Outline: Big Bang Nucleosynthesis

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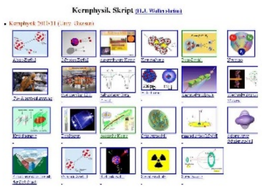
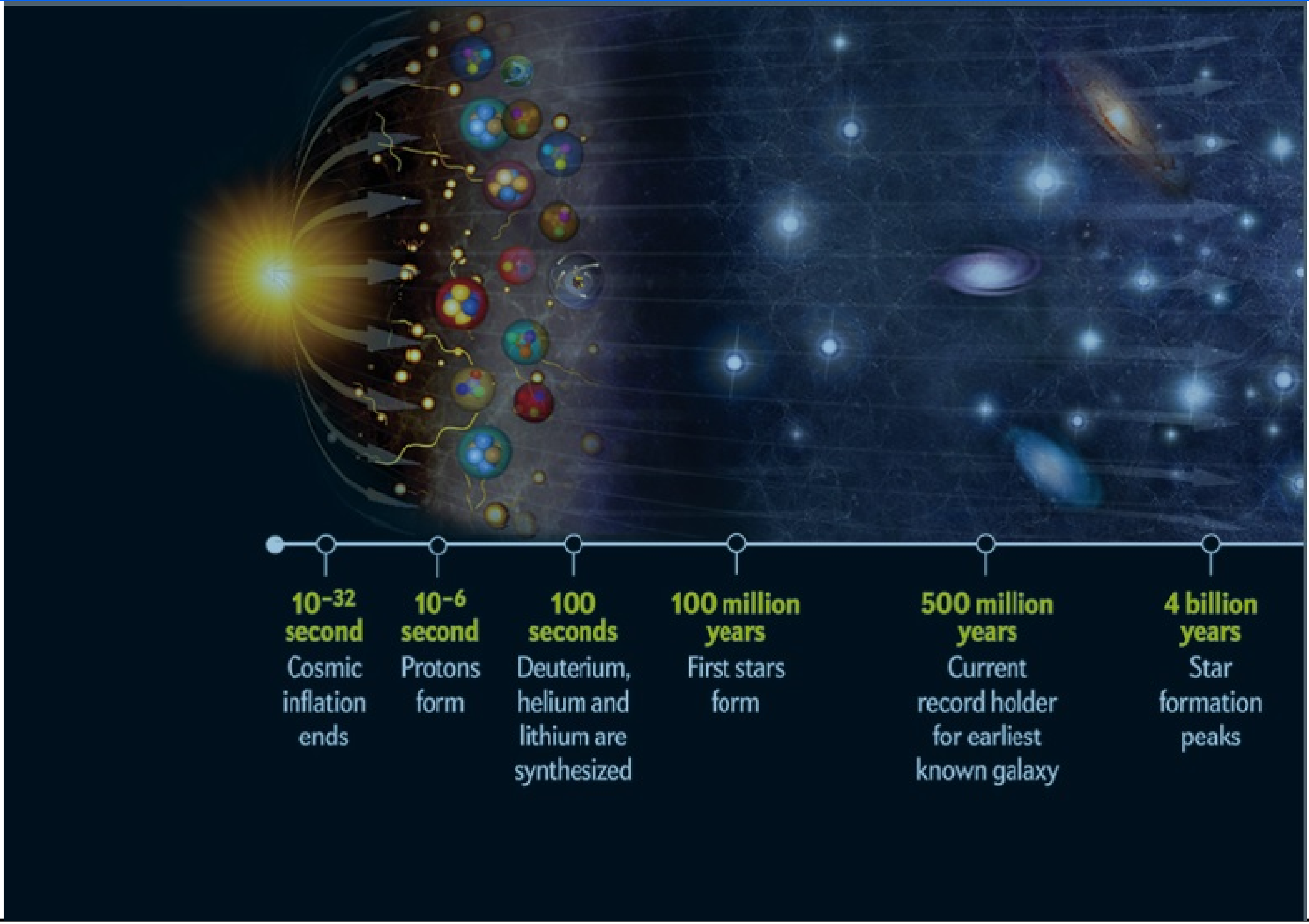
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. the first 3 minutes
2. neutron / proton ratio
3. deuteron bottleneck
4. Helium abundance

