Outline: Nuclear shell model with residual interaction

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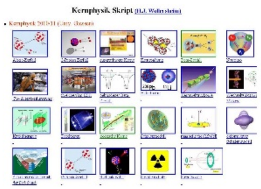
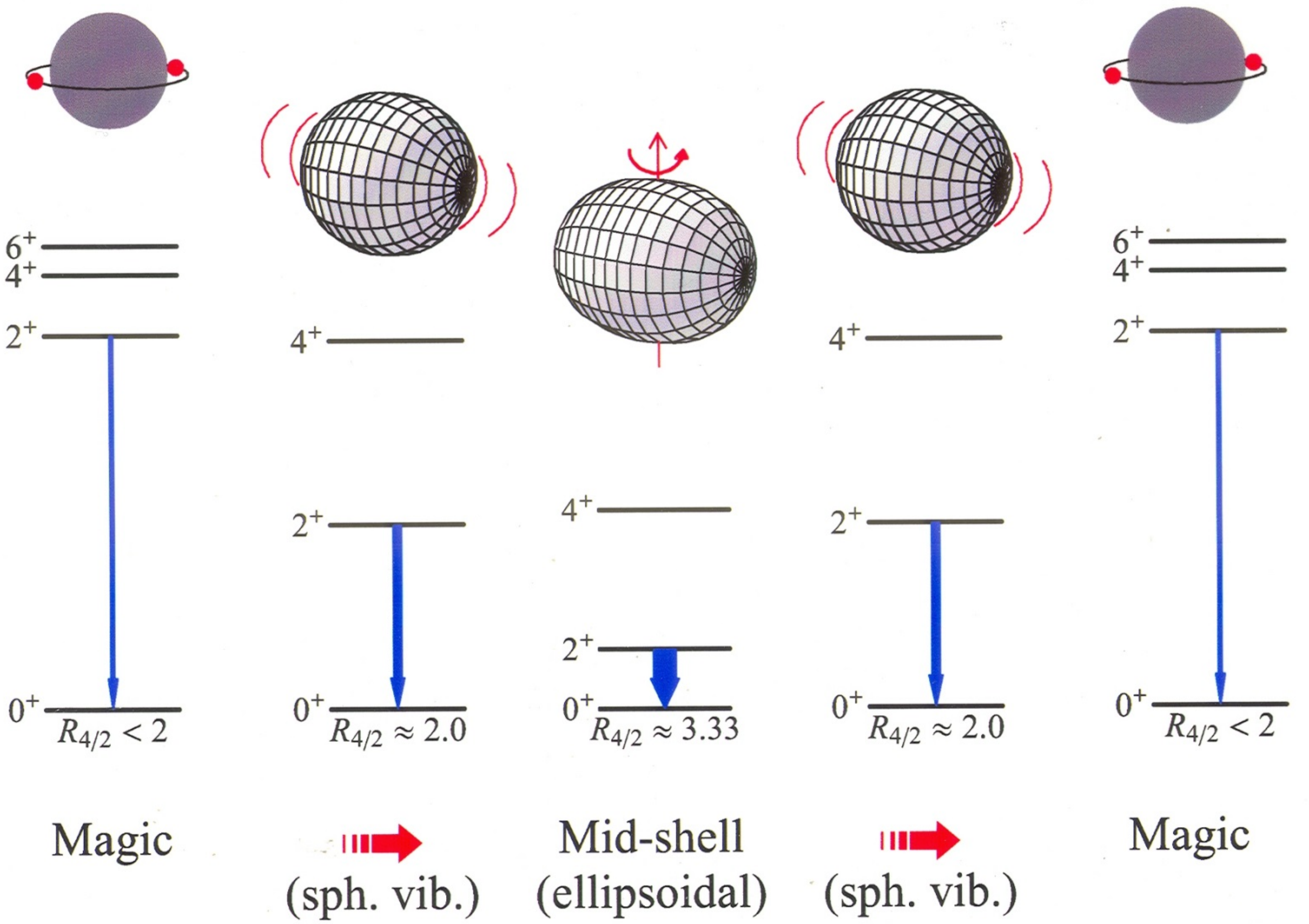
web-page: https://web-docs.gsi.de/~wolle/ and click on



1. experimental single-particle energies
2. coupling of two angular momenta
3. δ-interaction - pairing
4. generalized seniority scheme
5. signatures near closed shells

Evolution of nuclear structure (as a function of nucleon number)

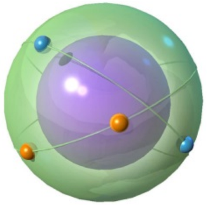


Shell model with residual interaction

$$H = H_0 + H_{residual}$$

Start with 2-particle system, that is a nucleus „doubly magic nucleus + 2 nucleons“

$$H_{residual} = H_{12}(r_{12})$$



Consider two identical valence nucleons with j_1 and j_2

Two questions:

What total angular momenta $j_1 + j_2 = J$ can be formed?

What are the energies of states with these J values?

Nuclear shell structure

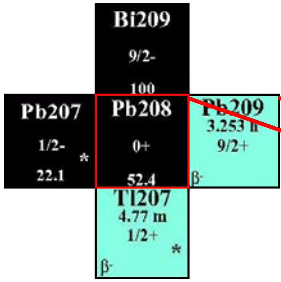


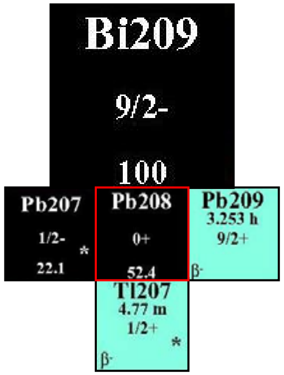
Table 1 -- Nuclear Shell Structure (from *Elementary Theory of Nuclear Shell Structure*, Maria Goeppert Mayer & J. Hans D. Jensen, John Wiley & Sons, Inc., New York, 1955.)

Angular Momentum ($h\Omega/2\pi$)		Spin-Orbit Coupling (1/2, 3/2, 5/2, 7/2...)	Number of Nucleons Shell	Total	Magic Number
7	1j				
		1j 15/2	16	[184]	{184}
		3d 3/2	4	[168]	
6	4s	4s 1/2	2	[164]	
6	3d	2g 7/2	8	[162]	
		1i 11/2	12	[154]	
		3d 5/2	6	[142]	
6	2g	2g 9/2	10	[136]	
6	1i				
		1i 13/2	14	[126]	{126}
		3p 1/2	2	[112]	
5	3p	3p 3/2	4	[110]	
		2f 5/2	6	[106]	
5	2f	2f 7/2	8	[100]	
		1h 9/2	10	[92]	
5	1h				
		1h 11/2	12	[82]	{82}
4	3s	3s 1/2	2	[70]	
		2d 3/2	4	[68]	
4	2d	2d 5/2	6	[64]	
		1g 7/2	8	[58]	
4	1g				
		1g 9/2	10	[50]	{50}
		2p 1/2	2	[40]	{40}
3	2p	1f 5/2	6	[38]	
		2p 3/2	4	[32]	
3	1f				
		1f 7/2	8	[28]	{28}
		1d 3/2	4	[20]	{20}
2	2s	2s 1/2	2	[16]	
2	1d	1d 5/2	6	[14]	
		1p 1/2	2	[8]	{8}
1	1p	1p 3/2	4	[6]	
0	1s	1s 1/2	2	[2]	{2}

