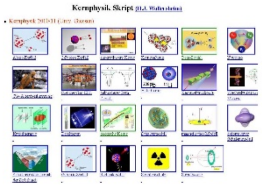
Outline: Cosmic clocks

Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

web-page: https://web-docs.gsi.de/~wolle/ and click on



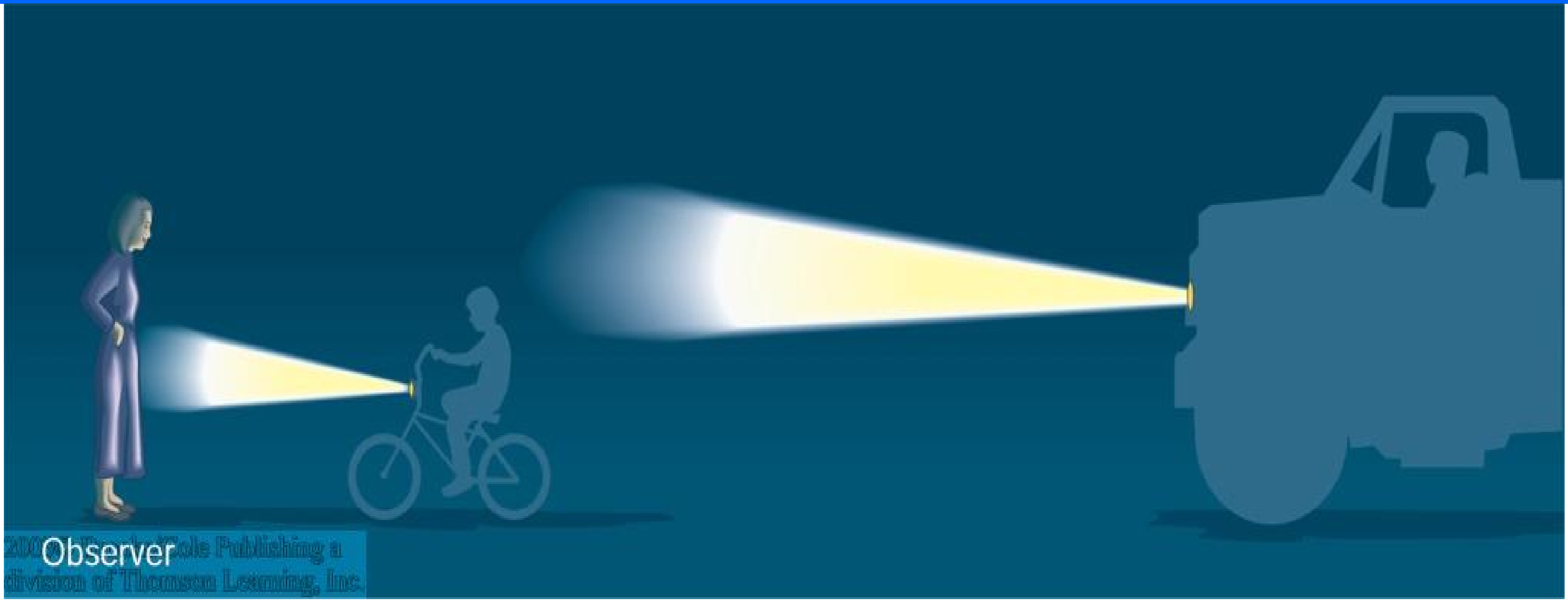
1. stellar and nuclear cosmic clocks
2. black body radiation
3. Hertzsprung-Russell-diagram
4. age of the Earth – Pb-Pb method
5. ^{187}Re/^{187}Os clock – ESR experiment

How old is the Universe?

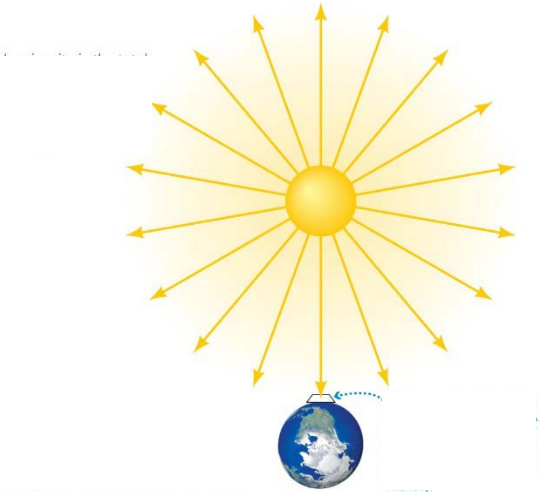
Hubble Ultra Deep Field, Hubble Space Telescope, advanced camera for surveys

Stellar and nuclear cosmic clocks Globular Cluster M13 in the constellation Hercules