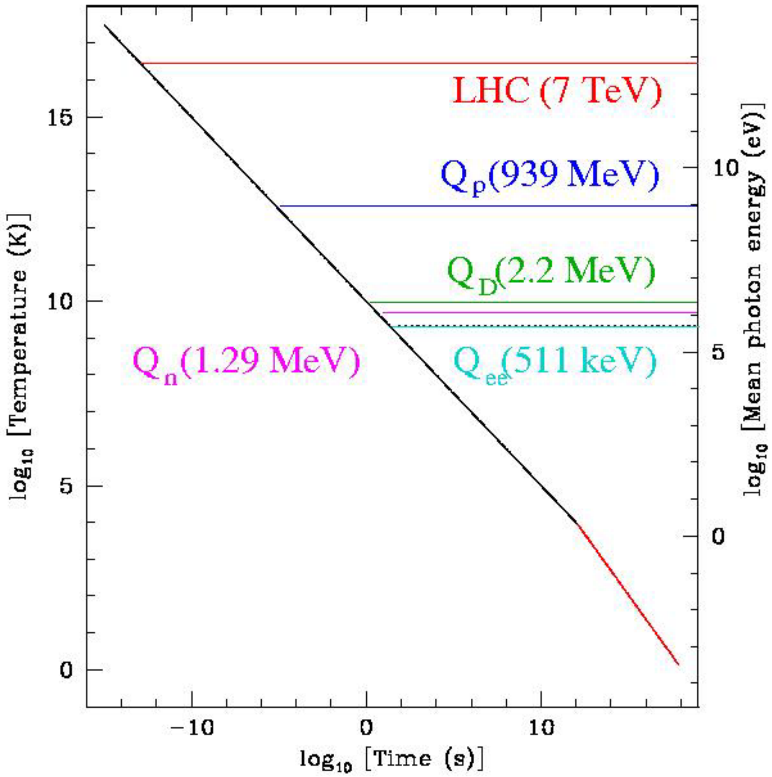
The first 3 minutes Steven Weinberg



Big Bang: Main steps

1) Universe started ~15 Ga, the size of an atom, at temperatures (or energy) too hot for normal matter > 10^{27} K – it start expanding extremely rapidly
2) Within 10^{-32} seconds, it cools enough to form a quark soup + electrons and other particles
3) At about 1 second, the universe was a hot and dense mixture of free electrons, protons, neutrons, neutrinos and photons.
4) At about 13.8 seconds, temperature has decreased to 3 x 10^{9} K and atomic nuclei began to form, but not beyond H and He. The universe was a rapidly expanding fireball!
5) 700 000 years later electrons became attached to nuclei of H and He – formation of true atoms. Matter became organized into stars, galaxies and clusters

The energy density in the early universe is dominated by radiation.



The radiation temperature/energy as a function of cosmic time

$$T(t) \cong 10^{10} K \left(\frac{t}{1s}\right)^{-1/2}$$

$$kT(t) \cong 1 MeV \left(\frac{t}{1s}\right)^{-1/2}$$

$$E_{mean} = 2.7 \cdot kT(t) \cong 3 MeV \left(\frac{t}{1s}\right)^{-1/2}$$

Hadron area ($t < 10^{-5}$ s): all matter, including electrons, protons, neutrons, neutrinos and their associated anti-particles are in thermal equilibrium with the photon radiation field.
Lepton area (t < 10 s): The temperature decreases such that *kT* is significantly lower than the rest mass energy of the proton (m_p = 938 MeV/c^2). The lepton era begins with photons in thermal equilibrium with electrons and positrons, muons, neutrinos and anti-neutrinos. The lepton era ends when the radiation temperature drops significantly below T~$5 \cdot 10^9$ K (i.e. *kT*~ $m_e c^2$ = 511 keV) leaving a small excess of electrons.

Time ~ 0.01 s

At t ~ 0.01 s, the temperature is T ~ 10^{11} K, and kT ~ 10 MeV, which is much larger than the electron mass. Neutrinos, electrons and positrons are easily produced and destroyed by means of weak interactions (i.e., interactions involving neutrinos)

$$i)\quad n + \nu_e \leftrightarrow p + e^-$$

$$ii)\quad n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$iii)\quad n \leftrightarrow p + e^- + \bar{\nu}_e$$

As long as the weak reactions are fast enough, the neutron-to-proton ratio is given by

$$[n/p] = \frac{number\ of\ neutrons}{number\ of\ protons} = \frac{N_n(T)}{N_p(T)} = exp\left[-\frac{\Delta mc^2}{kT}\right]$$

where m(n) = 939.5 MeV/c^2, m(p) = 938.3 MeV/c^2, and Δm = 1.294 MeV/c^2.
At T = 10^{11} K, kT = 8.62 MeV yielding **n/p = 0.86**

This temperature is far above the temperature of nucleosynthesis,
but the n/p ratio already begins to drop

Neutron production

In thermal equilibrium, the number of neutrons and protons are given by Maxwell-Boltzmann distribution

$$n_n = g_n \cdot \left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} \cdot exp\left(-\frac{m_n c^2}{kT}\right)$$

$$n_p = g_p \cdot \left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} \cdot exp\left(-\frac{m_p c^2}{kT}\right)$$

$$\frac{n_n}{n_p} = \frac{g_n}{g_p}\left(\frac{m_n}{m_p}\right)^{3/2} exp\left(-\frac{(m_n - m_p)c^2}{kT}\right)$$