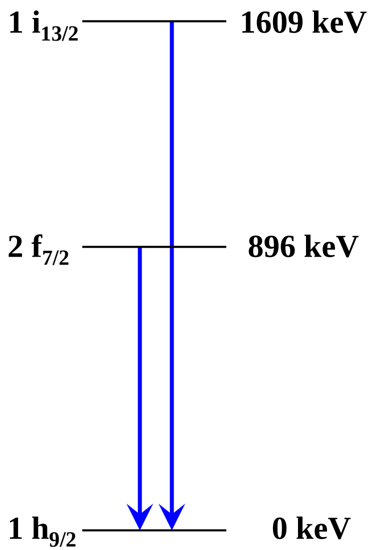
Maria Goeppert-Mayer

J. Hans D. Jensen

Experimental single-particle energies

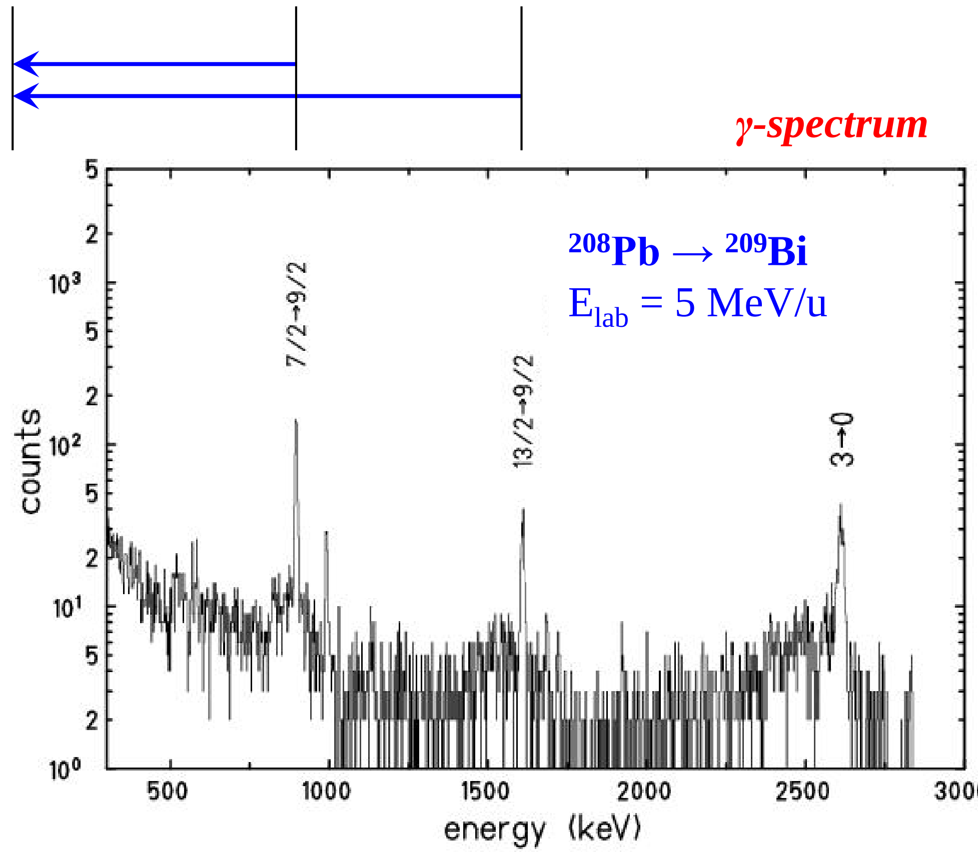


single-particle energies

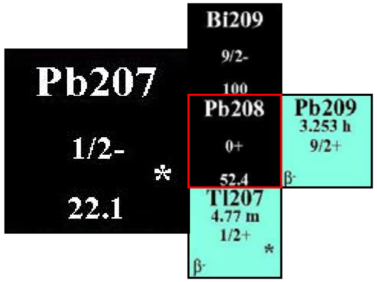


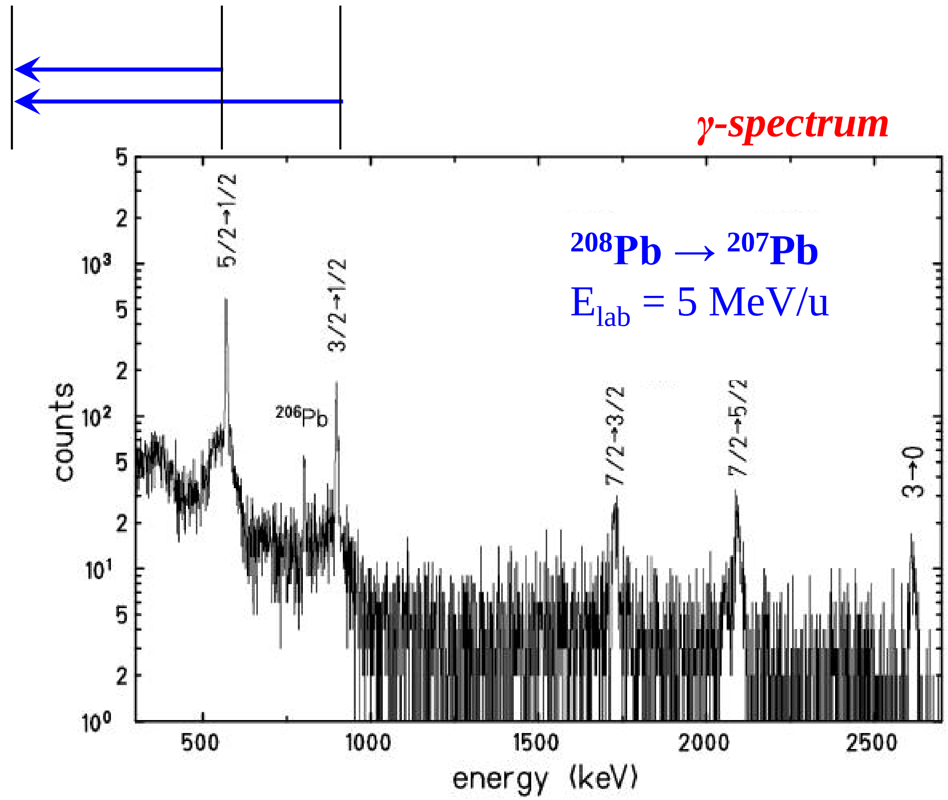
$^{209}_{83}Bi_{126}$

γ-spectrum

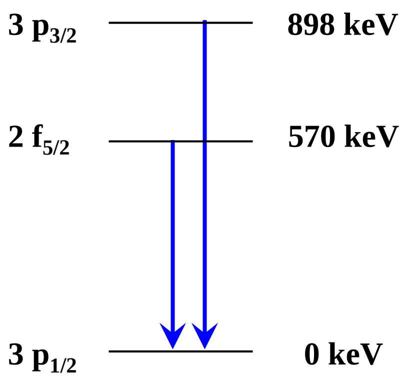


Experimental single-particle energies

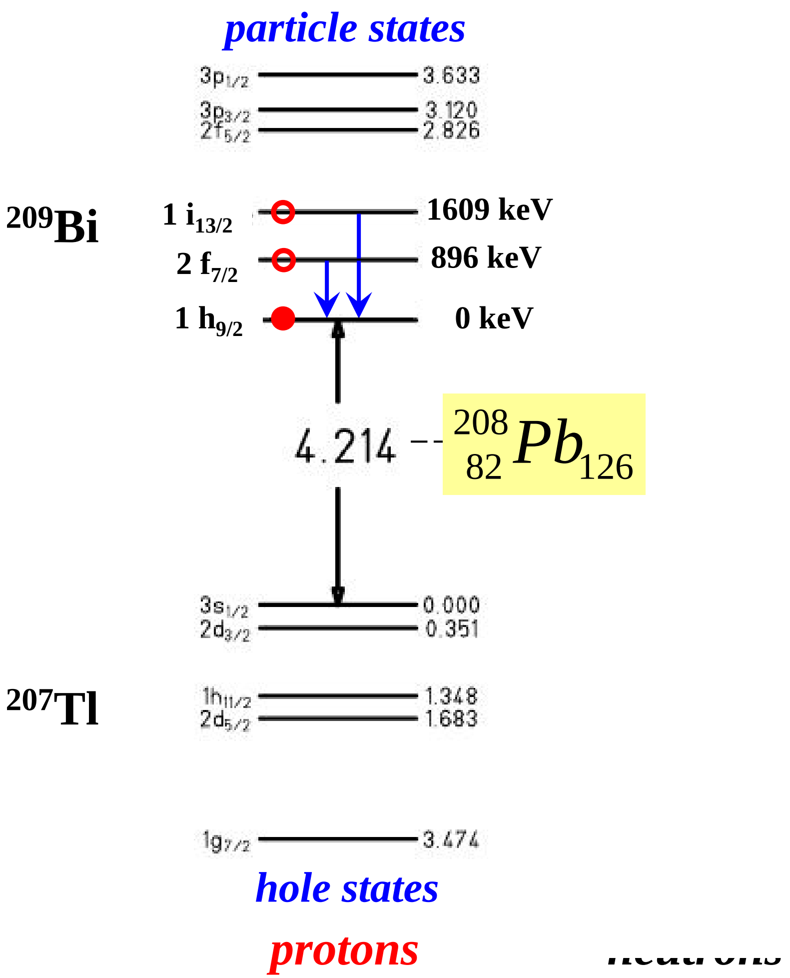




single-hole energies



$^{207}_{82}Pb_{125}$



^{209}Pb

^{207}Pb

energy of shell closure:

$$BE(^{209}Bi) - BE(^{208}Pb) = E(1h_{9/2})$$

$$BE(^{207}Tl) - BE(^{208}Pb) = -E(3s_{1/2})$$

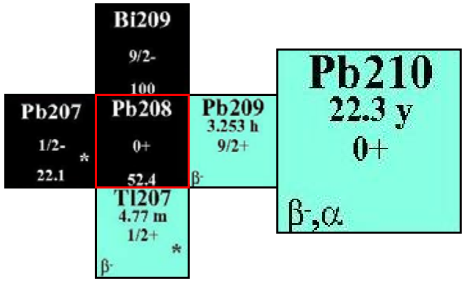
$$E(1h_{9/2}) - E(3s_{1/2}) = BE(^{209}Bi) + BE(^{207}Tl) - 2 \cdot BE(^{208}Pb)$$
$$= -4.211\,MeV$$

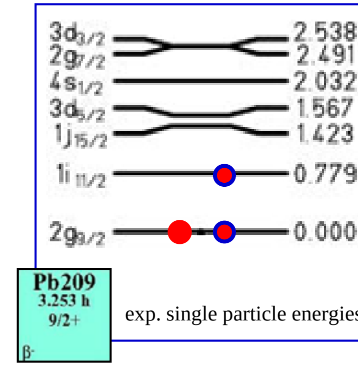
$$BE(^{209}Pb) - BE(^{208}Pb) = E(2g_{9/2})$$

$$BE(^{207}Pb) - BE(^{208}Pb) = -E(3p_{1/2})$$

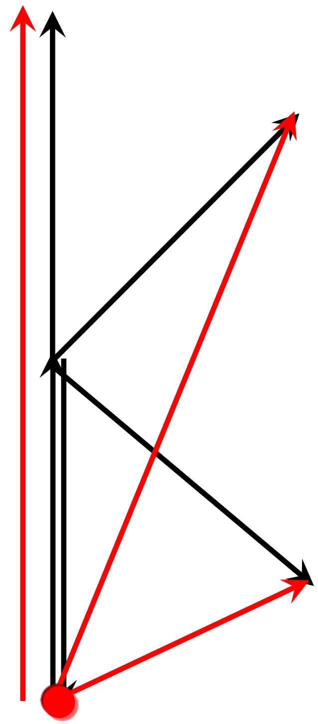
$$E(2g_{9/2}) - E(3p_{1/2}) = BE(^{209}Pb) + BE(^{207}Pb) - 2 \cdot BE(^{208}Pb)$$
$$= -3.432$$

Level scheme of ^{210}Pb





Coupling of two angular momenta



$j_1 + j_2$ all values from: $j_1 - j_2$ to $j_1 + j_2$ ($j_1 = j_2$)

Example: $j_1 = 3$, $j_2 = 5$: $J = 2, 3, 4, 5, 6, 7, 8$

BUT: For $j_1 = j_2$: $J = 0, 2, 4, 6, \ldots (2j - 1)$ (Why these?)

Coupling of two angular momenta

How can we know which total angular momenta J are observed for the coupling of two identical nucleons in the same orbit with angular momentum j?

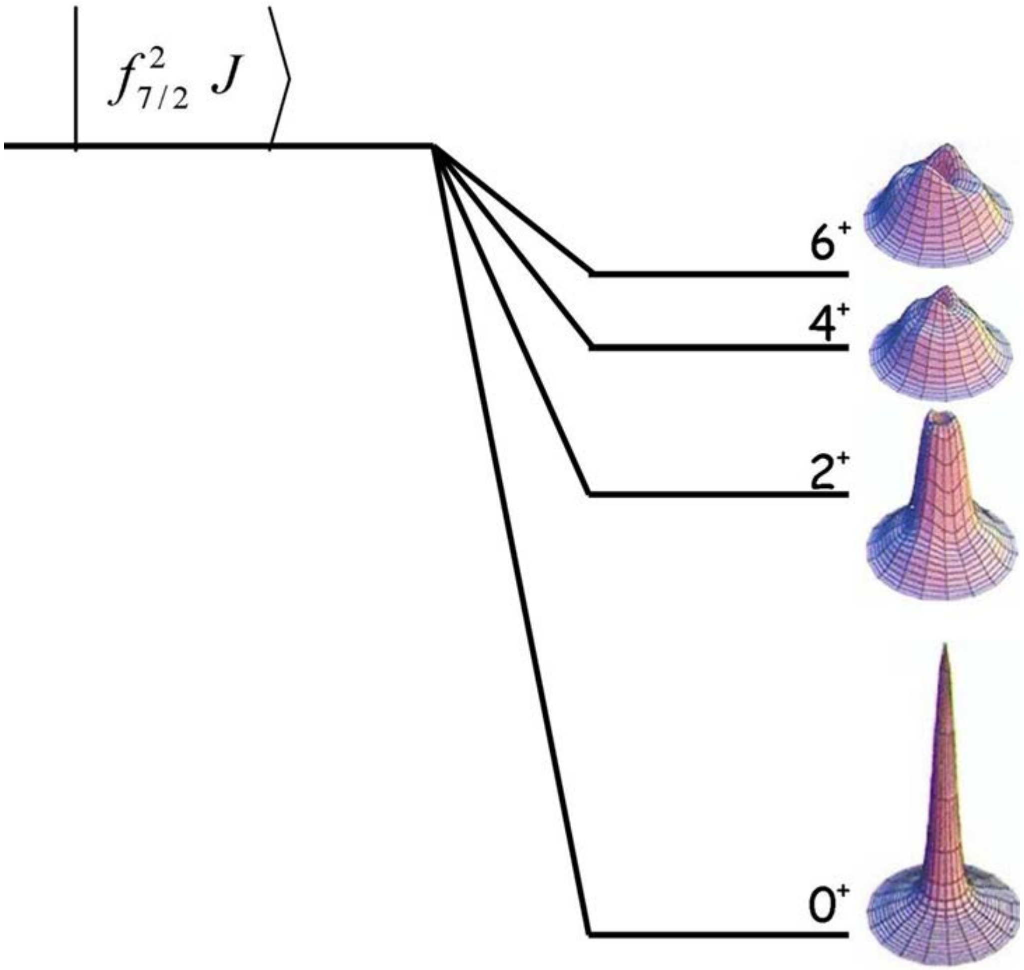
Several methods: easiest is the "m-scheme".

Table 5.1 *m scheme for the configuration* $|(7/2)^2 J\rangle$*

$j_1 = 7/2$ m_1	$j_2 = 7/2$ m_2	M	J
7/2	5/2	6	
7/2	3/2	5	
7/2	1/2	4	
7/2	–1/2	3	6
7/2	–3/2	2	
7/2	–5/2	1	
7/2	–7/2	0	
5/2	3/2	4	
5/2	1/2	3	
5/2	–1/2	2	4
5/2	–3/2	1	
5/2	–5/2	0	
3/2	1/2	2	
3/2	–1/2	1	2
3/2	–3/2	0	
1/2	–1/2	0	0

* Only positive total M values are shown. The table is symmetric for $M < 0$.

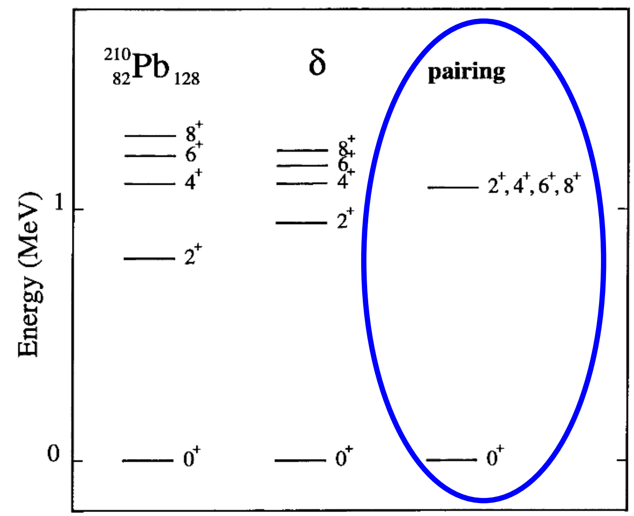
Coupling of two angular momenta



Residual interaction - pairing

➢ ***Spectrum of ^{210}Pb:*** $^{208}_{82}Pb_{126}$ core + 2 neutrons

$\left|g^2_{9/2}; J = 2,4,6,8\right\rangle$ $\nu = 2$ (two unpaired nucleons)

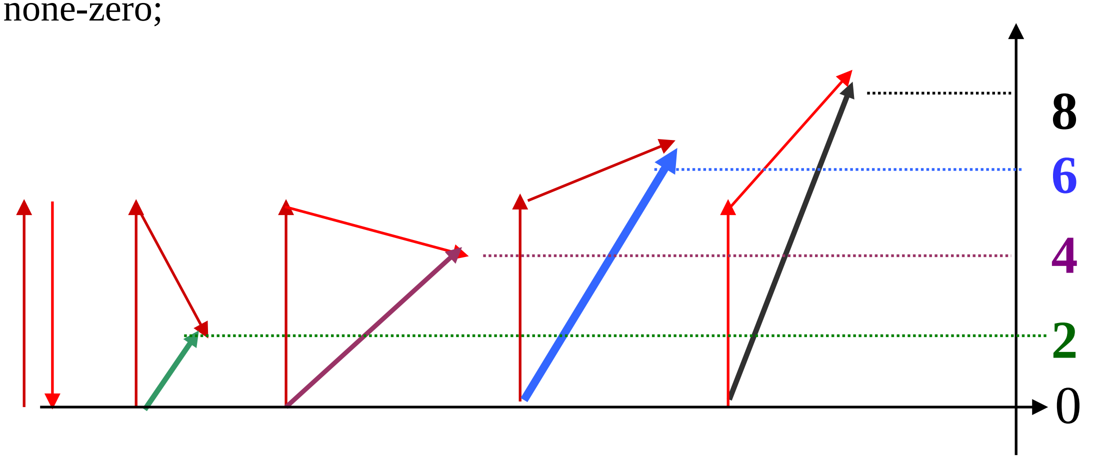


➢ ***Assume pairing interaction in a single-j shell***

$$\left\langle j^2\, JM_J \middle| V_{pairing}(r_1, r_2) \middle| j^2\, JM_J \right\rangle = \begin{cases} -\frac{1}{2}(2j+1)\cdot g & \nu = 0, J = 0 \\ 0, & \nu = 2, J \neq 0 \end{cases}$$

For the ground state the energy eigenvalue is none-zero;
all nucleons paired (ν=0) and spin J=0.