Stellar brightness



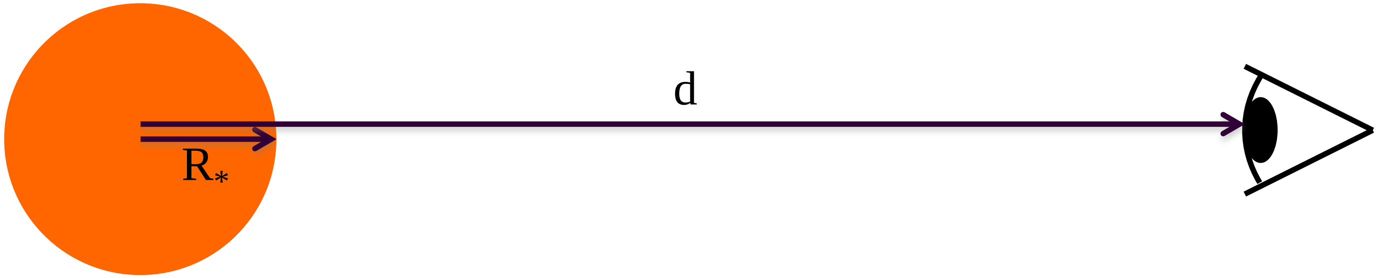
The brightness of an object depends on both **distance** and **energy output**.



Amount of energy output a star radiates is called the **Luminosity L**: the energy per second

Amount of starlight that reaches Earth is called the **apparent brightness (m)**

Stars show spectra very close to black-body radiation



J. Stefan & L. Boltzmann

flux at the surface:

$$F = \sigma_{SB} \cdot T_*^4$$

measured flux:

$$F = \left(\frac{R_*}{d}\right)^2 \cdot \sigma_{SB} \cdot T_*^4$$

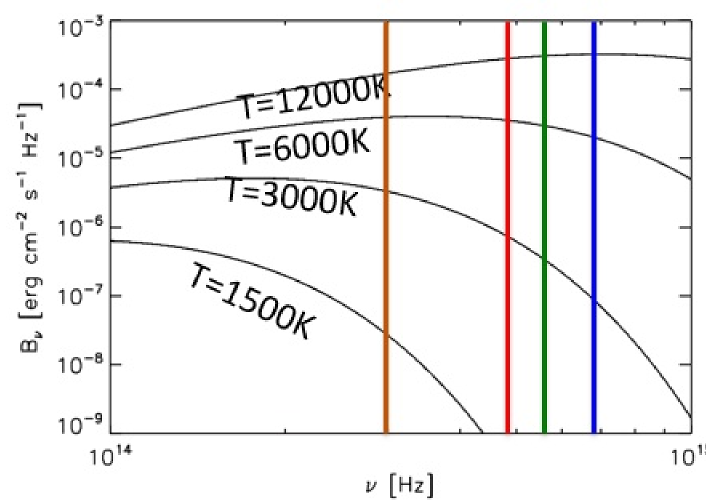
luminosity (Stefan-Boltzmann law):