We can employ a number of simplifications: $g_n = g_p = 2$, $(m_n/m_p)^{3/2} = 1.002$, $(m_n - m_p)c^2 = Q_n = 1.29\ MeV$

Therefore, in equilibrium the neutron to proton ratio is

$$\frac{n_n}{n_p} = exp\left(-\frac{Q_n}{kT}\right)$$

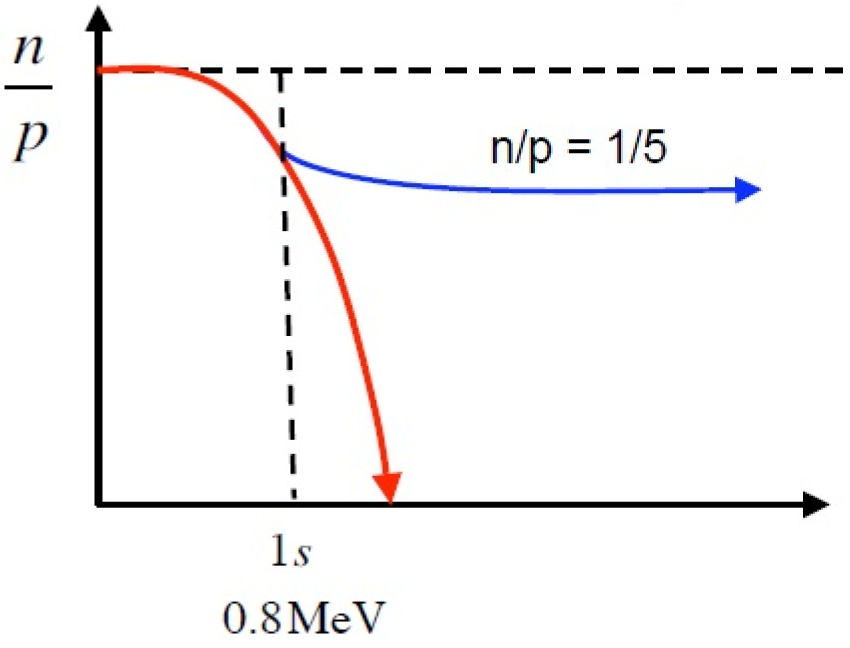
As Q_n = 1.29 MeV, this implies a corresponding value of kT ~ $1.5 \cdot 10^{10}$ K. Therefore, at T >> $1.5 \cdot 10^{10}$ K we expect $n_n \sim n_p$ and at T << $1.5 \cdot 10^{10}$ K we expect $n_n \ll n_p$

Neutron / Proton ratio

$$\gamma + \gamma \Leftrightarrow e^- + e^+$$

$$n + \nu_e \Leftrightarrow p + e^-$$

$$n + e^+ \Leftrightarrow p + \overline{\nu}_e$$



0.1% mass difference is critical!

LTE :

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{Q_n}{kT}\right)$$

$$m_n = 939.6\,\mathrm{MeV} \qquad m_p = 938.3\,\mathrm{MeV}$$

$$Q_n \equiv (m_n - m_p)c^2 = 1.29\,\mathrm{MeV}$$

Freeze-out:

$$\sigma_w \sim 10^{-47} m^2 (kT/1\mathrm{MeV})^2$$

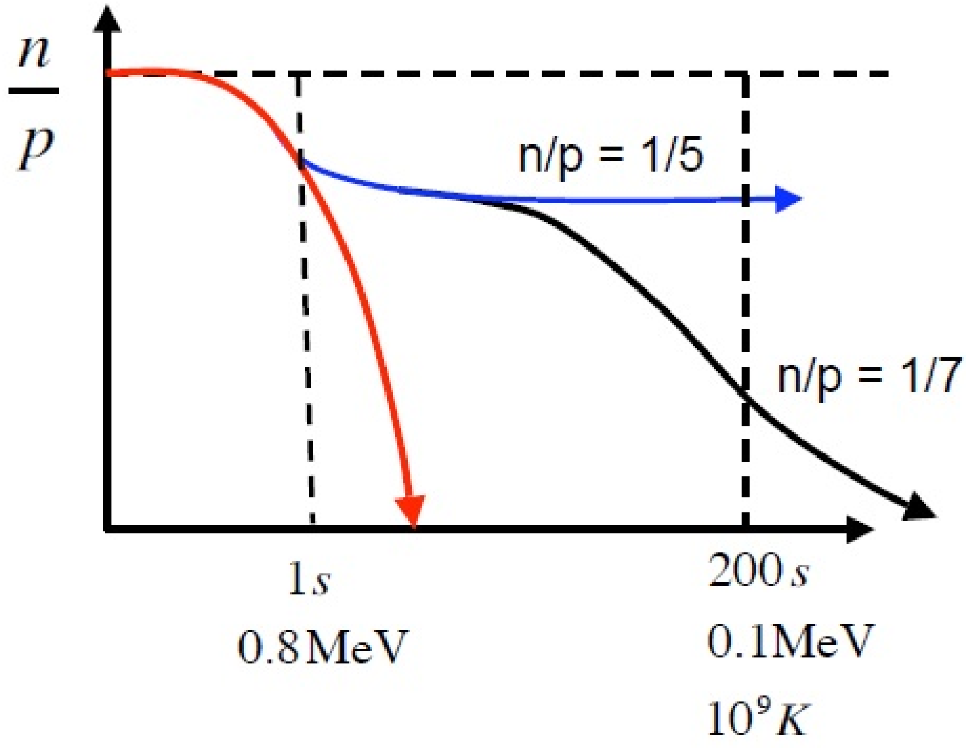
$$n\,\sigma_w\, c \sim H$$

$$t \approx 1\,\mathrm{s} \qquad kT \approx 0.8\,\mathrm{MeV}$$

$$\frac{n}{p} = \exp\left(-\frac{1.29}{0.8}\right) \approx \frac{1}{5}$$

Neutron / Proton → He / H

Deuterium production: $n + p \rightarrow D + \gamma$



$$B_D = 2.2\,\text{MeV} \qquad \eta = 10^9 \frac{\text{photons}}{\text{baryon}}$$

$$\ln\eta = \ln(10^9) \sim 20$$

$$t \approx 200s \qquad kT \approx \frac{B_D}{\ln\eta} = 0.1\,\text{MeV}$$

Neutron decay:

$$n_n = n_0 e^{-t/\tau} \qquad \tau = 890\text{s}$$

$$\frac{n}{p} = \frac{1}{5} e^{-\left(\frac{200}{890}\right)} \approx \frac{1}{7}$$

Primordial Abundances

$$X_p \equiv \frac{\text{mass in H}}{\text{total mass}} = 0.75 \qquad Y_p \equiv \frac{\text{mass in He}}{\text{total mass}} = 0.25$$

Big Bang Nucleosynthesis (BBN)

A summary of the BBN when the temperature of the universe allowed deuteron to be formed without being immediately destroyed by photons is:

1. The light elements (deuterium, helium, and lithium) were produced in the first few minutes after the Big Bang.

2. Elements heavier than ^{4}He were produced in the stars and supernovae explosions.

3. Helium and deuterium produced in stars do not to match observation because stars destroy deuterium in their cores.

4. Therefore, all the observed deuterium was produced around three minutes after the big bang, when T ~ 10^9 K

5. A simple calculation based on the n/p ratio shows that BBN predicts that 25% of the matter in the Universe should be helium

6. More detailed BBN calculations predict that about 0.001% should be deuterium

Onset of Big Bang nucleosynthesis (BBN)

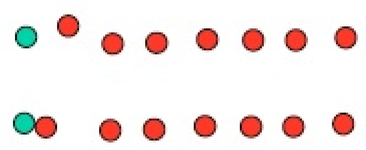
Deuterium production: $n + p \rightarrow D + \gamma$

delayed until the high energy tail of blackbody photons can no longer break up D. Binding energy: B_D = 2.2 MeV.

$$B_D/kT \sim ln(N_\gamma/N_B) = ln(10^9) \sim 20$$

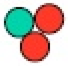
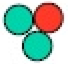
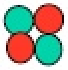
$$kT \sim 0.1\ MeV\ (T \sim 10^9\ K\ t \sim 200\ s)$$

Thermal equilibrium
+ neutron decay: $N_p/N_n \sim 7$
thus, at most, $N_D/N_p = 1/6$



Deuterium readily assembles into heavier nuclei

Key fusion reactions

Reaction	product:	binding energy:
$n + p \rightarrow D + \gamma$	Deuterium (pn) 	2.2 MeV
$D + D \rightarrow {}^3He^{++} + n$ $p + D \rightarrow {}^3He^{++} + \gamma$	${}^3He$ (ppn) 	7.72 MeV
$n + D \rightarrow T + \gamma$ $D + D \rightarrow T + p$ $n + {}^3He^{++} \rightarrow T + p$	Tritium (pnn) 	8.48 MeV
$n + {}^3He^{++} \rightarrow {}^4He^{++} + \gamma$ $D + {}^3He^{++} \rightarrow {}^4He^{++} + p$ $p + T \rightarrow {}^4He^{++} + \gamma$ $D + T \rightarrow {}^4He^{++} + n$ ${}^3He^{++} + {}^3He^{++} \rightarrow {}^4He^{++} + 2p$	${}^4He$ (ppnn) 	28.3 MeV

Deuterium bottleneck

As the temperature of the universe decreased, neutrons and protons started to interact and fuse to a deuteron

$$n + p \rightarrow d + \gamma$$

The binding energy of deuterons are small (E_B = 2.23 MeV). The baryon-to-photon ratio, called η, at this time is also very small ($< 10^{-9}$). As a consequence, there are many high-energy photons to dissociate the formed deuterons, as soon as they are produced.

