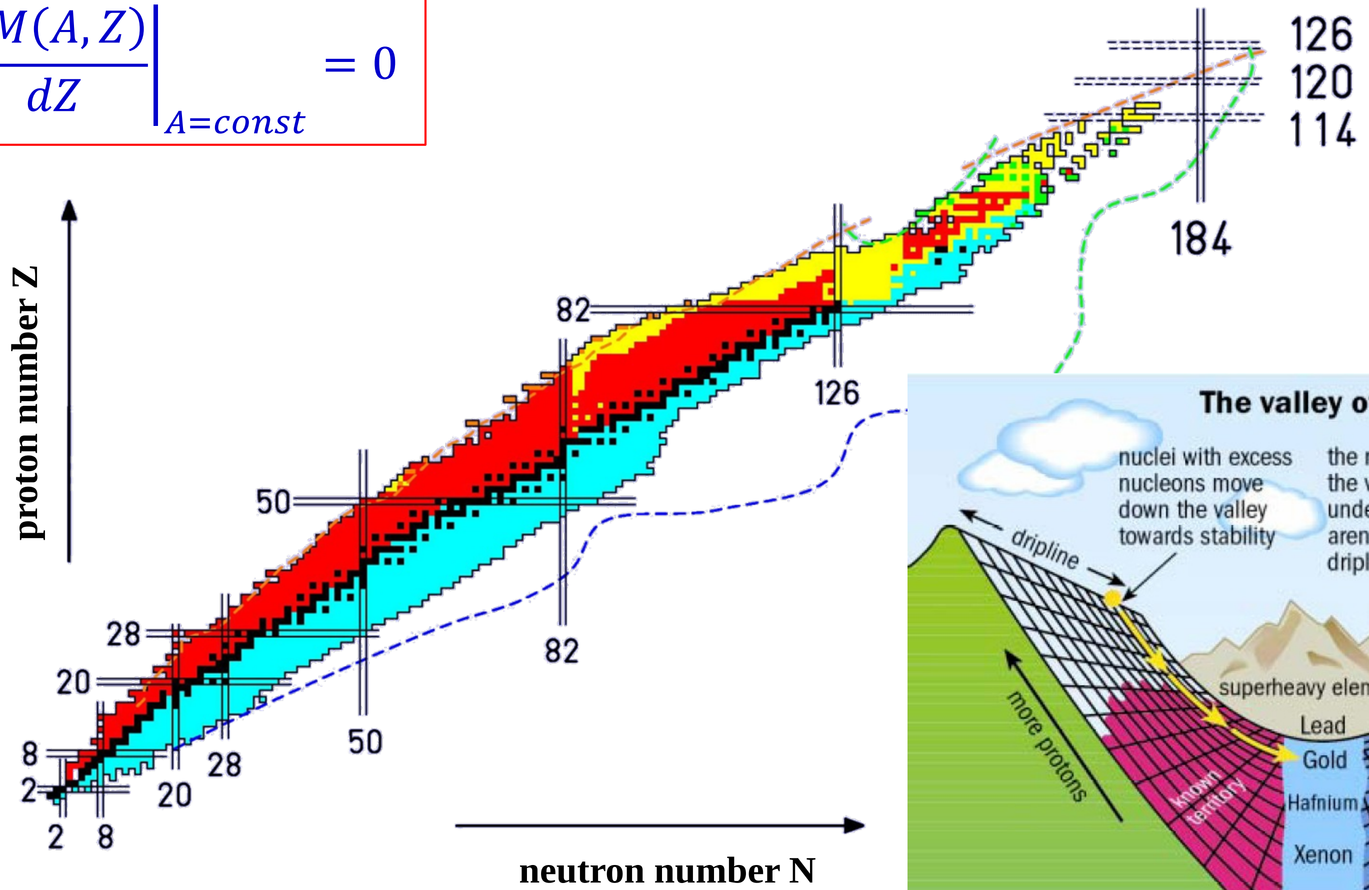
Liquid drop model's contribution

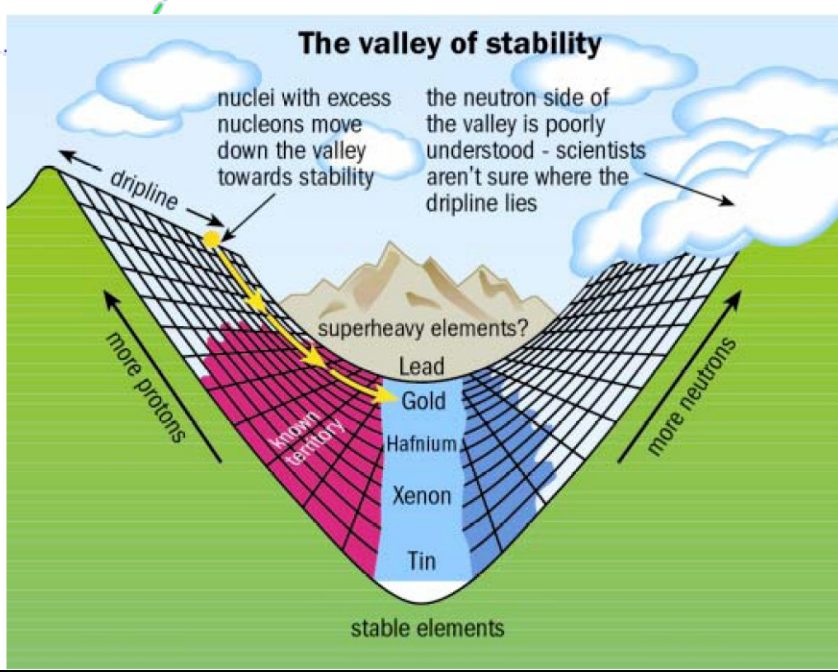


Liquid drop model line of stability

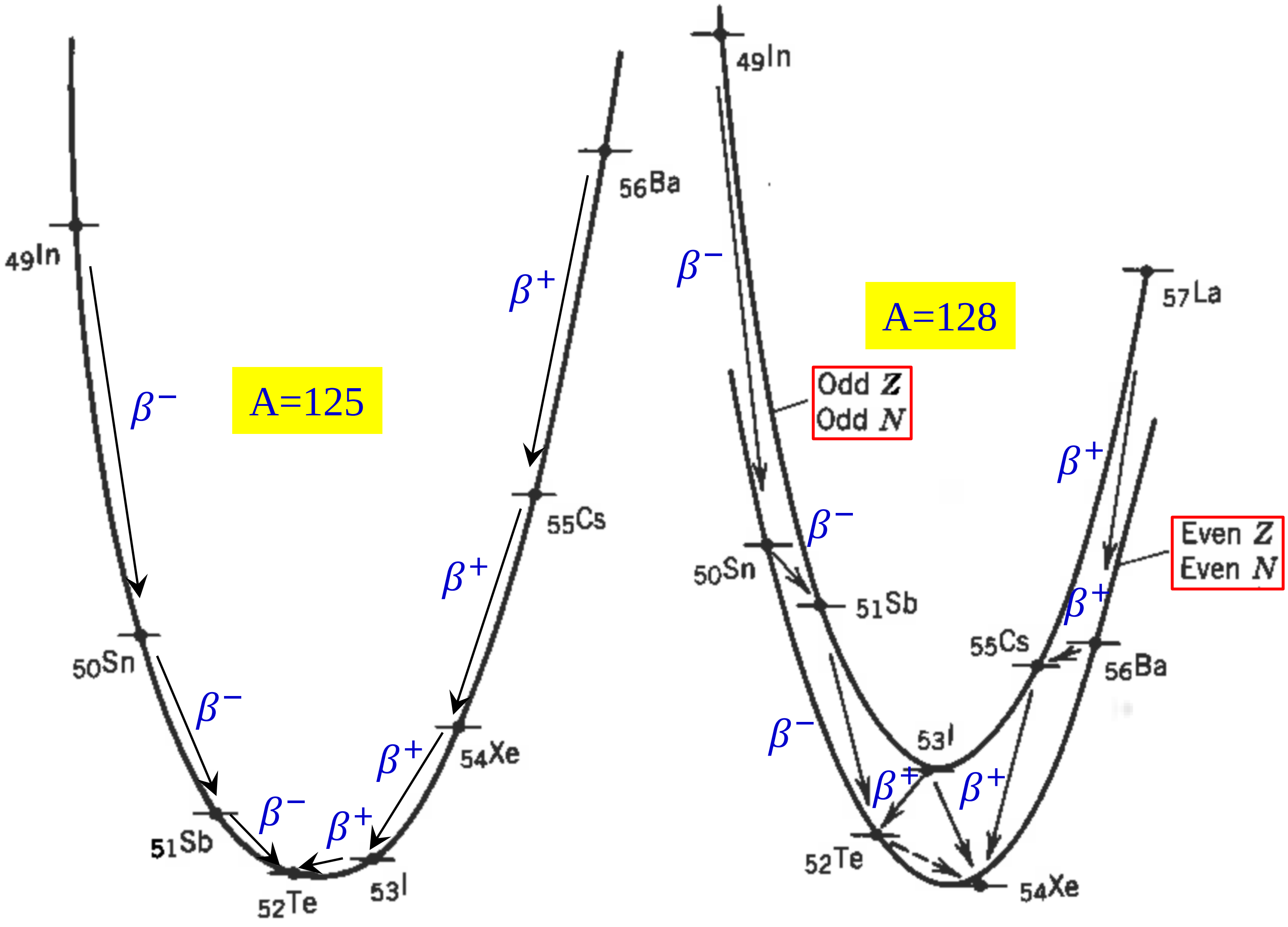
$$M(A,Z) = Z^2 \cdot \left[\frac{4 \cdot a_{asym}}{A} + \frac{a_C}{A^{\frac{1}{3}}}\right] + Z \cdot \left[M(\,^1H) - M(n) - 4 \cdot a_{asym}\right] + A \cdot \left[M(n) - a_V + \frac{a_S}{A^{\frac{1}{3}}} + a_A\right] + \delta(A,Z)$$