$$L = 4\pi \cdot R_*^2 \cdot \sigma_{SB} \cdot T_*^4$$

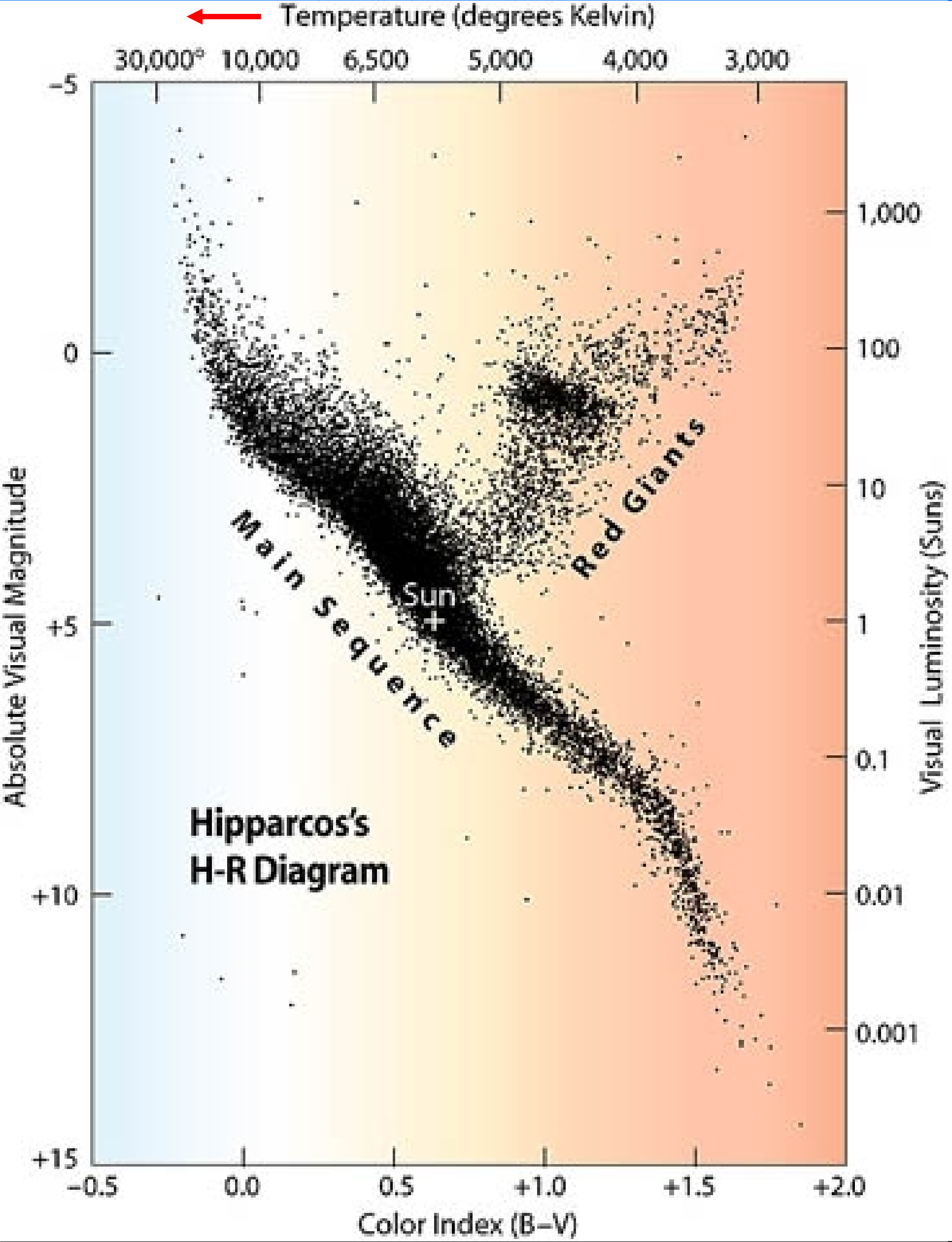
spectrum of a black-body:

$$F_\nu = \pi \cdot B_\nu$$

$$B_\nu = 2 \cdot \frac{\nu^2}{c^2} \cdot h\nu \cdot \frac{1}{exp(h\nu/k_B T) - 1}$$