➢ ***The δ-interaction yields a simple geometrical expression for the coupling of two nucleons***



Pairing – δ-interaction

$$\Delta E(j_1 j_2\, J) = \langle j_1 j_2\, JM|V_{12}|j_1 j_2\, JM\rangle = \frac{1}{\sqrt{2J+1}}\langle j_1 j_2\, J\|V_{12}\|j_1 j_2\, J\rangle$$

wave function: $\varphi(n\ell m) = \frac{1}{r} R_{n\ell}(r) \cdot Y_{\ell m}(\theta,\phi)$

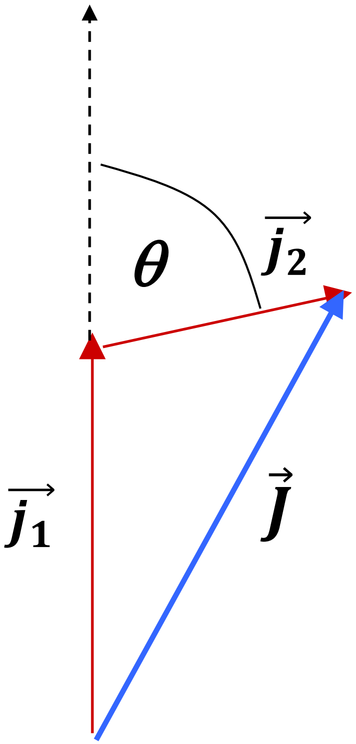
interaction: $V_{12}(\delta) = \frac{-V_0}{r_1 r_2}\,\delta(r_1 - r_2)\,\delta(cos\theta_1 - cos\theta_2)\,\delta(\phi_1 - \phi_2)$

$$\Delta E(j_1 j_2\, J) = -V_0 \cdot F_R(n_1\ell_1 n_2\ell_2) \cdot A(j_1 j_2\, J)$$

with $F_R(n_1\ell_1 n_2\ell_2) = \frac{1}{4\pi}\int \frac{1}{r^2} R^2_{n_1\ell_1}(r) R^2_{n_2\ell_2}(r)\, dr$

and $A(j_1 j_2\, J) = (2j_1+1)\cdot(2j_2+1)\cdot\begin{pmatrix} j_1 & j_2 & J \\ 1/2 & -1/2 & 0\end{pmatrix}^2$

A. de-Shalit & I. Talmi: Nuclear Shell Theory, p.200



$$J^2 = j_1^2 + j_2^2 + 2|j_1||j_2|cos\theta$$

$$cos\theta = \frac{J^2 - j_1^2 - j_2^2}{2|j_1||j_2|} = \frac{J(J+1) - j_1(j_1+1) - j_2(j_2+1)}{2\sqrt{j_1(j_1+1)j_2(j_2+1)}}$$

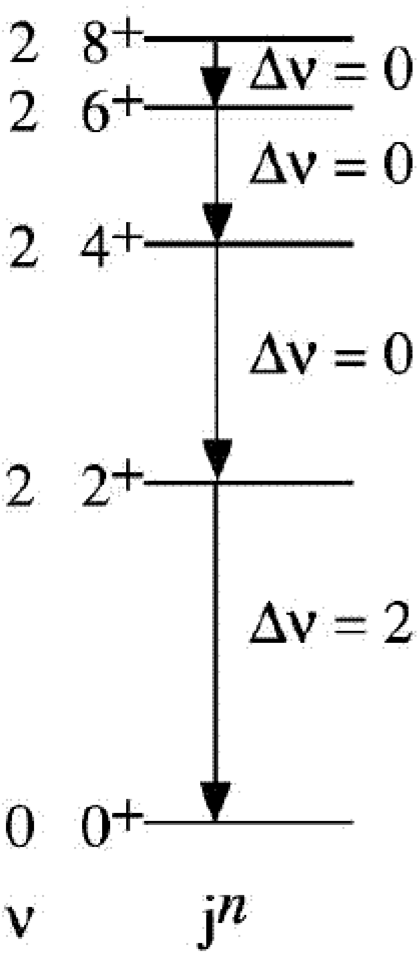
$$cos\theta \cong \frac{J^2 - 2j^2}{2j^2} \quad for \quad j_1 = j_2 = j \quad and \quad j, J \gg 1$$

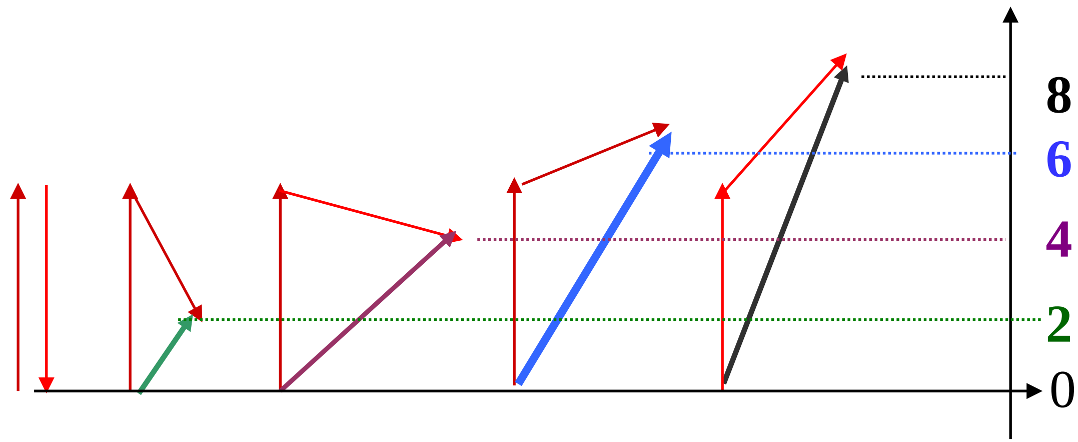
$\theta = 0^0$ belongs to **large J**, **$\theta = 180^0$** belongs to **small J**

example $h^2_{11/2}$: J=0 θ =180⁰, J=2 θ ~159⁰, J=4 θ ~137⁰,
J=6 θ ~114⁰, J=8 θ ~ 87⁰, J=10 θ ~ 49⁰

$$sin\theta = \sqrt{1 - cos^2\theta} = \frac{J}{j}\left[1 - \frac{J^2}{4j^2}\right]^{1/2} \qquad sin\frac{\theta}{2} = [(1 - cos\theta)/2]^{1/2} = \left(1 - \frac{J^2}{4j^2}\right)^{1/2}$$

$$\begin{pmatrix} j & j & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \approx \left(1 - \frac{J^2}{4j^2}\right)\frac{1}{\pi}\frac{1}{Jj\left(1 - \frac{J^2}{4j^2}\right)^{1/2}} = \frac{sin^2(\theta/2)}{\pi \cdot j^2 \cdot sin\theta} = \frac{tan(\theta/2)}{\pi \cdot j^2}$$





δ-interaction yields a simple geometrical explanation for **Seniority-Isomers:**

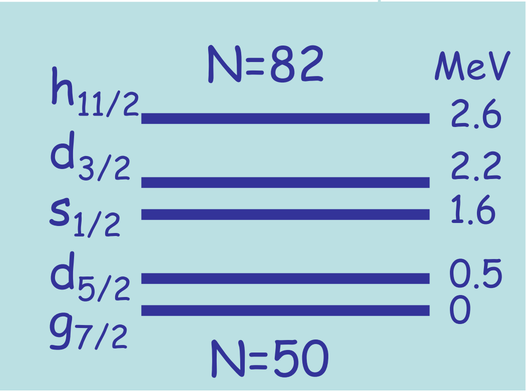
$$\Delta E \sim -V_o \cdot F_R \cdot \tan\left(\theta/2\right)$$

for $T=1$, even J

energy intervals between states 0^+, 2^+, 4^+, ...$(2j\text{-}1)^+$ decrease with increasing spin.

The ^{100}Sn / ^{132}Sn region, a brief background

Single particle energies



The ^{100}Sn / ^{132}Sn region

Z = 50

$g_{7/2}$

Sn102	Sn103	Sn104	Sn105	Sn106	Sn107	Sn108	Sn109	Sn110	Sn111
	7 s	20.8 s	31 s	115 s	2.90 m	10.30 m	18.0 m	4.11 h	35.3 m
0+		0+		0+	(5/2+)	0+	5/2(+)	0+	7/2+
	EC	EC	ECp	EC	EC	EC	EC	EC	EC

$d_{5/2}$ $s_{1/2}$ $d_{3/2}$

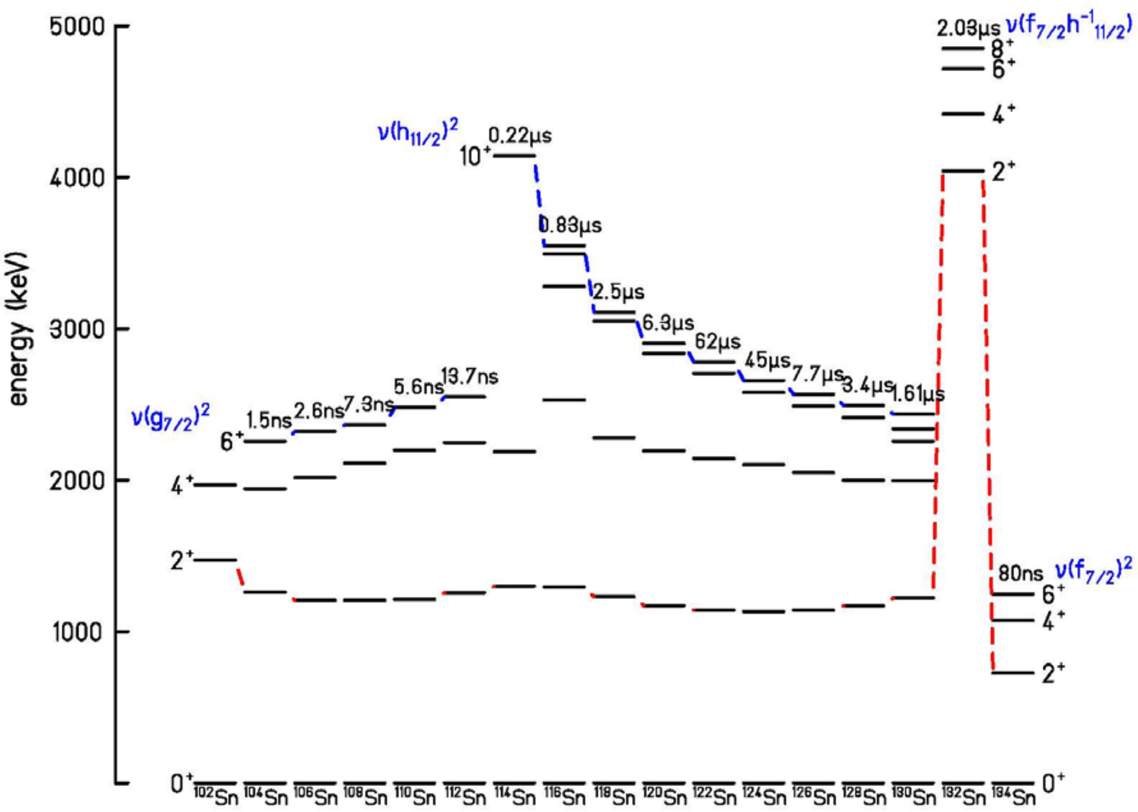
Sn112	Sn113	Sn114	Sn115	Sn116	Sn117	Sn118	Sn119	Sn120
	115.09 d							
0+	1/2+	0+	1/2+	0+	1/2+	0+	1/2+	0+
0.97	EC	0.65	0.34	14.53	7.68	24.23	8.59	32.59

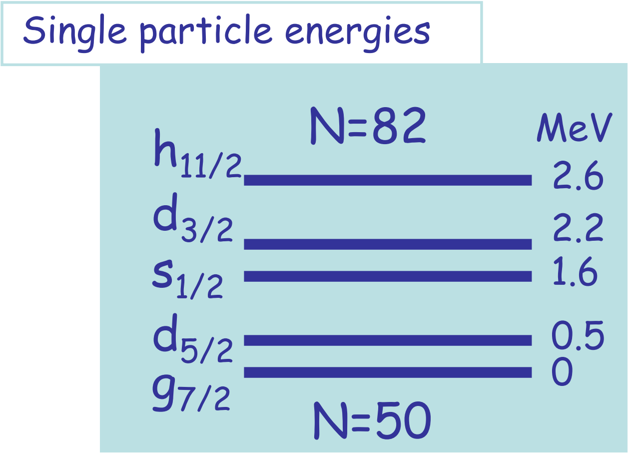
Naïve single particle filling

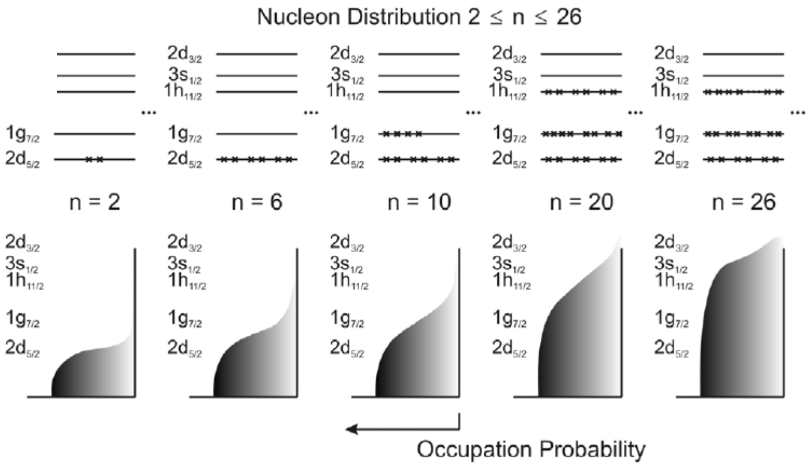
$h_{11/2}$

Sn121	Sn122	Sn123	Sn124	Sn125	Sn126	Sn127	Sn128	Sn129	Sn130	Sn131	Sn132
27.06 h		129.2 d		9.64 d	1E+5 y	2.10 h	59.07 m	2.23 m	3.72 m	56.0 s	39.7 s
3/2+	0+	11/2-	0+	11/2-	0+	(11/2-)	0+	(3/2+)	0+	(3/2+)	0+
β-	4.63	β-	5.79	β-	β-	β-	β-	β-	β-	β-	β-