The temperature for nucleosynthesis at the start is about 100 keV, when we would have expected ~ 2 MeV, the binding energy of deuterium. The reason is the very small value of η. The BBN temperature, ~ 100 keV, corresponds to timescales less than about 200 sec. The cross-section and reaction rate for the reaction is

$$\sigma \cdot v \sim 5 \cdot 10^{-20} \quad cm^3/sec$$

So, in order to achieve appreciable deuteron production rate we need $\rho \sim 10^{-17}\ cm^{-3}$. The density of baryons today is known approximately from the density of visible matter to be $\rho_0 \sim 10^{-7}\ cm^{-3}$ and since we know that the density ρ scales as $R^{-3} \sim T^3$, the temperature today must be $T_0 = (\rho_0/\rho)^{1/3} T_{BBN} \sim 10\ K$, which is a good estimate.

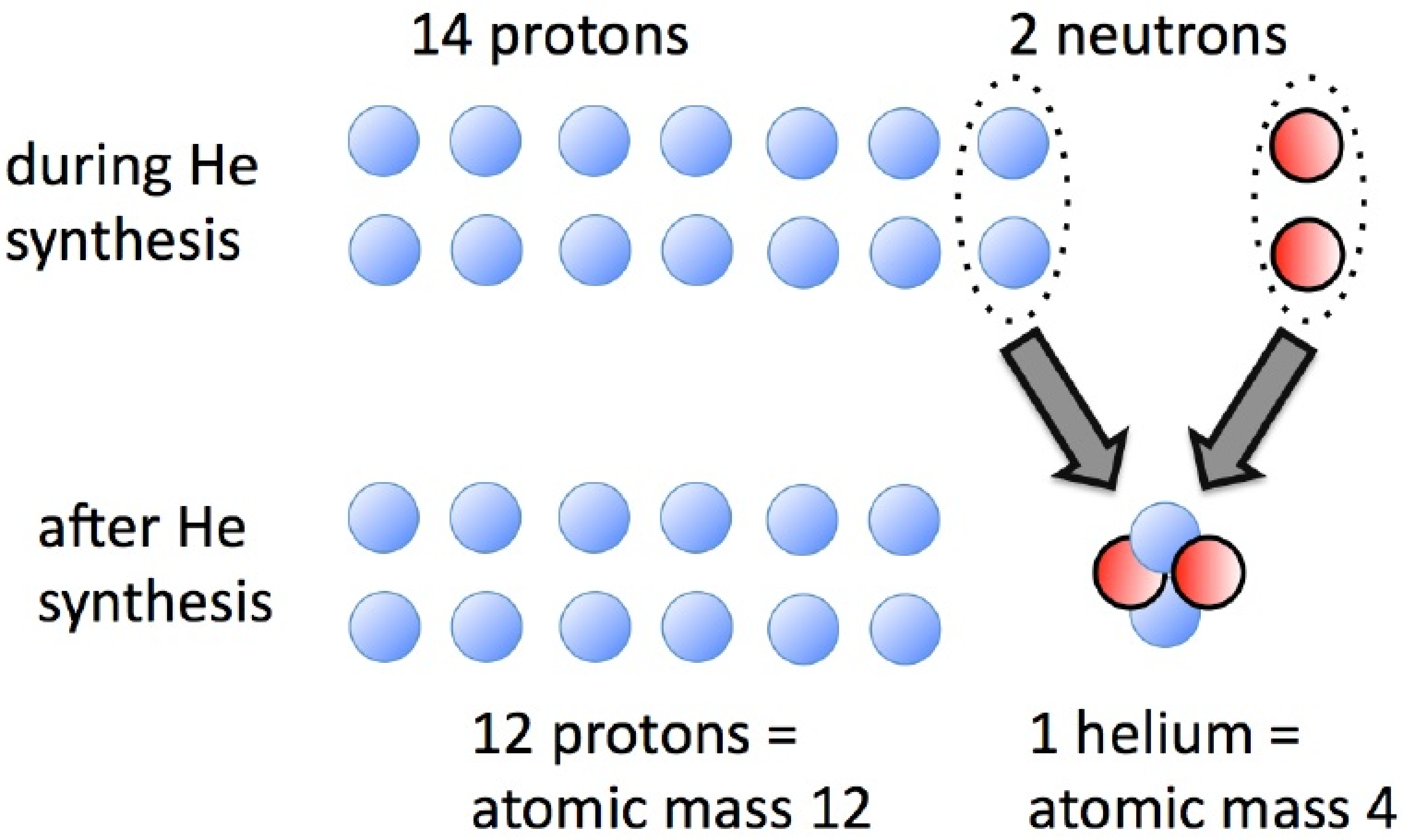
Deuterium bottleneck

The bottleneck implies that there would be no significant abundance of deuterons before the universe cooled to about 10^9 K.