Hertzsprung-Russell-diagram of all stars at a range of 300 light years



Ejnar Hertzsprung, Henry Norris Russell

absolute luminosity versus temperature

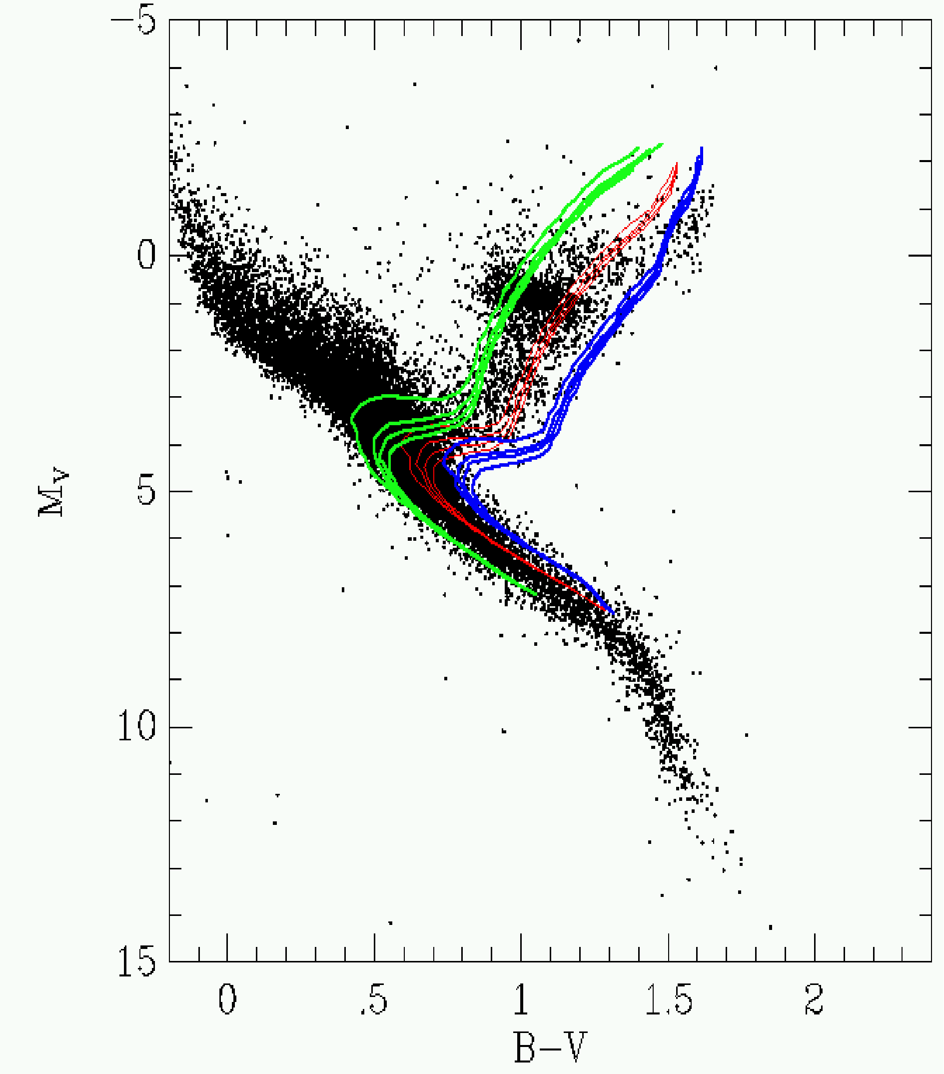
The stars are stationary on the

´main sequence´,

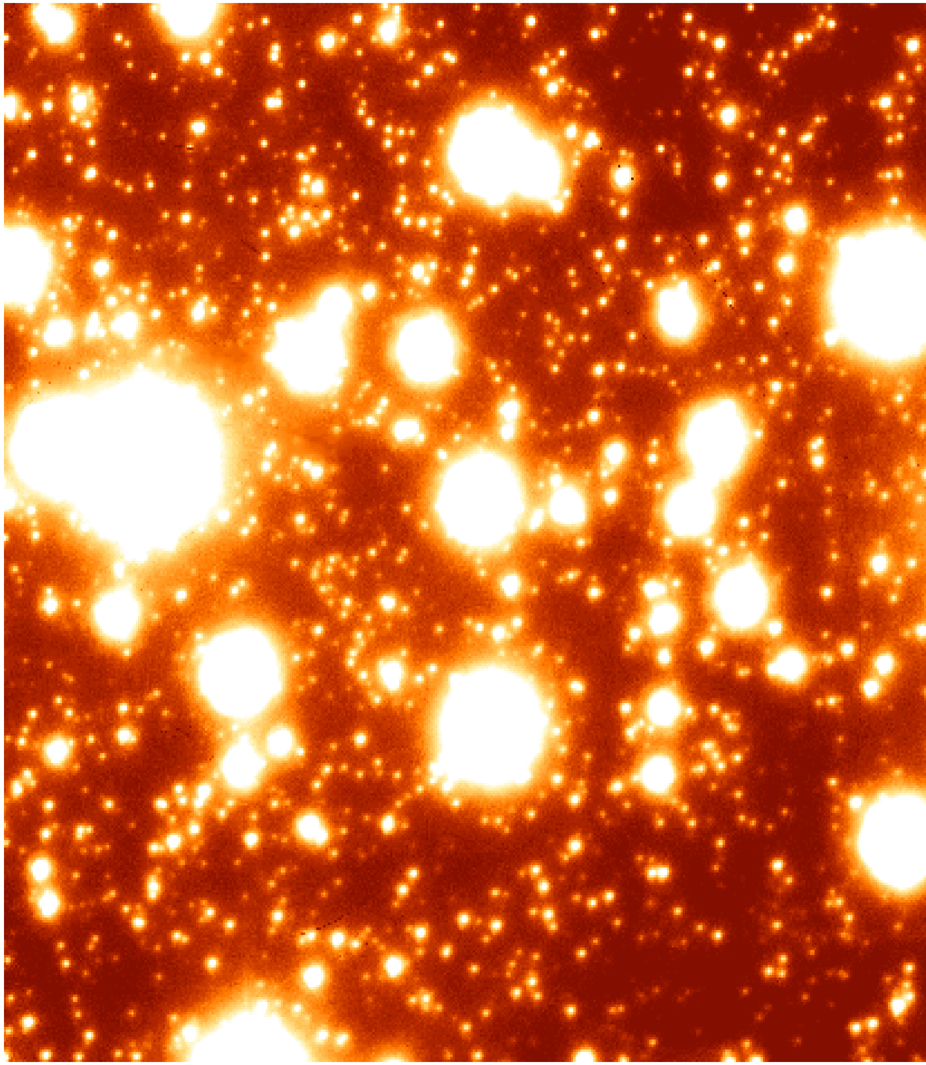
as long as the fusion of protons to helium persists

This time depends very sensitively on the mass of the individual star

Where is a GC leaving the main sequence?