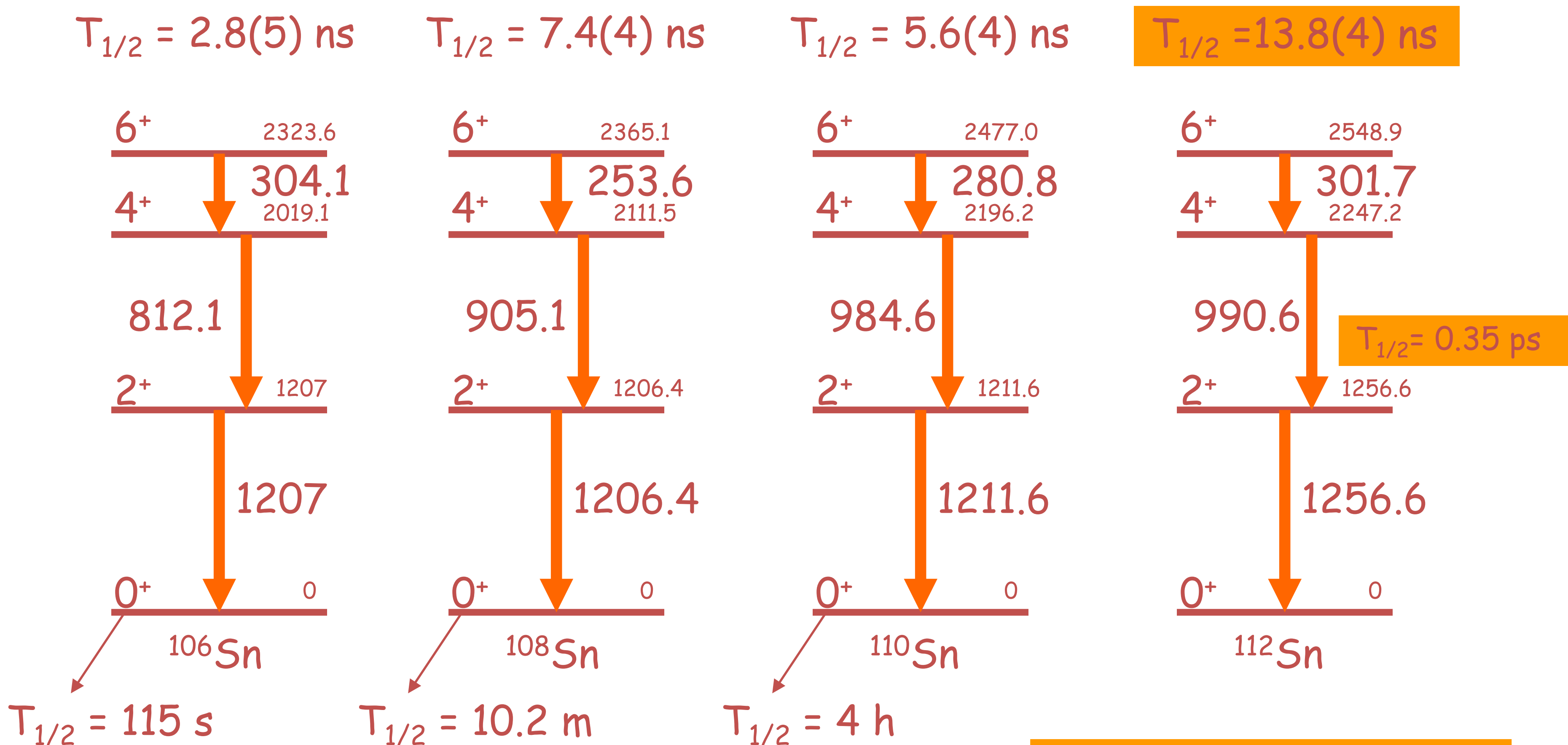
The ^{100}Sn / ^{132}Sn region, isomeric states







Isomeric states in ^{106}Sn – ^{112}Sn

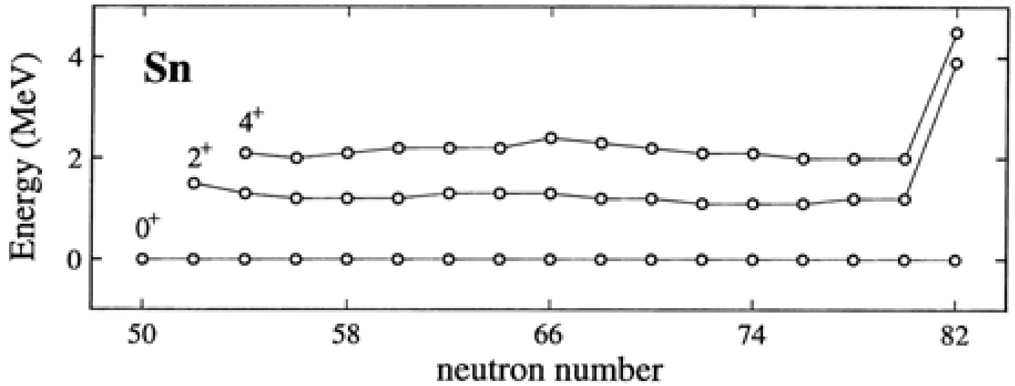


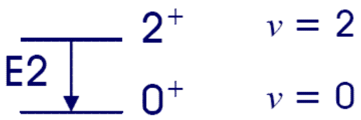
$B_{exp}(E2,6^+\text{->}4^+)$ = 0.49(+-0.02) W.u

$B_{exp}(E2,2^+\text{->}0^+)$ = 16.0 W.u

Generalized seniority scheme

Seniority quantum number **ν** is equal to the number of unpaired particles in the **j^n** configuration, where **n** is the number of valence nucleons.





energy spacing between ν=2 and ground state (ν=0, J=0):

$$E(j^n, \nu = 2, J) - E(j^n, \nu = 0, J = 0) = \langle j^2\, J|V|j^2\, J\rangle + \frac{n-2}{2}\cdot V_0 - \frac{n}{2}\cdot V_0$$

$$= \langle j^2\, J|V|j^2\, J\rangle - V_0$$

independent of **n**

energy spacing within ν=2 states:

$$E(j^n, \nu = 2, J) - E(j^n, \nu = 2, J') = \left[\langle j^2\, J|V|j^2\, J\rangle + \frac{n-2}{2}\cdot V_0\right] - \left[\langle j^2\, J'|V|j^2\, J'\rangle + \frac{n-2}{2}\cdot V_0\right]$$

$$= \langle j^2\, J|V|j^2\, J\rangle - \langle j^2\, J'|V|j^2\, J'\rangle$$

independent of **n**

G. Racah et al., Phys. Rev. 61 (1942), 186 and Phys. Rev. 63 (1943), 367

Seniority quantum number **v** is equal to the number of unpaired particles in the $\mathbf{j^n}$ configuration, where **n** is the number of valence nucleons.

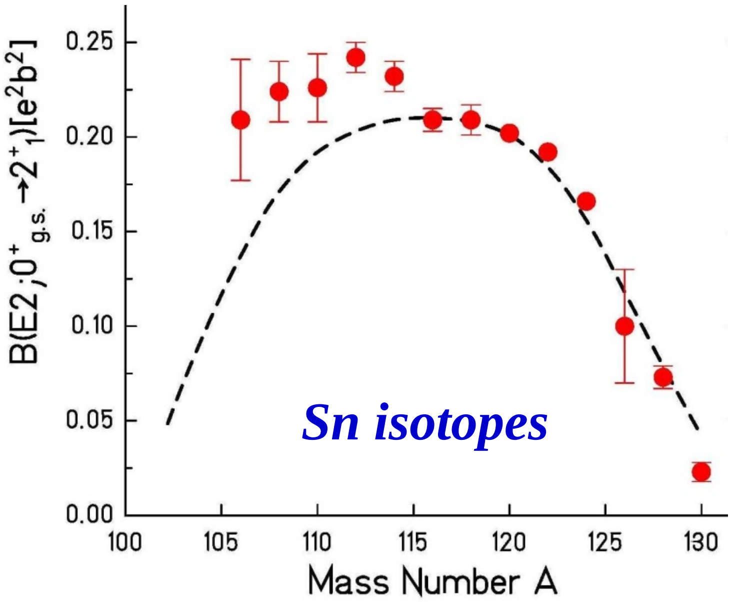
E2 transition rates: $B(E2; J_i \to J_f) = \frac{1}{2 \cdot J_i + 1} \cdot \langle J_f \| Q \| J_i \rangle^2$

$$\langle j^n J = 2 \| Q \| j^n J = 0 \rangle^2 = \left[\frac{n \cdot (2j + 1 - n)}{2 \cdot (2j - 1)} \right] \cdot \langle j^2 J = 2 \| Q \| j^2 J = 0 \rangle^2$$

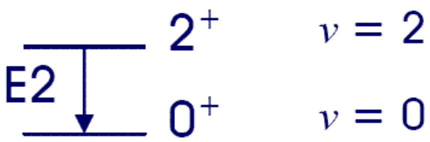
$$= \left[\frac{(2j + 1)^2}{2 \cdot (2j - 1)} \right] \cdot f \cdot (1 - f) \cdot \langle j^2 J = 2 \| Q \| j^2 J = 0 \rangle^2 \quad f = \frac{n}{2j + 1} \to 1 \quad for\ large\ n$$

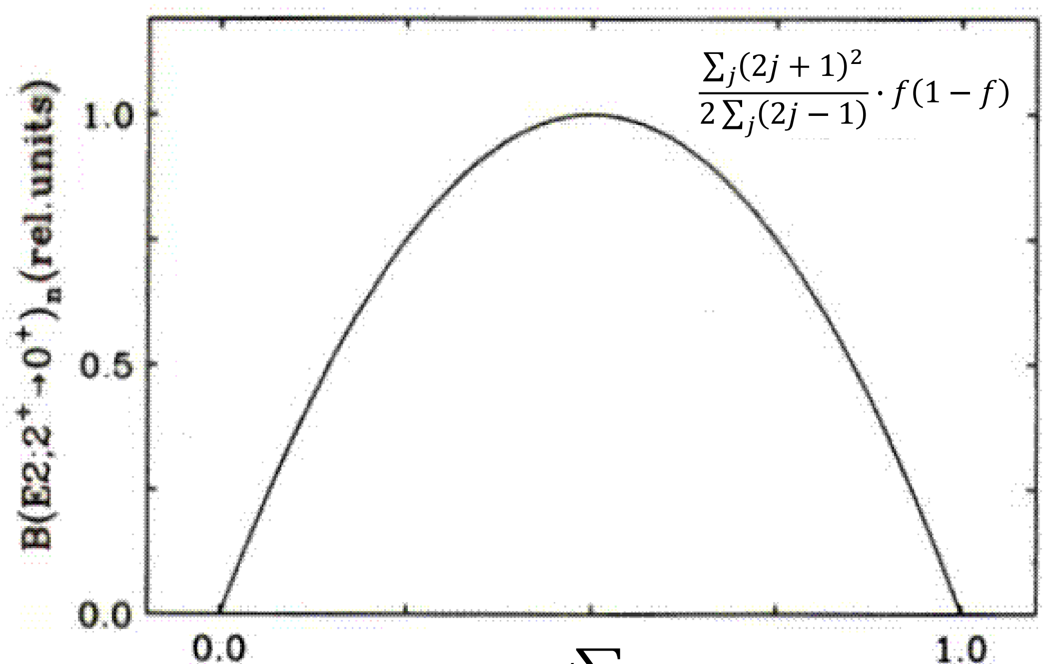
$B(E2; 2_1^+ \to 0_1^+) \approx f \cdot (1 - f)$

$\approx \mathbf{N_{particles} * N_{holes}}$



Seniority quantum number **v** is equal to the number of unpaired particles in the **j^n** configuration, where **n** is the number of valence nucleons.





$$f = n / \sum_j (2j+1)$$

$$B(E2; 2_1^+ \rightarrow 0_1^+) \approx f \cdot (1-f)$$

$$\approx \mathbf{N_{particles} * N_{holes}}$$

$$\sum_j (2j+1) \equiv$$ number of nucleons between shell closures

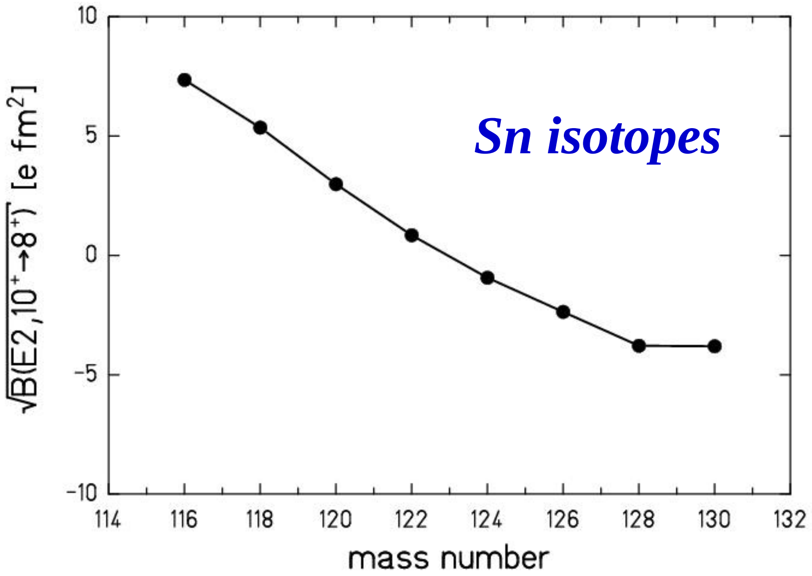
Generalized seniority scheme

Seniority quantum number **ν** is equal to the number of unpaired particles in the $\mathbf{j^n}$ configuration, where **n** is the number of valence nucleons.