Other important facts are:

1. The nucleon composition during BBN was proton-rich
2. The most tightly bound light nucleus is ^{4}He
3. There is no stable nucleus with mass numbers A = 5 and A = 8
4. The early universe was too cold and not dense enough to overcome the Coulomb barriers to produce heavier nuclides
5. The BBN network is active until all neutrons are bound in ^{4}He. As the BBN mass fraction of neutrons was $X_n = N_n / (N_n + N_p) = 1/8$, it follows that the mass fraction of ^{4}He after BBN is about $X_{4He} = 2 \cdot X_n = 25\%$

BBN prediction – the Helium abundance



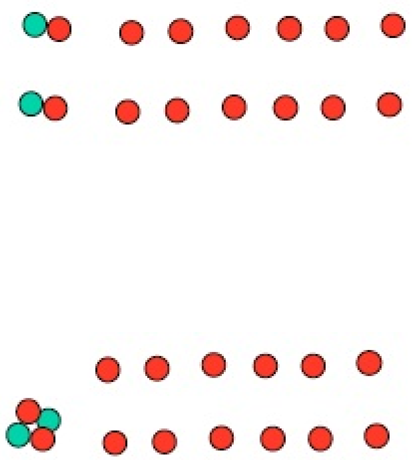
BBN predicts that when the universe had T = 10^9 K (1 minute old), protons outnumbered neutrons by 7:1. When ^{2}H and He nuclei formed, most of the neutrons formed He nuclei. That is, one expects 1 He nucleus for every 12 H nuclei, or 75% H and 25% He. This is the fraction of He and ^{2}H we observe today.

Primordial abundances

Because ^{4}He is so stable, all fusion pathways lead to ^{4}He, and further fusion is rare.

Thus almost all neutrons end up in ^{4}He, and residual protons remain free.
(p + p → ^{2}He does not occur)

To first order, with $N_p/N_n \sim 7$



$$X_p \equiv \frac{\text{mass in H}}{\text{total mass}} = \frac{N_p - N_n}{N_p + N_n} = \frac{6}{8} = 0.75$$

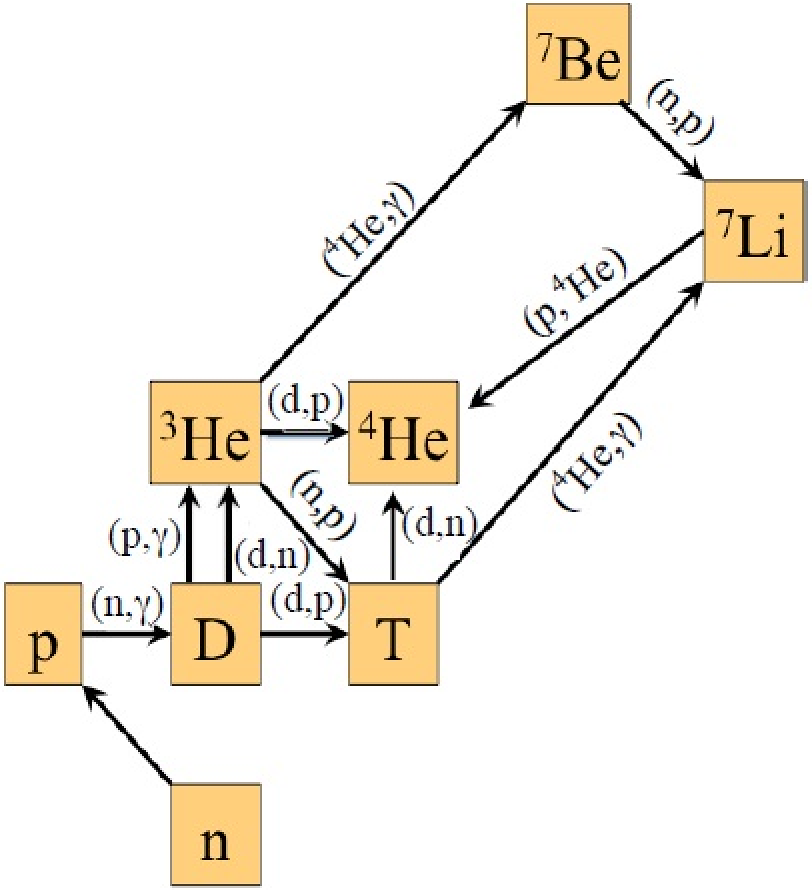
$$Y_p \equiv \frac{\text{mass in He}}{\text{total mass}} = \frac{2N_n}{N_p + N_n} = \frac{2}{8} = 0.25$$