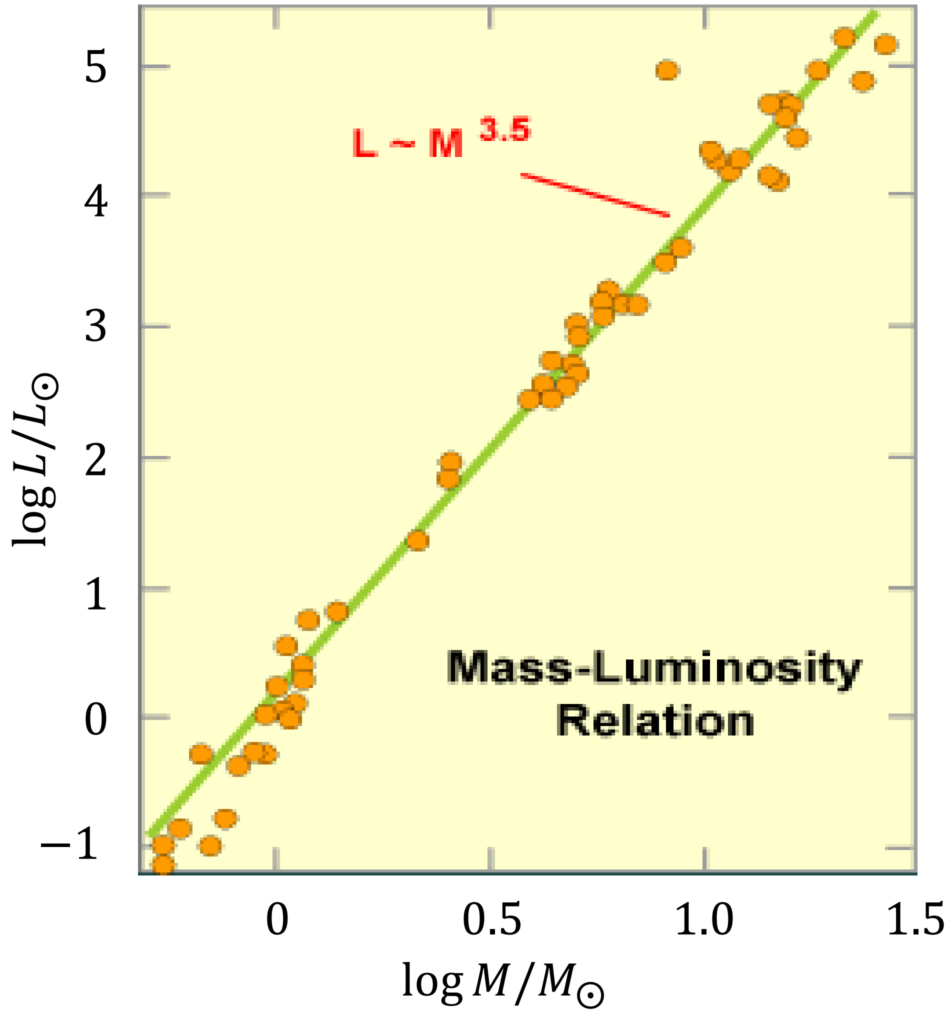


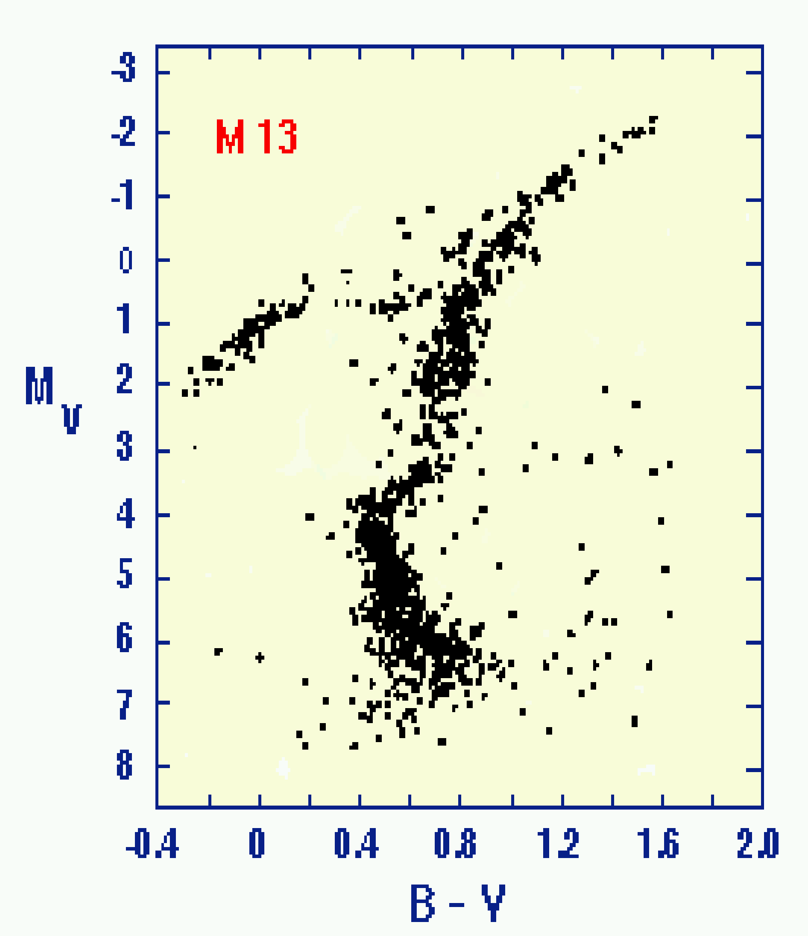
Hertzsprung-Russel of all stars nearby



Globular Cluster (GC) M13 in Hercules