$$\left.\frac{dM(A,Z)}{dZ}\right|_{A=const} = 0$$

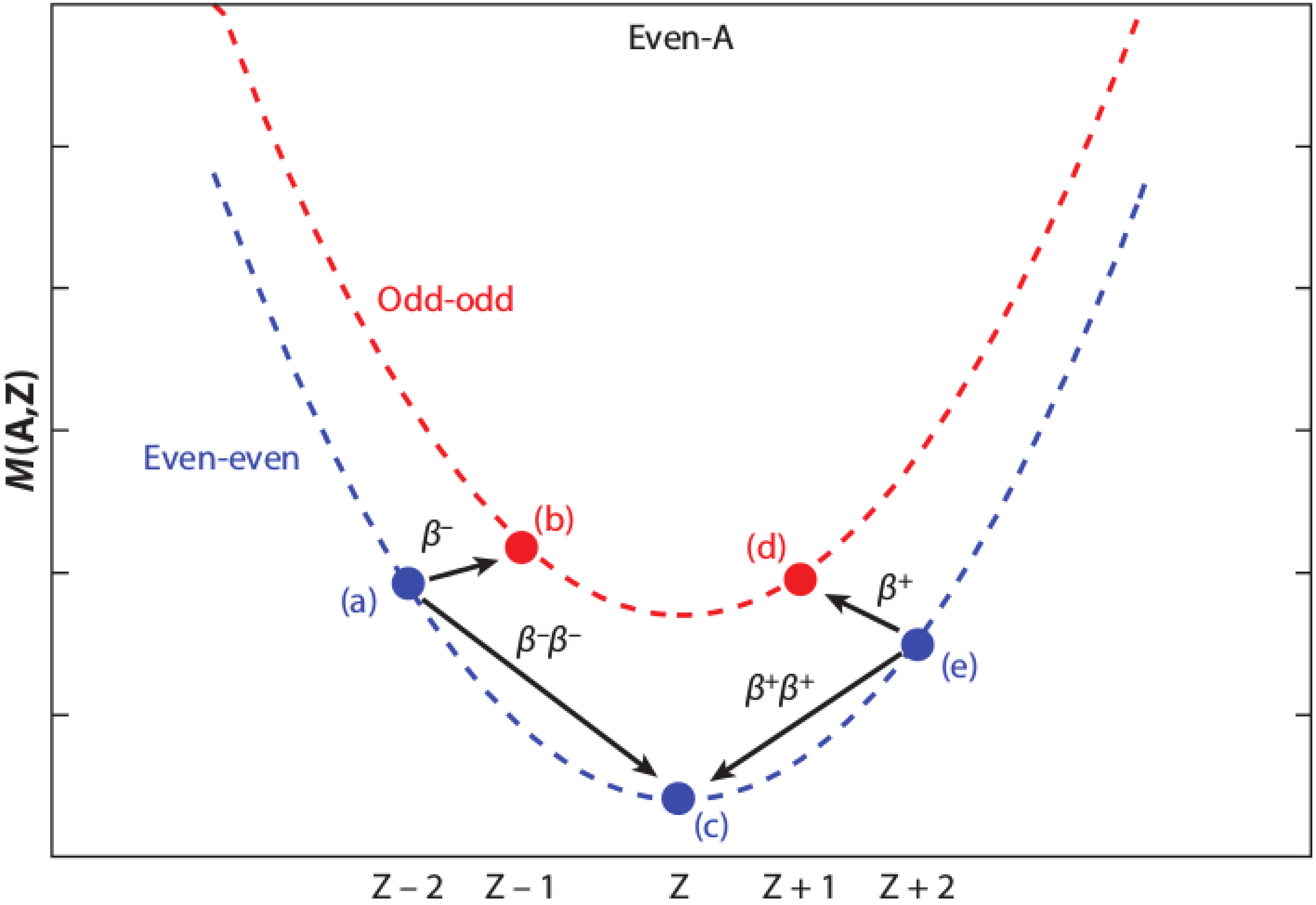




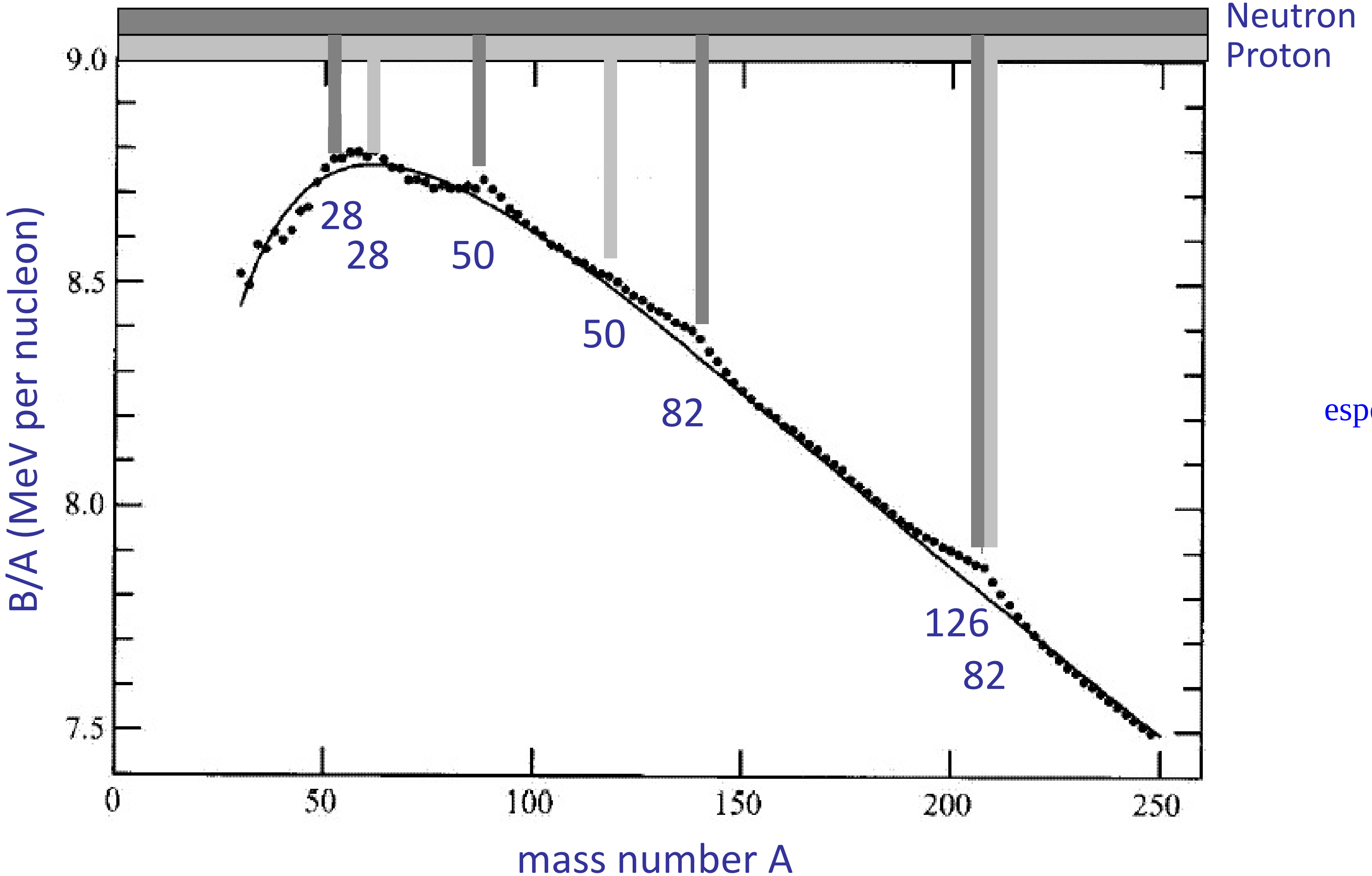
Mass parabola



Stable and radioactive nuclei



Deviation from liquid drop model



especially stabil:

$^{4}_{2}He_{2}$

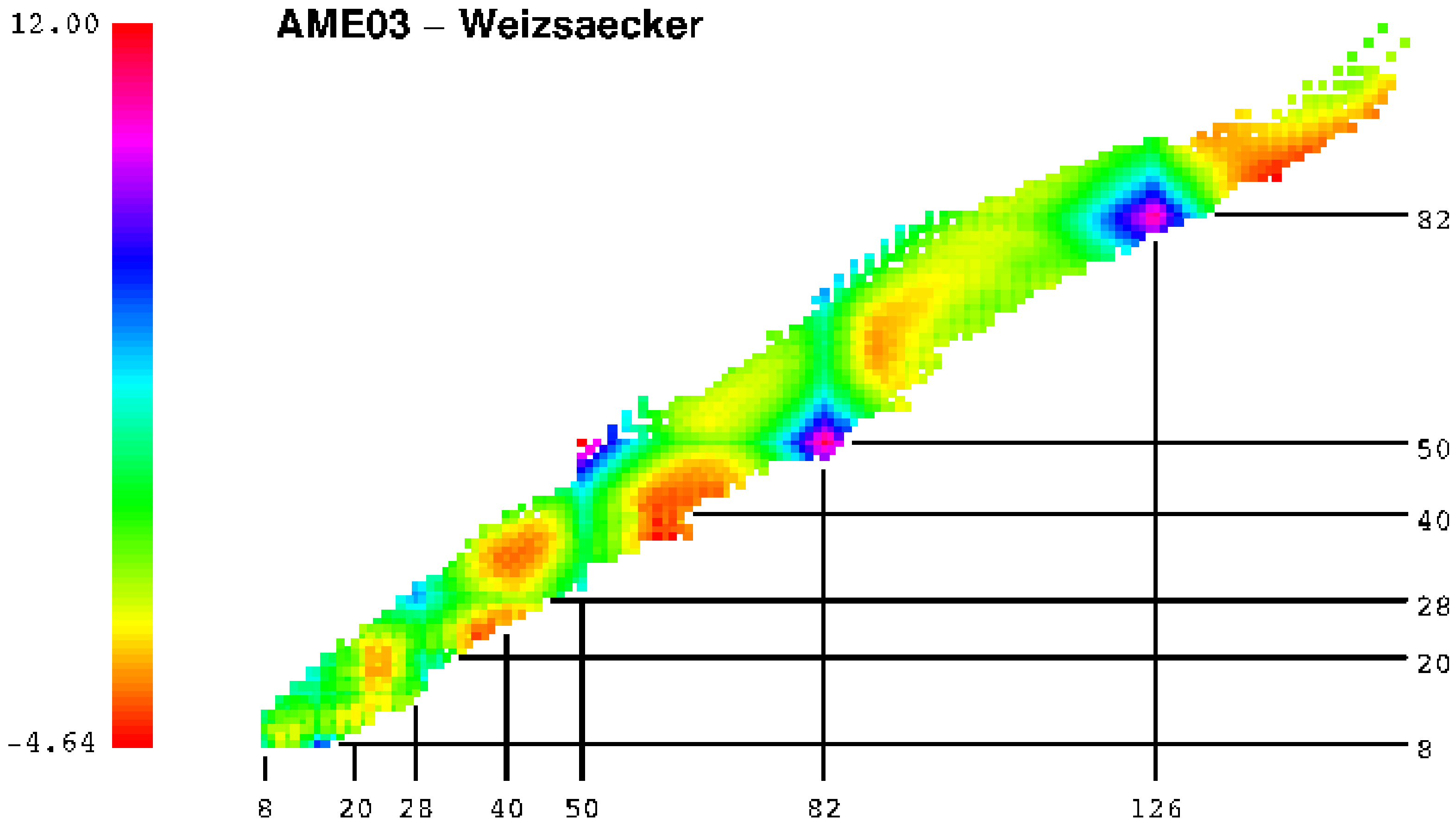
$^{16}_{8}O_{8}$

$^{40}_{20}Ca_{20}$

$^{48}_{20}Ca_{28}$

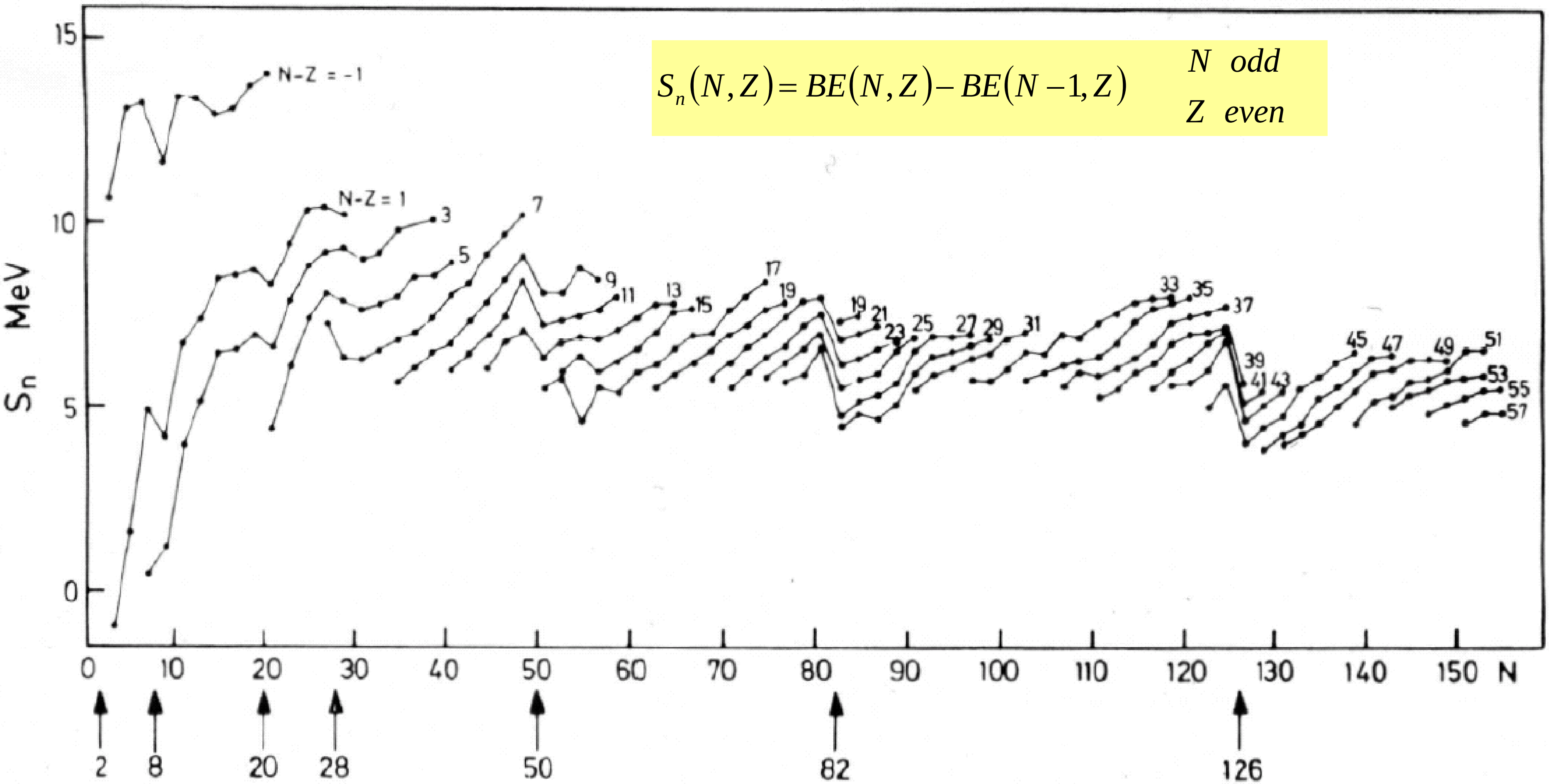
$^{208}_{82}Pb_{126}$

Deviation from liquid drop model



Neutron separation energy

$$S_n(N,Z) = BE(N,Z) - BE(N-1,Z) \qquad \begin{matrix} N & odd \\ Z & even \end{matrix}$$



2-neutron separation energy

$$S_{2n} = M({}^{Z+N-2}_{Z}X) - M({}^{Z+N}_{Z}X) + 2m_n$$