Primordial abundances of H and He (by mass, not number)

The BBN reaction network

After deuterons are produced at T ~ 10^9 K, a successive chain of nuclear reactions occur. The most important are

1:	$n \rightarrow p$	7:	$^4He(^3H,\gamma)^7Li$
2:	$n(p,\gamma)d$	8:	$^3He(n,p)^3H$
3:	$d(p,\gamma)^3He$	9:	$^3He(d,p)^4He$
4:	$d(d,n)^3He$	10:	$^4He(^3He,\gamma)^7Be$
5:	$d(d,p)^3H$	11:	$^7Li(p,^4He)^4He$
6:	$^3H(d,n)^4He$	12:	$^7Be(n,p)^7Li$



Except for ^7Be electron capture, all reactions are fast. The binding energies of ^3He, ^3H, ^4He are significantly larger than the one of deuterons. Thus these nuclei are not dissociated again.

At T ~ 10^8 K BBN terminates because

- the temperature and density are too low
- the Coulomb barriers are too high

BBN nuclei in stars

Deuteron

- In stellar processes deuteron is quickly converted to ^{3}He.
- Astronomers look at quasar: bright atomic nuclei of active galaxies, ten billion light years away

^{3}He

- star account for only 0.1% of all He.
- The ^{3}He abundance in stars is difficult to deduce. Its abundance is increasing in stellar fusion.
- Scientists look to our own galaxy.