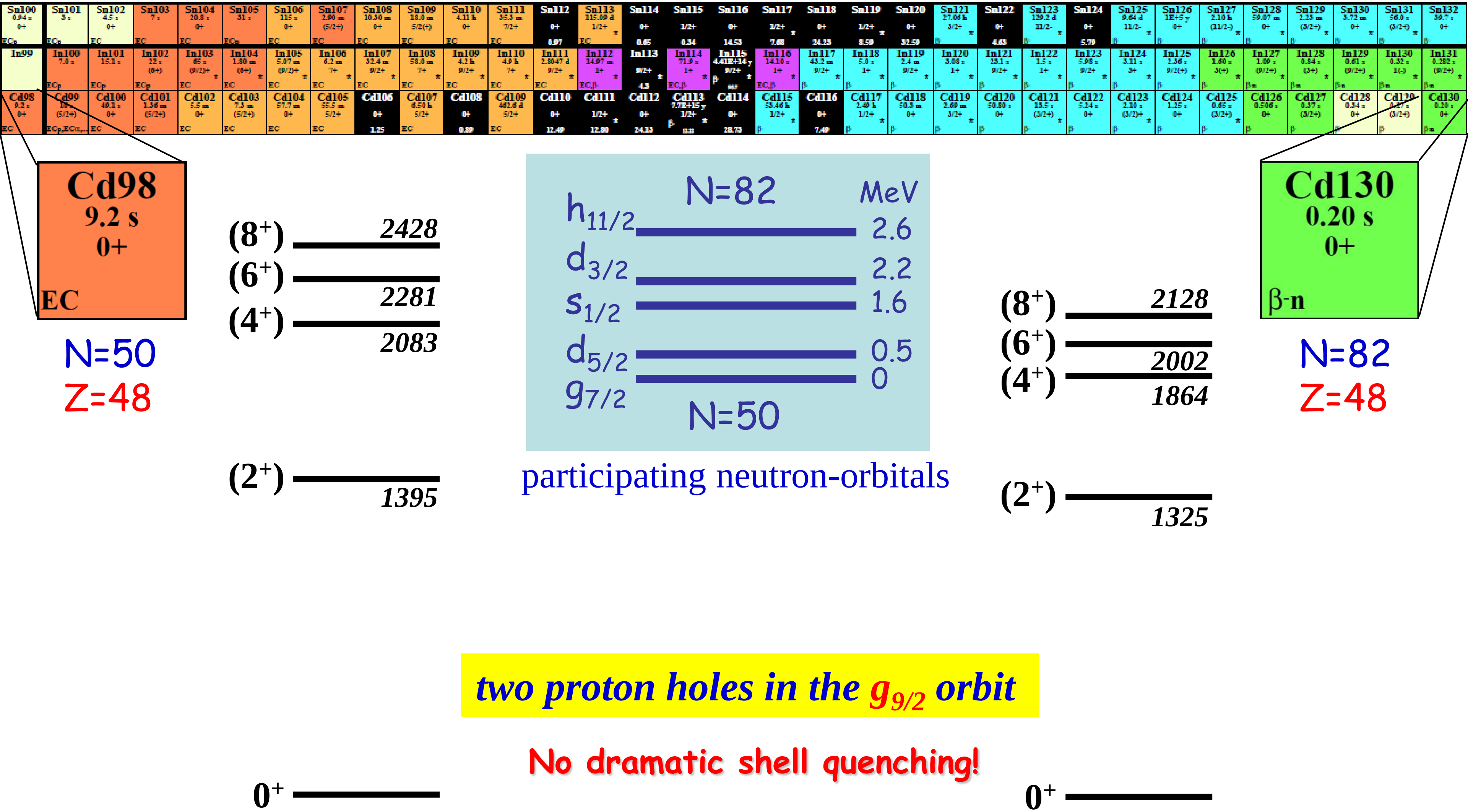
E2 transition rates that do not change seniority (ν=2):

$$\langle j^n\, J \| Q \| j^n\, J' \rangle = \left[\frac{2j+1-2n}{2j-3}\right] \cdot \langle j^2\, J \| Q \| j^2\, J' \rangle$$

$$= \frac{2j+1}{2j-3} \cdot [1-2f] \cdot \langle j^2\, J \| Q \| j^2\, J' \rangle$$



$8^+(g_{9/2})^{-2}$ seniority isomers in ^{98}Cd and ^{130}Cd



A. Blazhev et al., Phys. Rev. C69 (2004) 064304

A. Jungclaus et al., Phys. Rev. Lett. 99 (2007), 132501

Spin isomer in ^{98}Cd

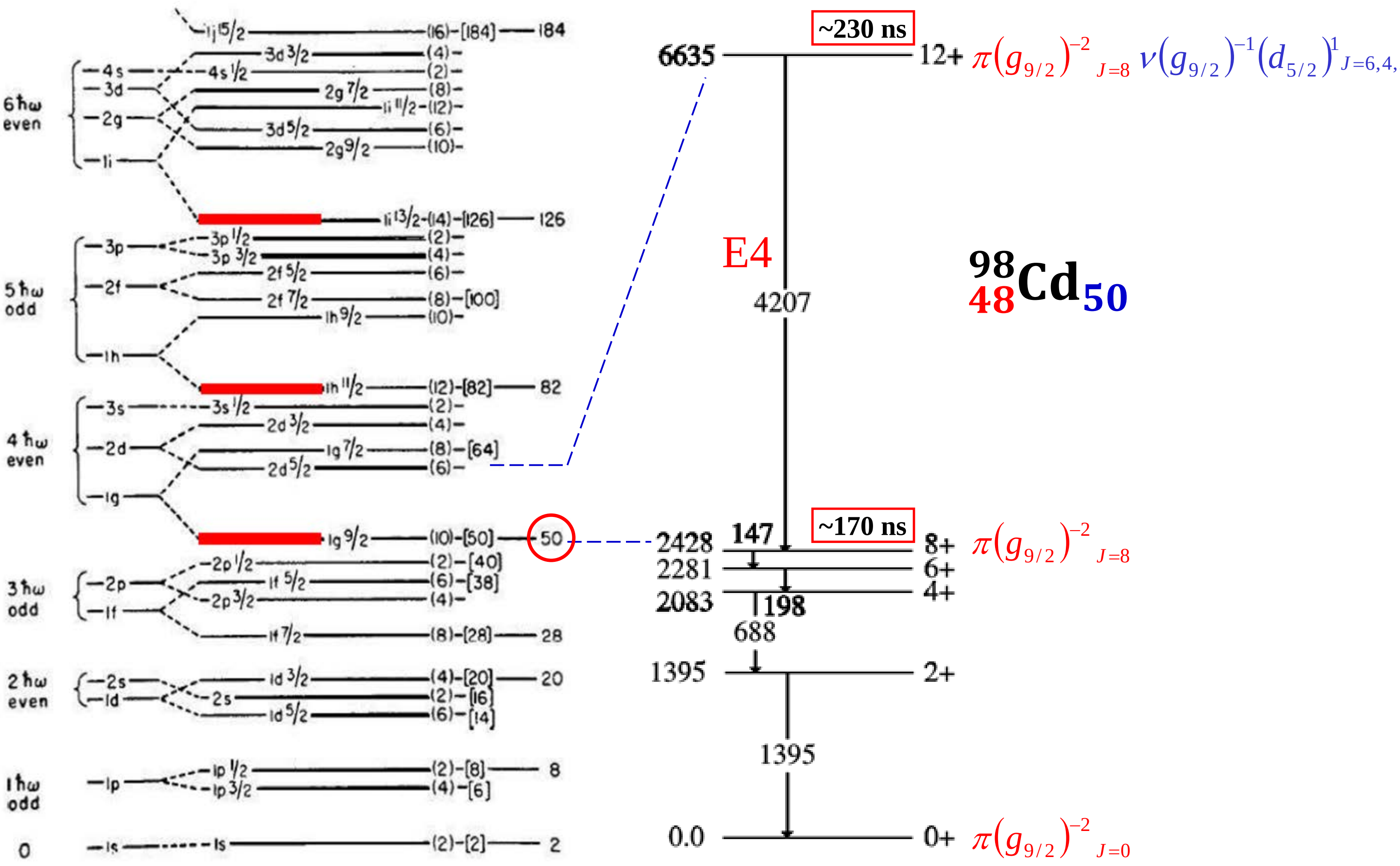
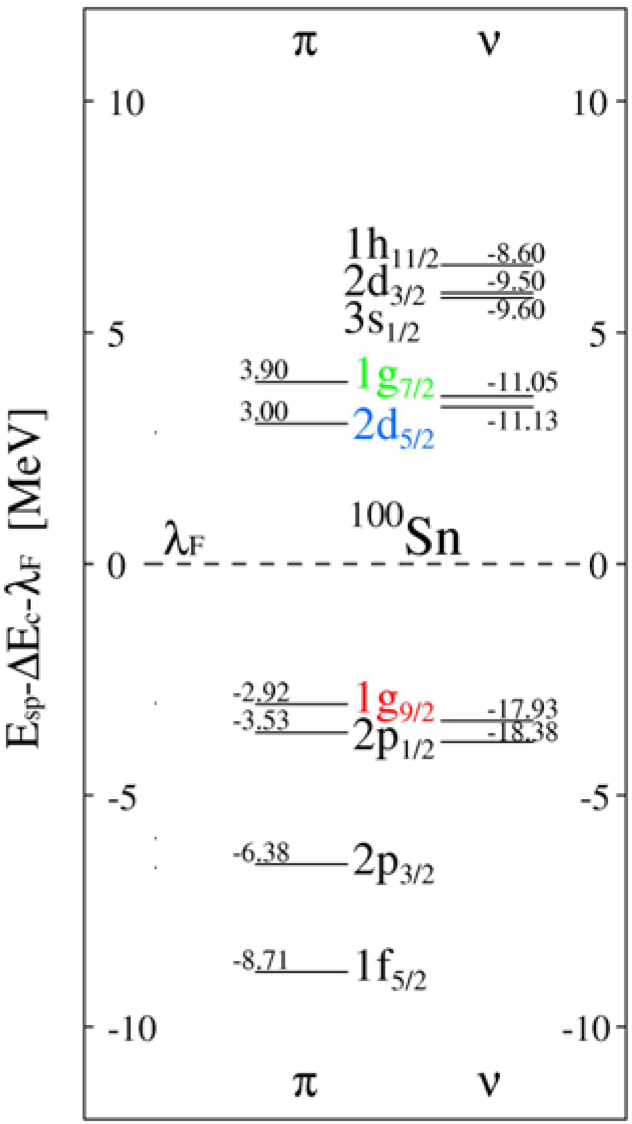
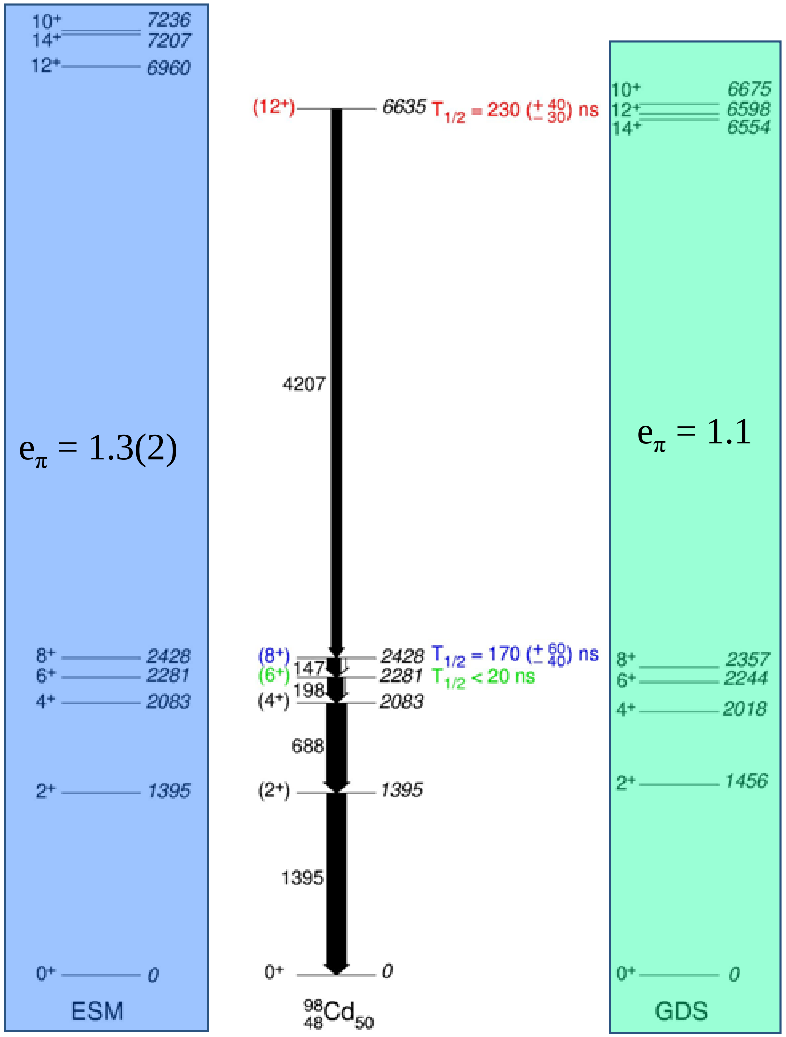


Fig. 7. Realistic level diagram for protons.