Age of GC from ´kink´ at main sequence

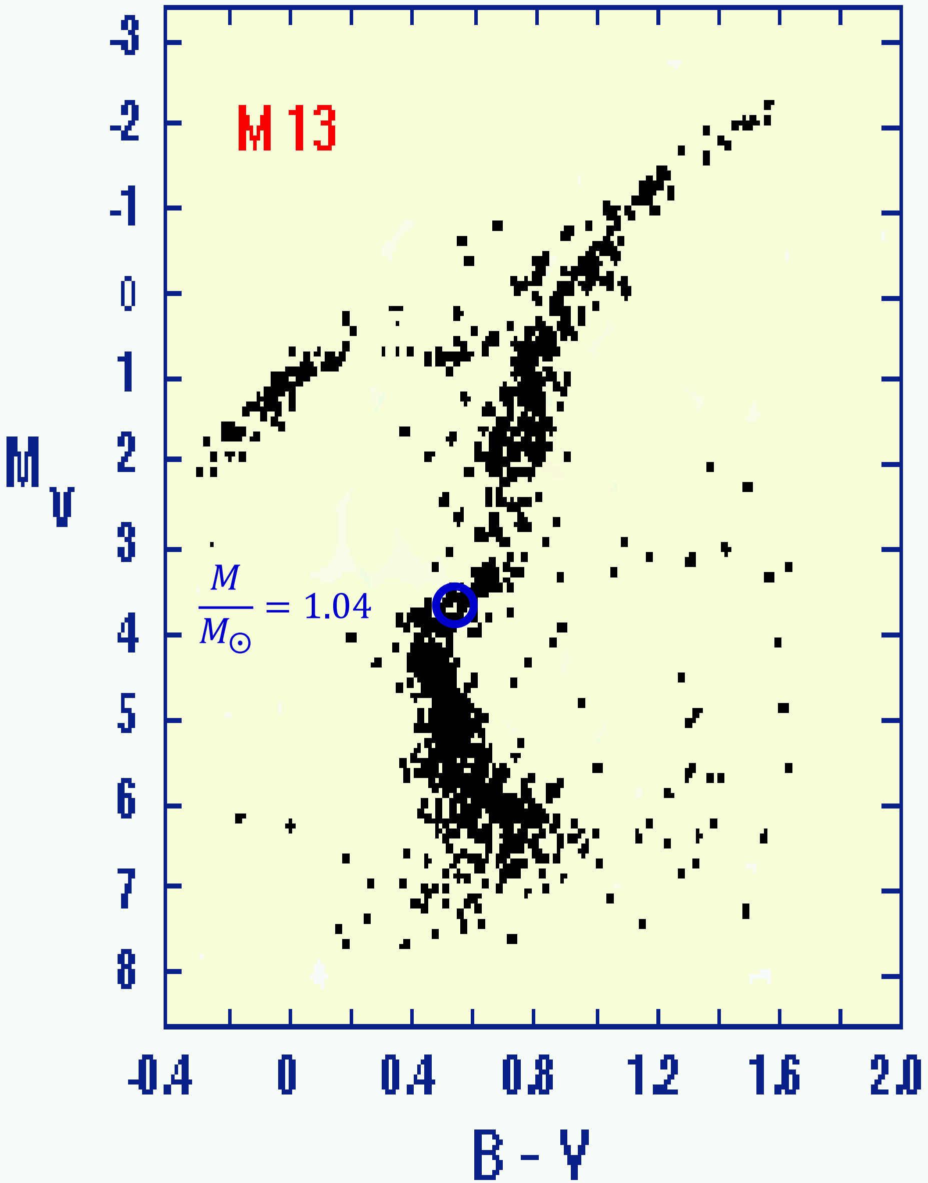




$T_{MS} = M/L \propto M^{-2.5}$

mass (M_{sun})	lifetime (years)
1	$\sim 10^{10}$
5	$\sim 10^{8}$
10	$\sim 10^{7}$