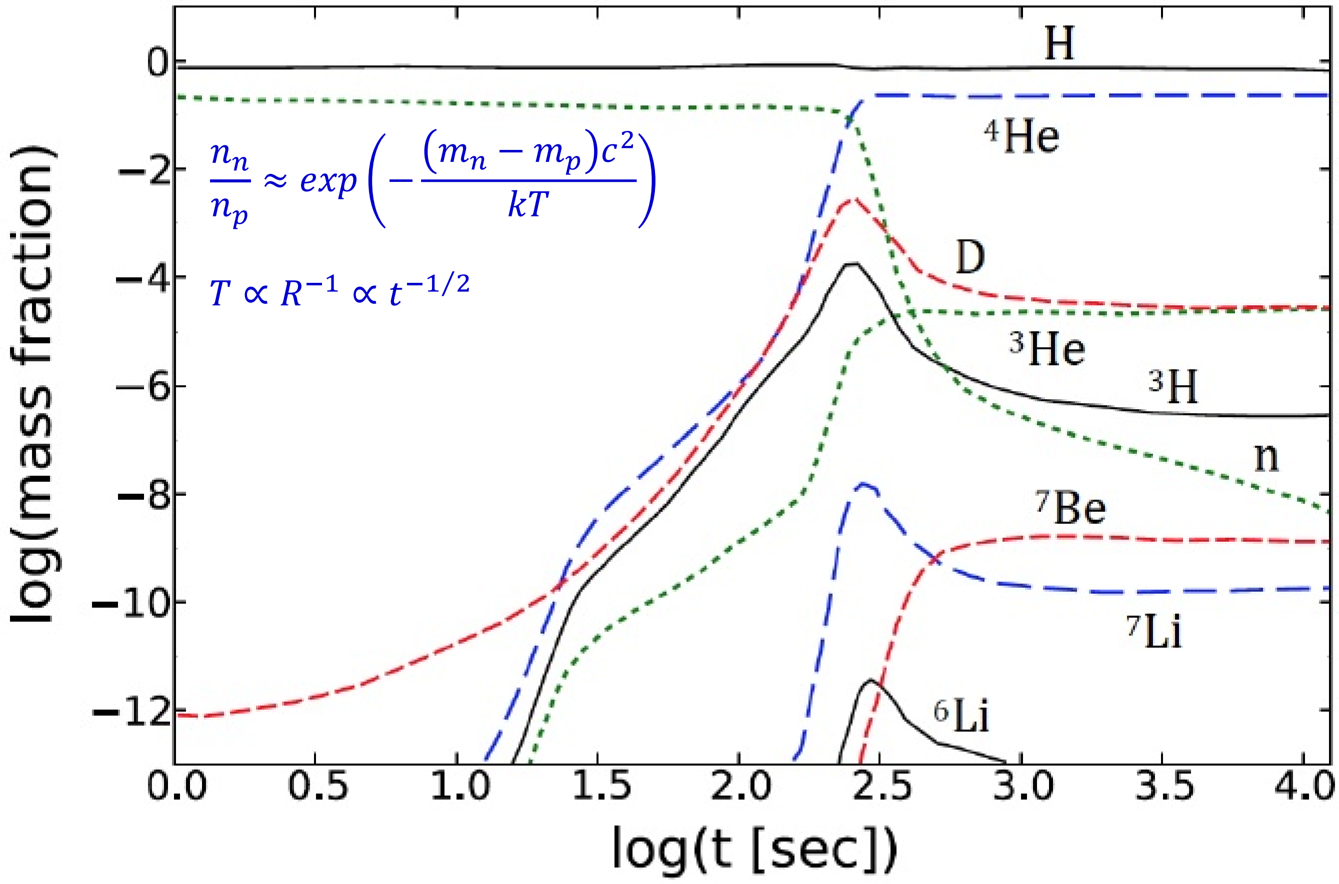
^{7}Li

- ^{7}Li can form when "cosmic rays" collide with stellar gas.
- Observations can be made on old, cool stars in our own galaxy.
- ^{7}Li is destroyed more that it is created inside stars.
- Very old stars have low oxygen content, and their outermost layers still contain mostly primordial ^{7}Li

Time evolution of BBN – mass fractions

thermal equilibrium

reactions "freeze out" due to expansion



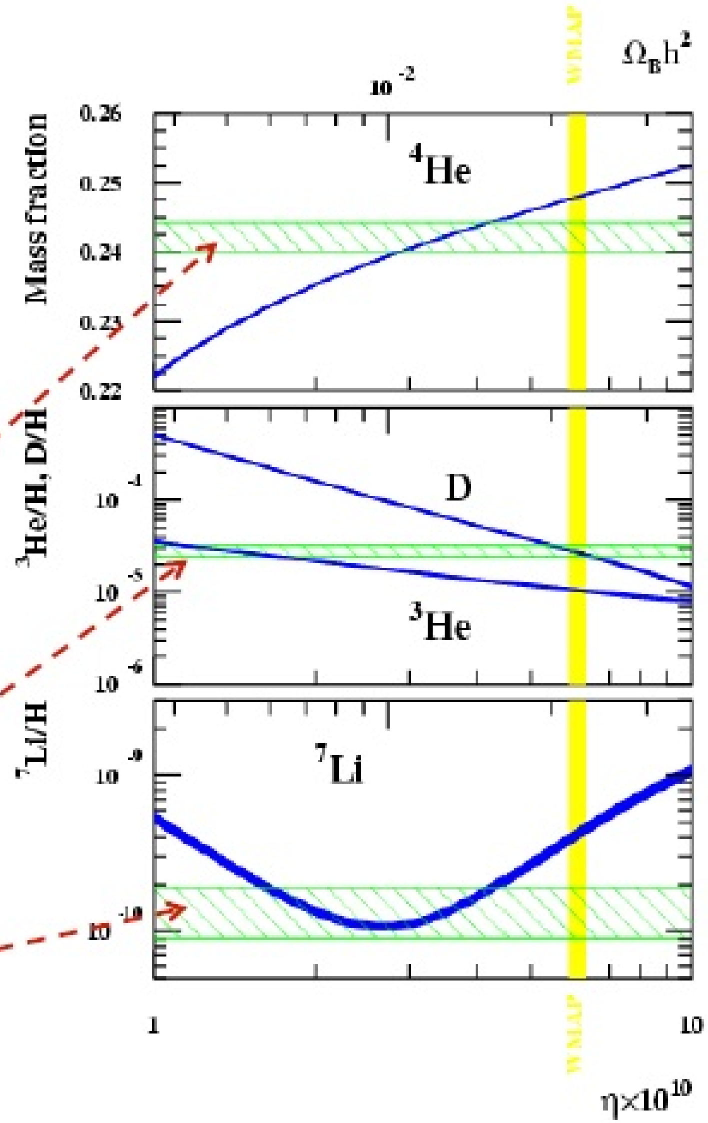
Mass fractions of light nuclei as a function of time during the BBN

Primordial abundances

Note: Light elements have been made and destroyed since the Big Bang

- Some are made in stars:
  - stars (^{3}He, ^{4}He)
  - spallation (scattering) (^{6,7}Li, Be, B)
  - supernova explosions (^{7}Li, ^{11}B)
- Some are destroyed
  - d, Li, Be, B are very fragile
    they are destroyed in the center of stars

→ observed abundances (at surface of stars)
do not reflect the destruction side



^{4}He : Y_p = 0.2421 ± 0.0021

D : D/H = (2.78 + 0.44 - 0.38) x 10^{-5}

^{7}Li : Li/H = (1.23 + 0.68 - 0.32) x 10^{-10}

$\Omega_B h^2$ = 0.0224 ± 0.0009