A. Blazhev et al., Phys.Rev.C69 (2004) 064304

Core excited states in ^{98}Cd





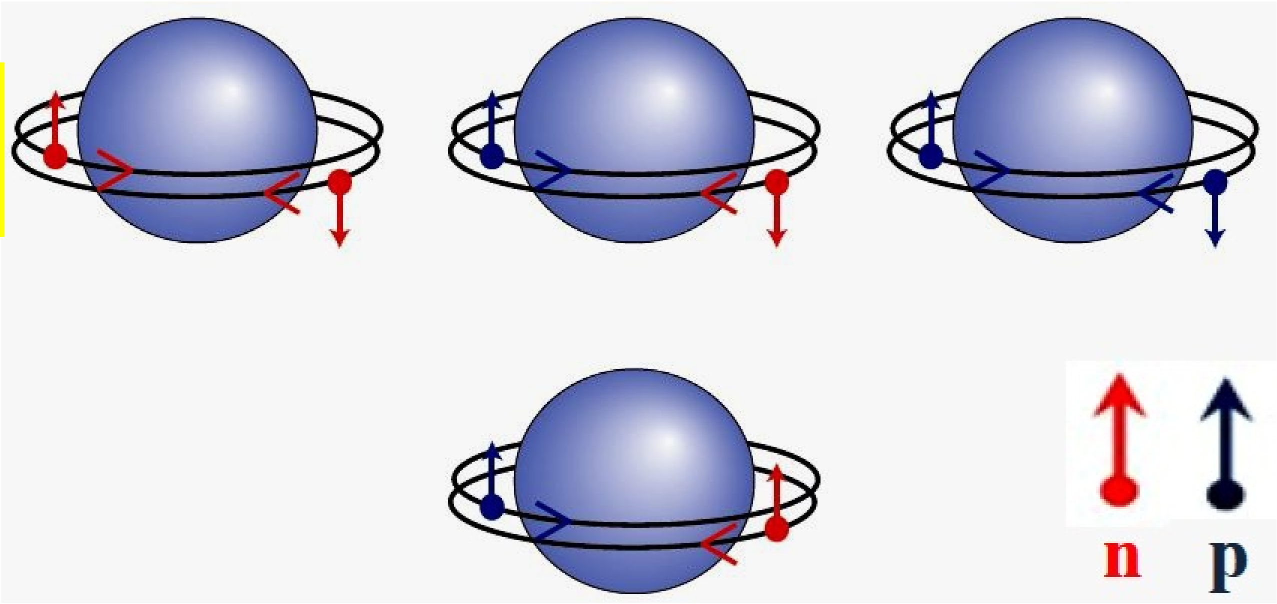
ESM: H. Grawe et al., NS98 AIP CP 481 (1999) 177

GDS: F.Nowacki, Nuc. Phys. A 704 (2002) 223c

A. Blazhev et al., Phys.Rev.C69 (2004) 064304

Nature of nucleon pair correlations

T=1 S=0
isovector pairing



T=0 S=1
isoscalar pairing

Deuteron-like pair condensate

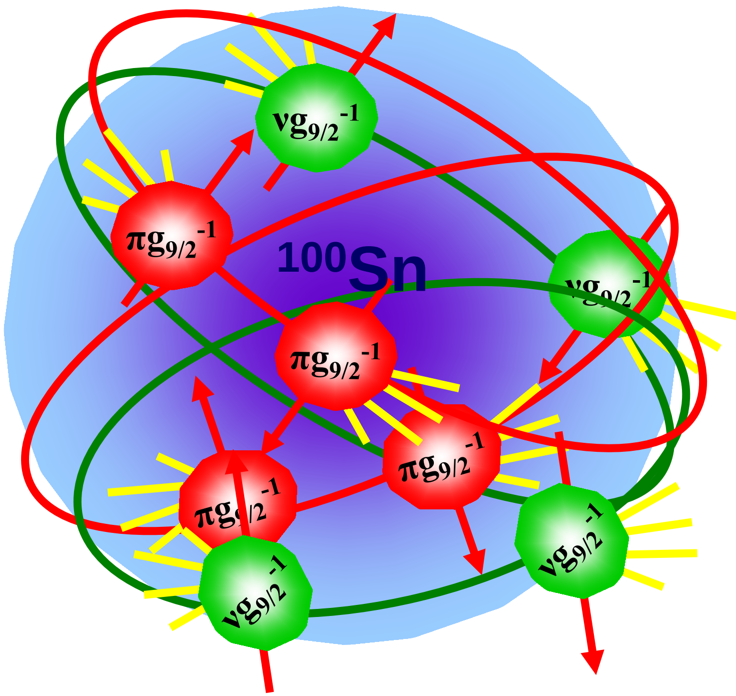
Does T=0 pairing exist?

- The strong nuclear force is observed to be roughly equally strong between a **proton-proton(pp) pair** and a **neutron-neutron(nn) pair** (charge symmetry) and
- on average equally strong between a proton-neutron(pn) pair as between pp and nn pairs (charge independence).

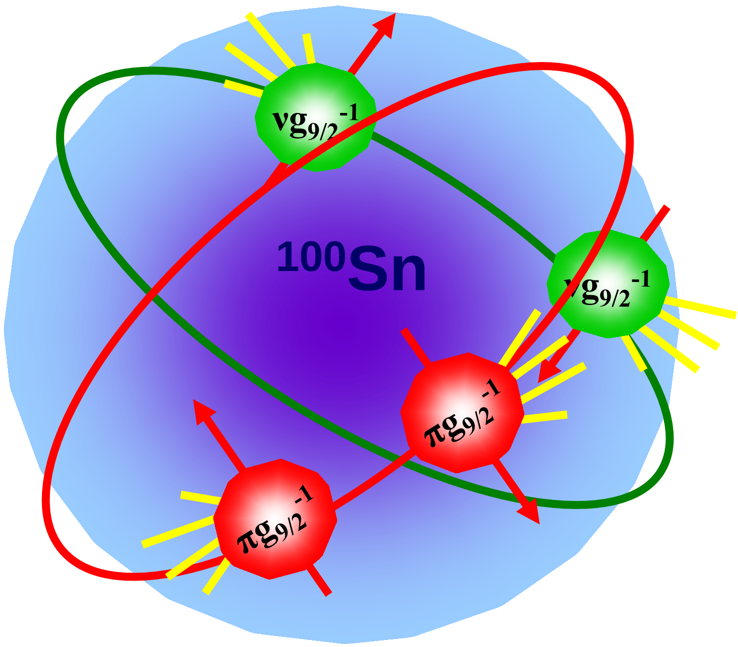
What is the ground state structure of N=Z nuclei below ^{100}Sn

For ^{92}Pd and ^{96}Cd neutrons and protons mainly occupy the g $_{9/2}$ subshell

^{92}Pd



^{96}Cd



The conventional picture :

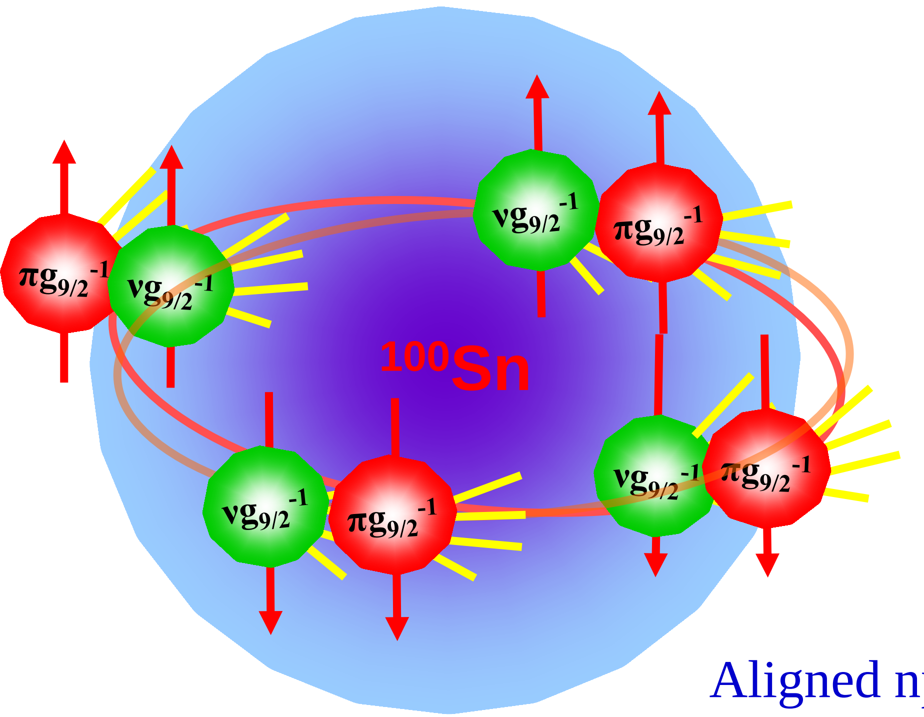
$$\Psi=(\{\nu g_{9/2}^{-2}\}_{0+})^n \times (\{\pi g_{9/2}^{-2}\}_{0+})^n$$

This would lead to a normal seniority type spectrum of low-lying excited states

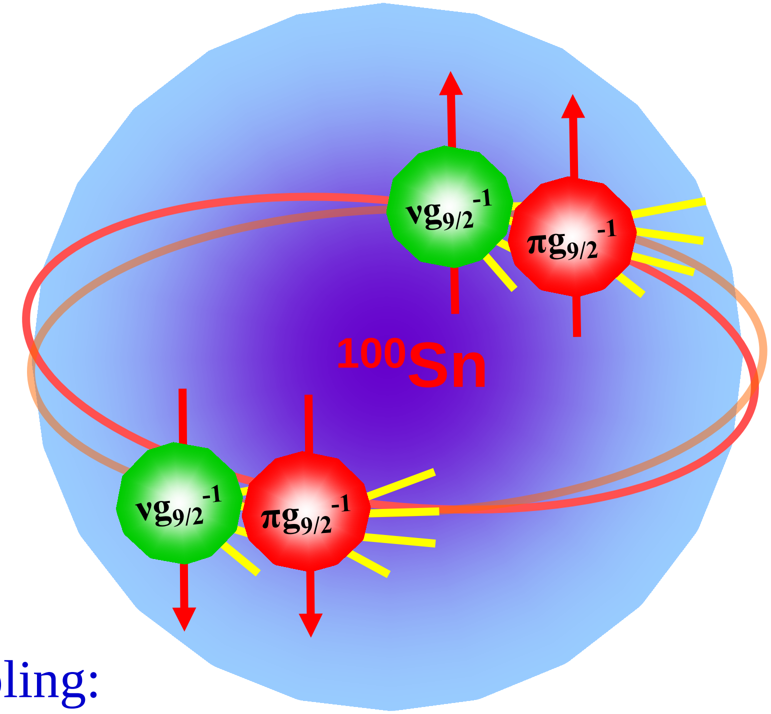
Shell Model calculations predict strong np-interactions

Model space: $g_{9/2}$ (and $f_{5/2}$, $p_{3/2}$, $p_{½}$)

^{92}Pd



^{96}Cd



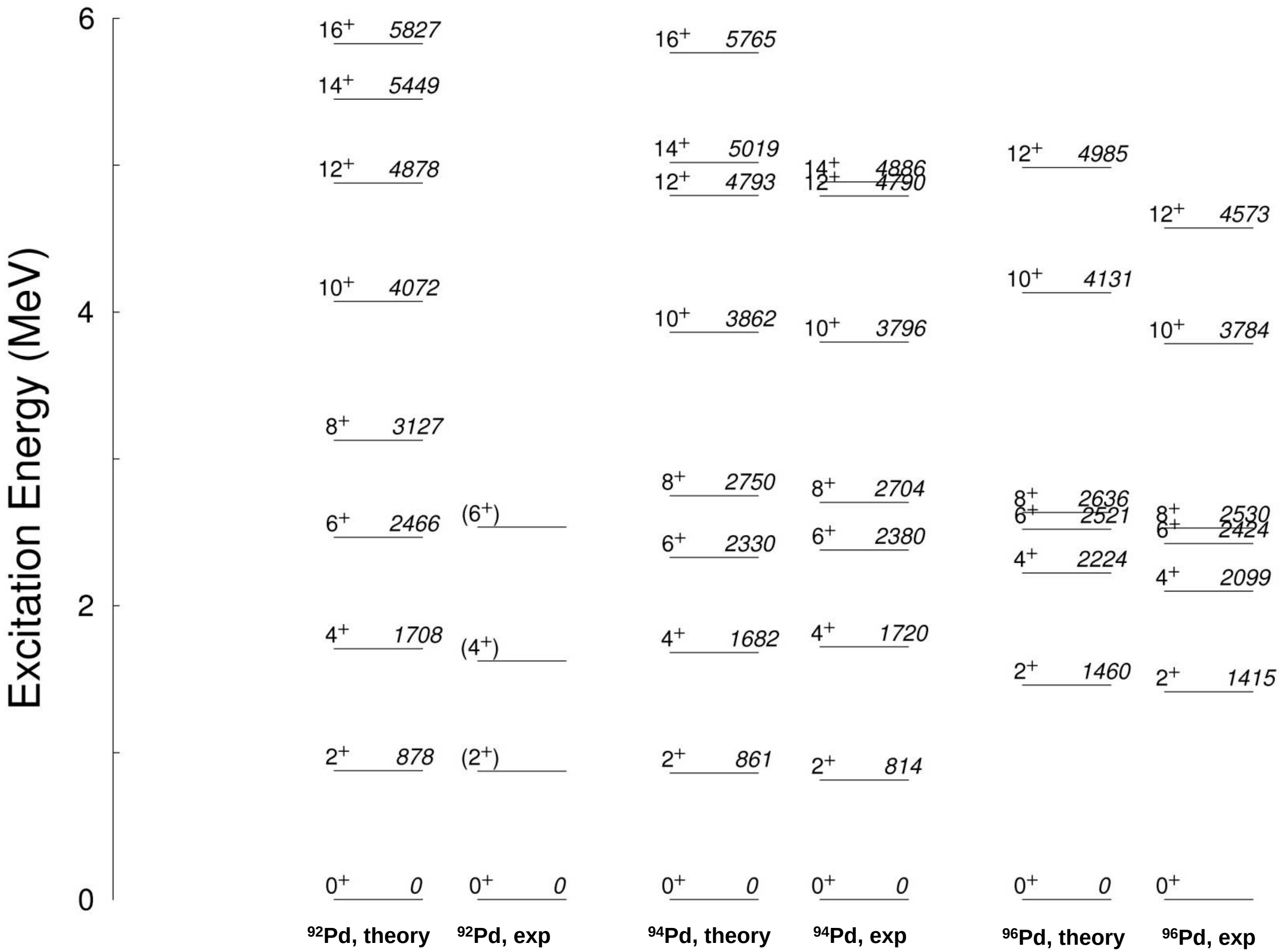
Aligned np coupling:

$$\Psi=[(\{\nu g_{9/2}^{-1} \times \pi g_{9/2}^{-1}\}_{9+})^2]_{0+} \times [(\{\nu g_{9/2}^{-1} \times \pi g_{9/2}^{-1}\}_{7+})^2]_{0+}$$

$$\Psi=[(\{\nu g_{9/2}^{-1} \times \pi g_{9/2}^{-1}\}_{9+})^2]_{0+}$$

J. Blomqvist et al.

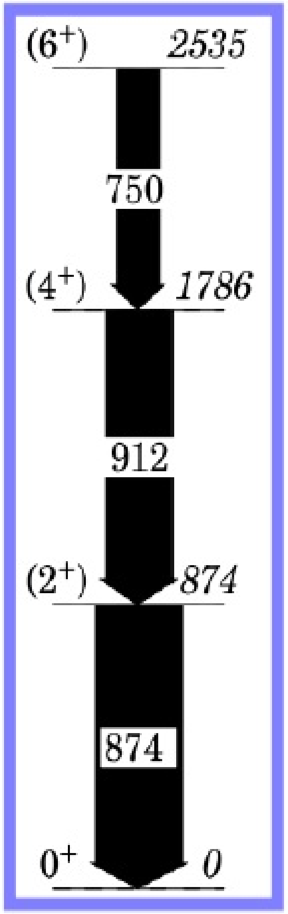
Pd level systematics near N=Z



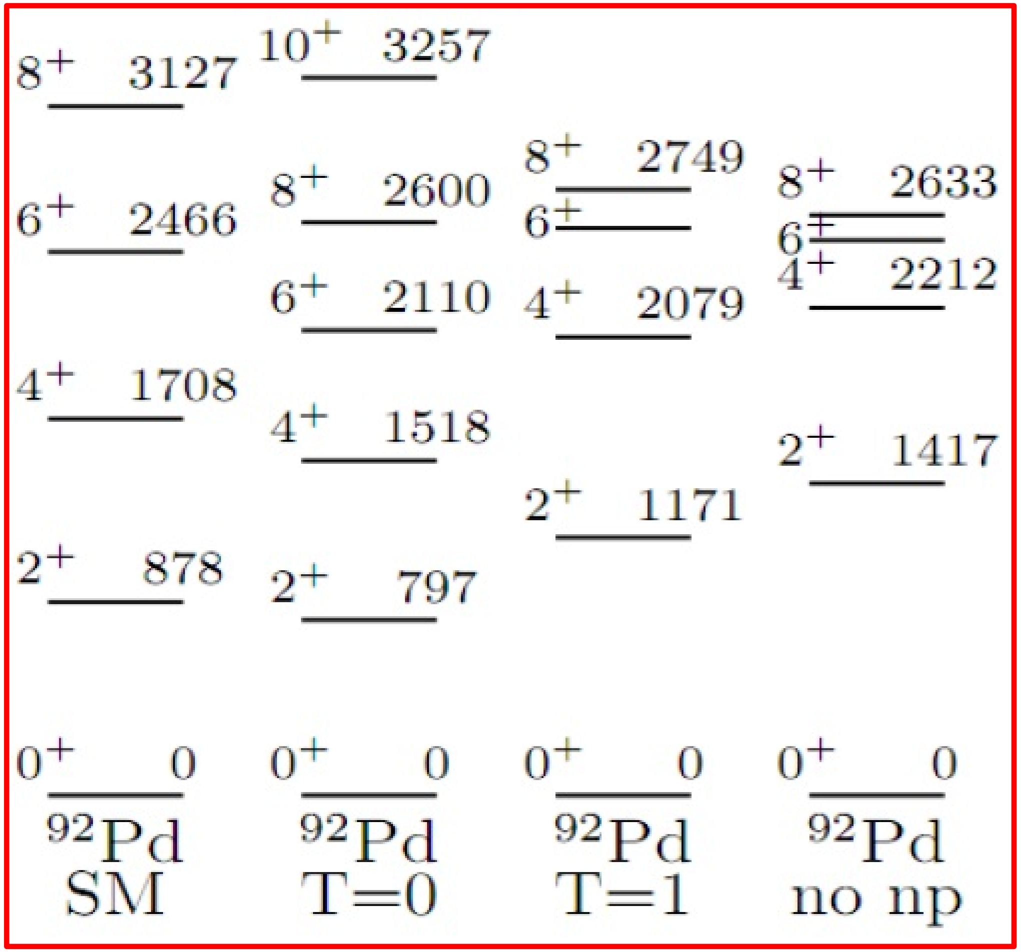
Shell model calculations by J. Blomqvist, R. Liotta, C. Qi

Experimental results and shell model calculations for ^{92}Pd

experimental results



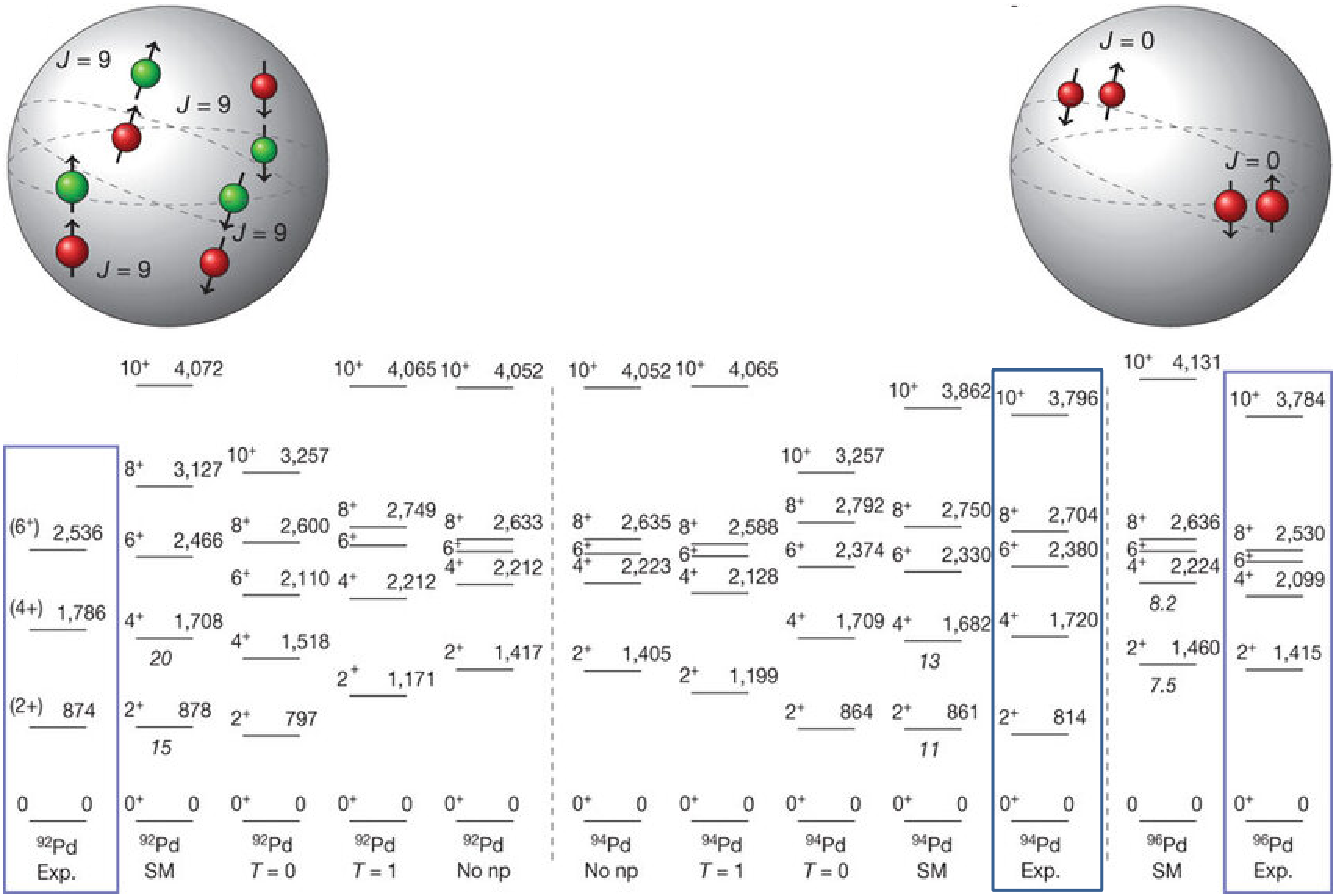
shell model calculations by Stockholm group



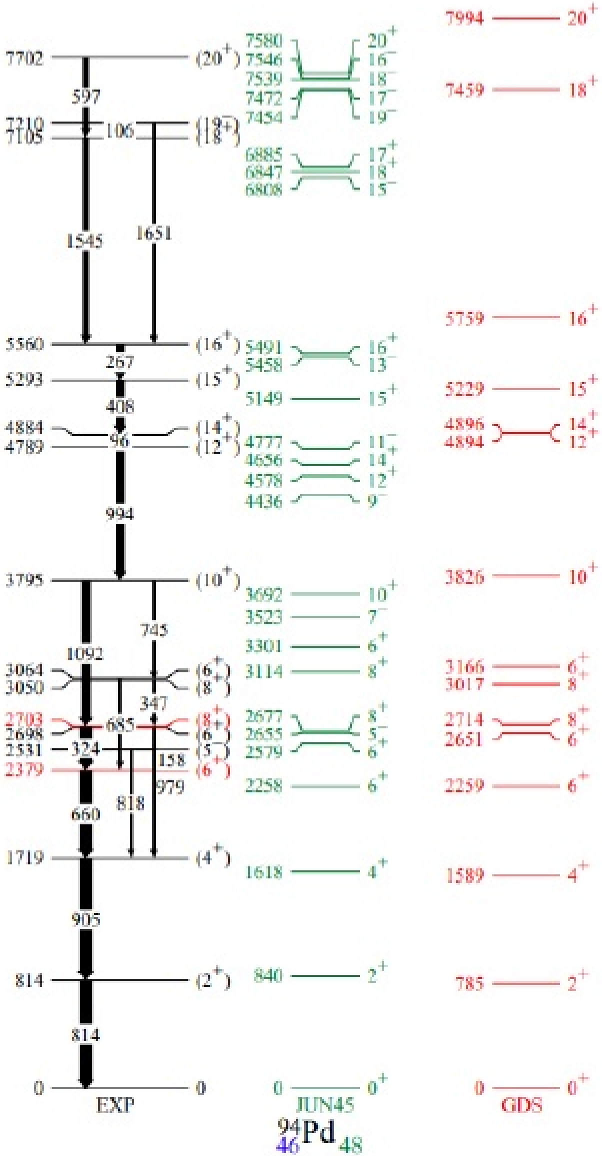
B. Cederwall et al., Nature 469 (2011) 68

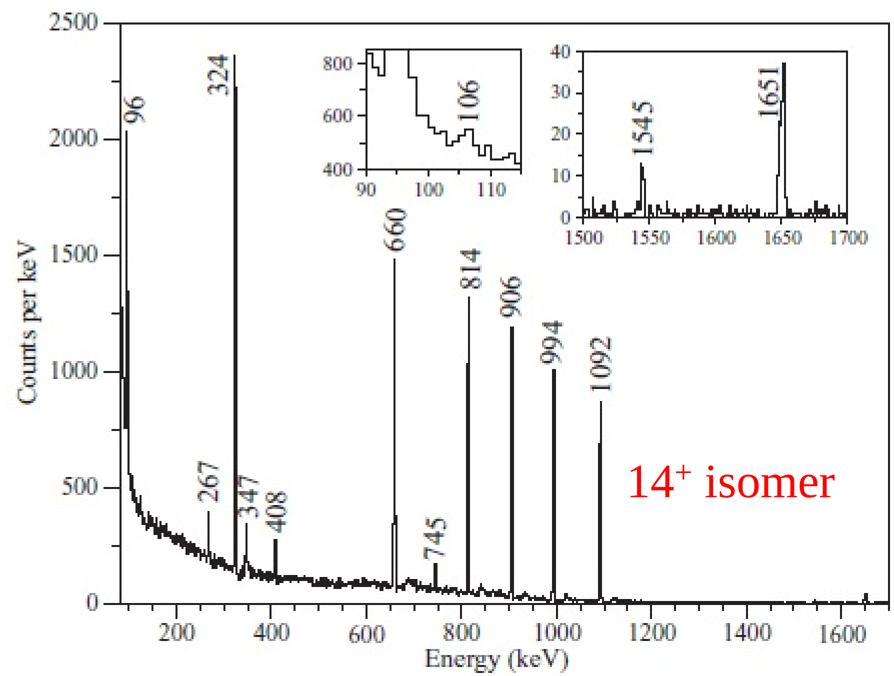
evidence that T=0 mode of the np-interactions play a role in ^{92}Pd?