$T_{GC}/T_\odot = [M_\odot/M_{GC}]^{2.5}$



For our Sun this time
is about 9 billion years (Gyr)
for lighter stars longer,
for heavier ones shorter

$$T_{main\ sequence} = 9\ Ga \cdot [M_\odot / M]^{2.5}$$

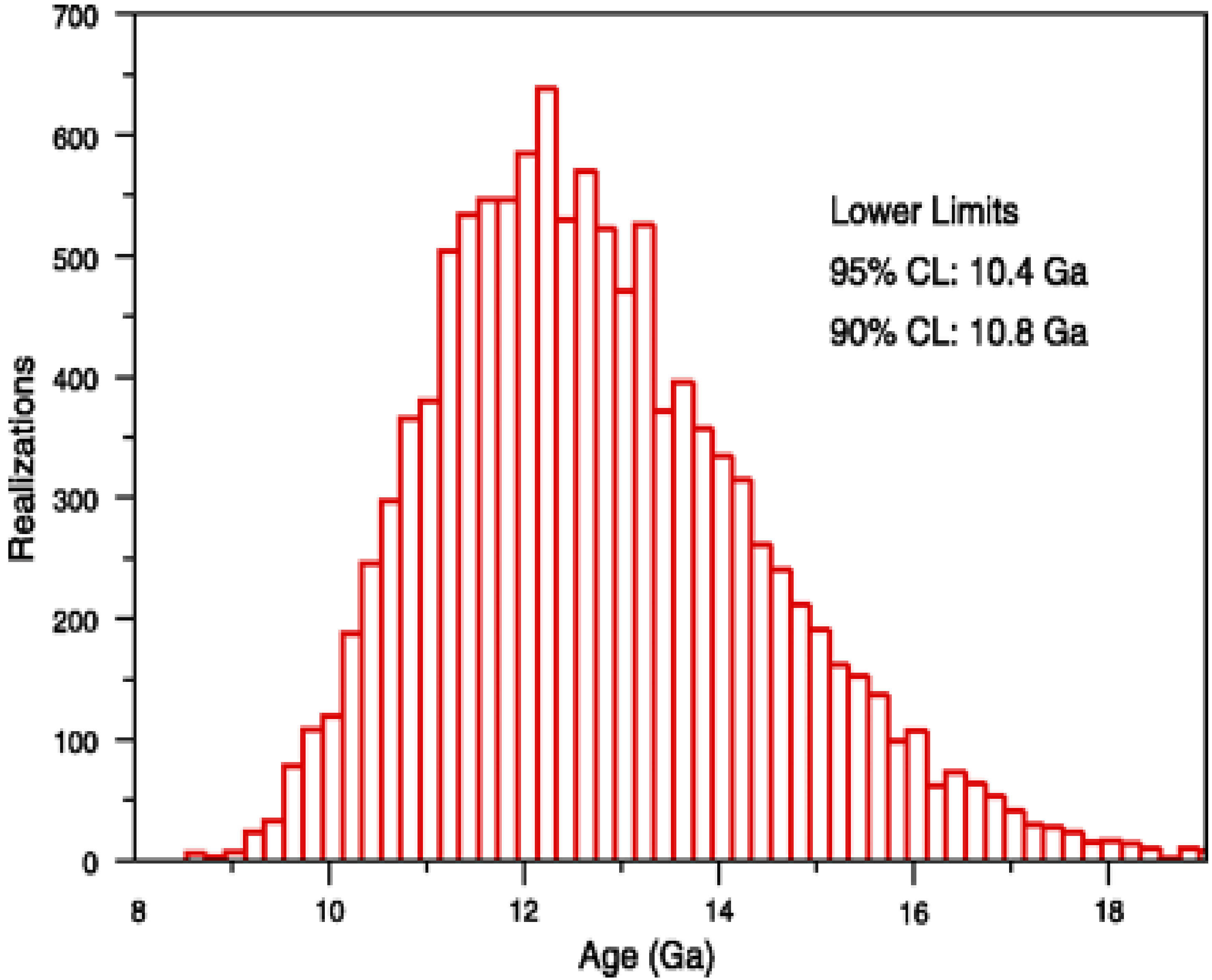
when observing at which mass M the stars of M13 are leaving the main sequence

one can determine the age of M13
- and therewith the
minimum age T_G of our galaxy

from $M = 1.04 \cdot M_\odot$

$$\rightarrow T_G > 8\ Ga$$

Lower limit of the age of our galaxy ~ 11 Ga



B. Charboyer, L. M. Krauss 2003

Nuclear cosmic clocks independent on stellar evolution models!??

only four at most:

1. ^{87}Rb/^{87}Sr (β) $T_{1/2}$ = 50 Ga Q_β = 273 keV ($3/2^- \rightarrow 9/2^+$)