Experimental results and shell model calculations



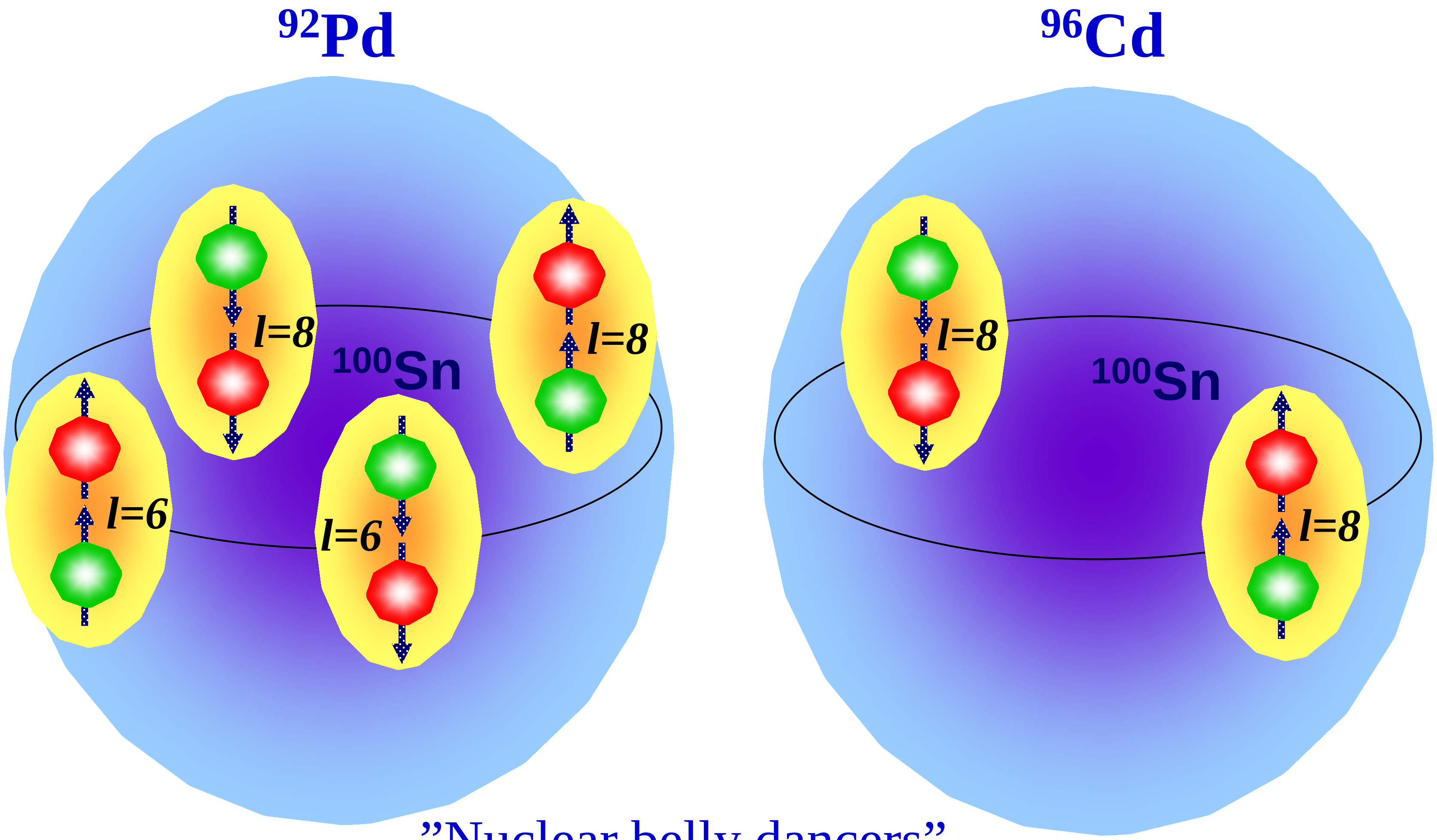
$T_z = +1$ nucleus ^{94}Pd





$T_{z,\pi} = -1/2$ for protons $T_{z,\nu} = +1/2$ for neutrons

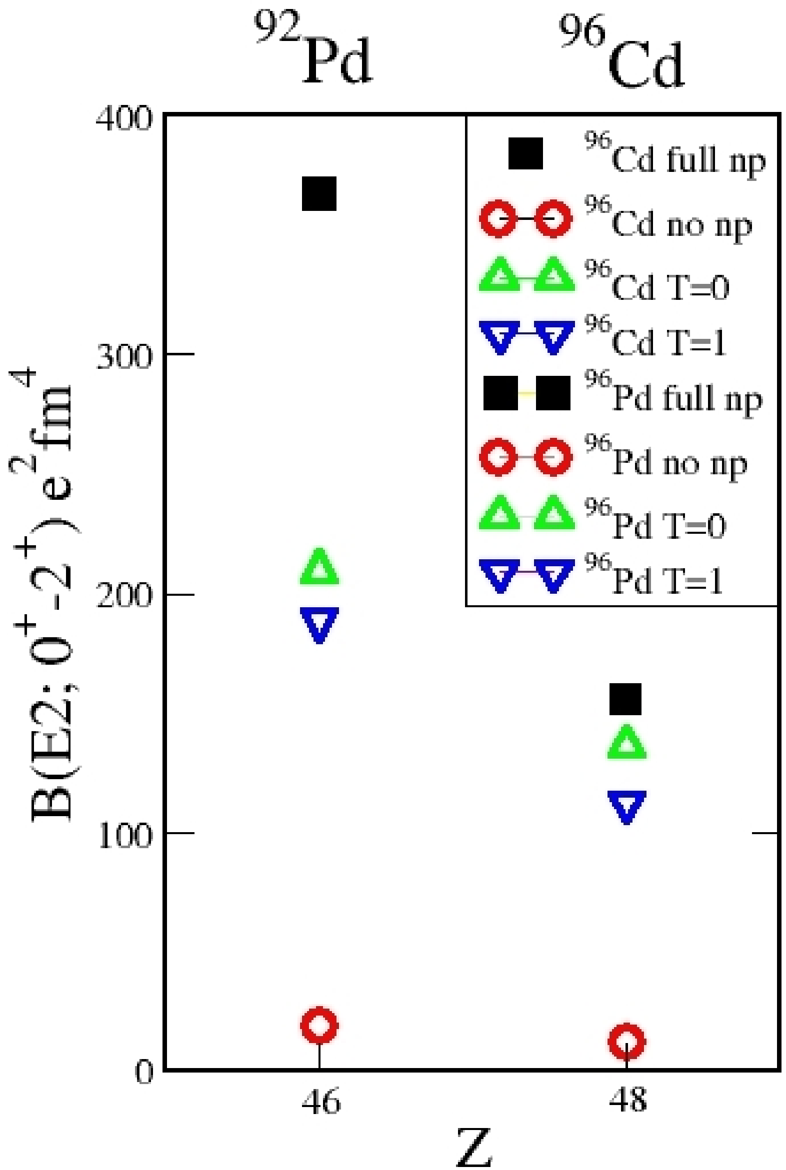
New manifestation of T=0 np "pairing"?



”Nuclear belly dancers”

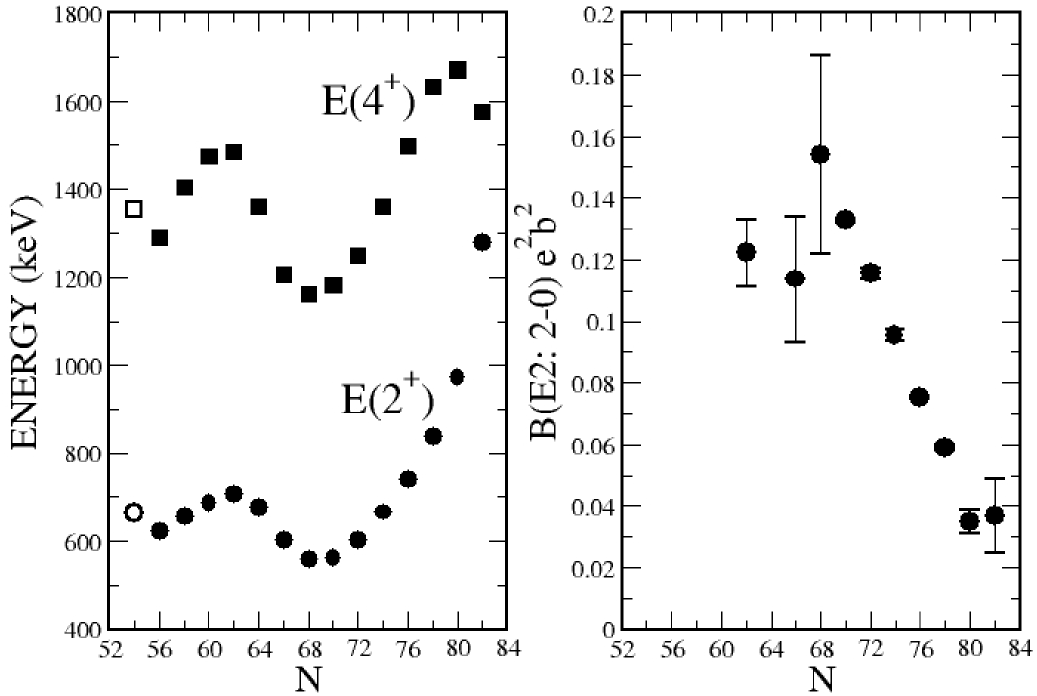
Special deuteron-hole cluster like ground states imply significant deformation

B(E2;0⁺→2⁺) – sensitive probe of neutron-proton interactions



Shell model calculations by J. Blomqvist, R. Liotta, C. Qi

Te experimental E(2^+) and B(E2; $2^+ \rightarrow 0^+$)



Quadrupole collectivity
The traditional view of the mechanism behind deformation and collectivity: long range np QQ interactions

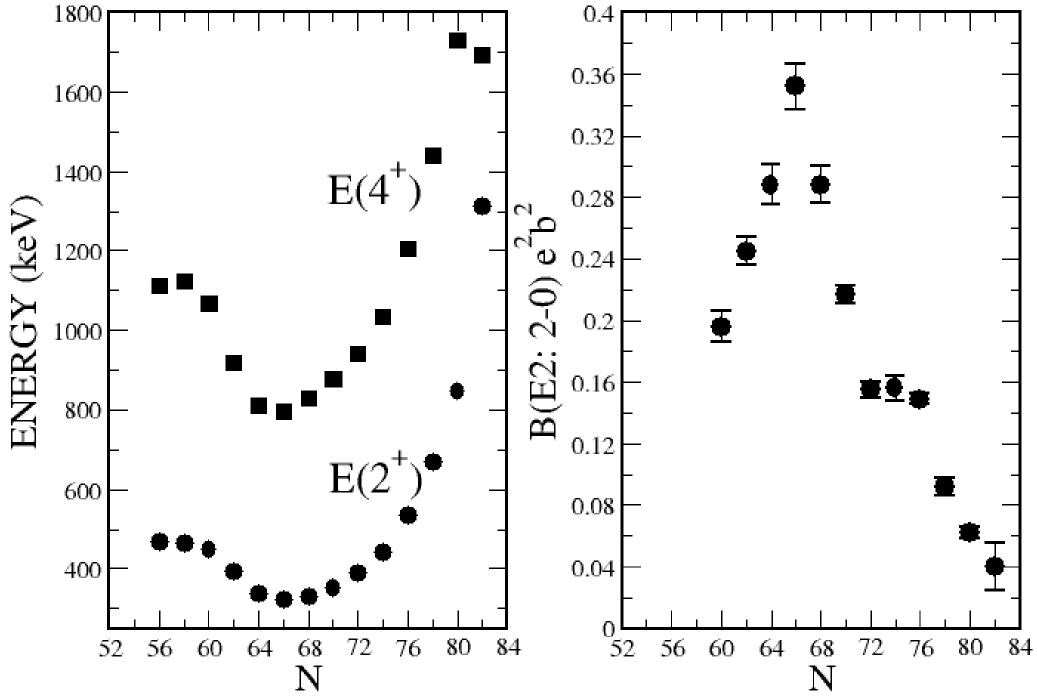
Enhancement of collectivity due to np (short-range) pairing correlations predicted in new calculations by D.S Delion, R. Liotta *et al*.

❖ Evidence for onset of collectivity, possibly induced by np pairing in neutron deficient Te and Xe nuclei

T_z=1 nucleus ^{106}Te : B. Hadinia, B. Cederwall *et al.*, Phys. Rev. C 72, 041303 (2005)

Evidence for enhanced collective strength in Te and Xe nuclei approaching N=Z

Xe experimental $E(2^+)$ and $B(E2; 2^+ \rightarrow 0^+)$



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- Evidence for onset of collectivity, possibly induced by np pairing in neutron deficient Te and Xe nuclei

T_Z=1 nucleus ^{110}Xe; M. Sandzelius, B. Hadinia,, B. Cederwall *et al*., Phys. Rev. Lett. 99, 022501 (2007)

Appendix

$$B(E2; J_i \rightarrow J_f) = \frac{1}{2 \cdot J_i + 1} \cdot \langle J_f \| Q \| J_i \rangle^2$$

Seniority changing: Δv = 2

$$\langle j^n\, J = 2 \| Q \| j^n\, J = 0 \rangle^2 = \left[\frac{n \cdot (2j + 1 - n)}{2 \cdot (2j - 1)} \right] \cdot \langle j^2\, J = 2 \| Q \| j^2\, J = 0 \rangle^2$$

$$= \left[\frac{(2j+1)^2}{2 \cdot (2j - 1)} \right] \cdot f \cdot (1 - f) \cdot \langle j^2\, J = 2 \| Q \| j^2\, J = 0 \rangle^2 \quad f = \frac{n}{2j+1} \rightarrow 1 \;\; for\ large\ n$$

Seniority conserving: v ⟶ v

$$\langle j^n\, J \| Q \| j^n\, J' \rangle = \left[\frac{2j + 1 - 2n}{2j + 1 - 2v} \right] \cdot \langle j^2\, J \| Q \| j^2\, J' \rangle$$

$$= \frac{2j+1}{2j + 1 - 2v} \cdot [1 - 2f] \cdot \langle j^2\, J \| Q \| j^2\, J' \rangle \quad f = \frac{n}{2j+1} \rightarrow 1 \;\; for\ large\ n$$

Example: quadrupole moments this is why nuclei are prolate at the beginning of a shell and oblate at the end.