2. ^{176}Lu/^{176}Hf (β) $T_{1/2}$ = 30 Ga Q_β = 1186 keV ($7^- \rightarrow 0^+$)

3. ^{187}Re/^{187}Os (β) $T_{1/2}$ = 42 Ga Q_β = 2.6 keV ($5/2^+ \rightarrow 1/2^-$)

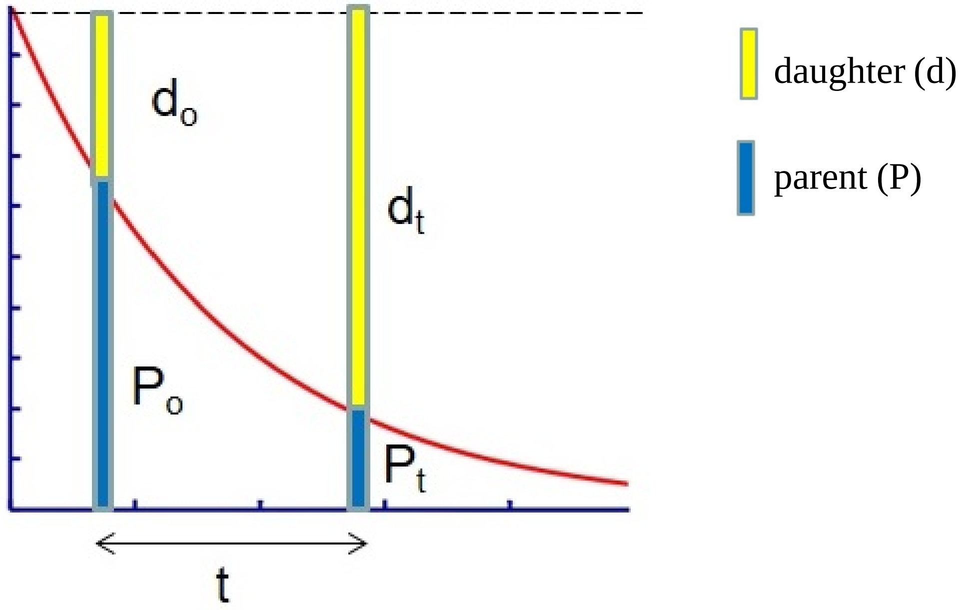
4. ^{238}U...^{206}Pb (α, β) $T_{1/2}$ = 4.5 Ga

4a. ^{235}U…^{207}Pb (α,β) $T_{1/2}$ = 0.7 Ga

4b. ^{232}Th...^{208}Pb (α, β) $T_{1/2}$ = 14 Ga

from measured mother/daughter abundance ratio and known half-life
age of the sample

How can radioactive decay be used to estimate dates in the past?



$$P_t = P_0 \cdot e^{-\lambda \cdot t}$$

$$P_0 + d_0 = P_t + d_t$$

$$R_0 = \frac{d_0}{P_0} \qquad R_t = \frac{d_t}{P_t}$$

$$1 + \frac{d_0}{P_0} = \frac{P_t}{P_0} + \frac{d_t}{P_0} = \frac{P_0 \cdot e^{-\lambda \cdot t}}{P_0} + \frac{d_t}{P_t \cdot e^{\lambda \cdot t}}$$

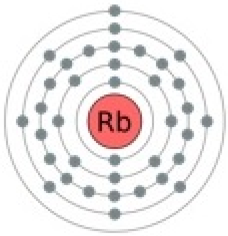
$$1 + R_0 = e^{-\lambda \cdot t} + R_t \cdot e^{-\lambda \cdot t}$$

$$\frac{1 + R_0}{1 + R_t} = e^{-\lambda \cdot t}$$

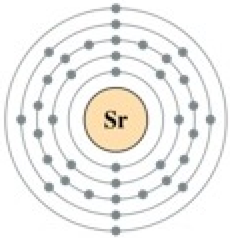
$$t = \frac{ln\left[\frac{1 + R_t}{1 + R_0}\right]}{\lambda}$$

If $R_0 = 0$ then t can be found as R_t can be measured

Rubidium strontium dating



Rubidium and strontium are both reactive metals



This uses a very simple decay process, which has very long half life. Therefore the dating method is only suitable for rocks which are thought to be very old indeed

$${}^{87}_{37}Rb \rightarrow {}^{87}_{38}Sr + \beta^{-} + \bar{\nu}$$ half life = 48 800 000 000 years

The problem with using Rb/Sr dating is that the rocks in question already have strontium present before cooling. Therefore the initial conditions are not known.

This means that a special technique has to be used which is called

ISOCHRON DATING

Isochron dating

The initial conditions of rock can never be measured – only assumed.

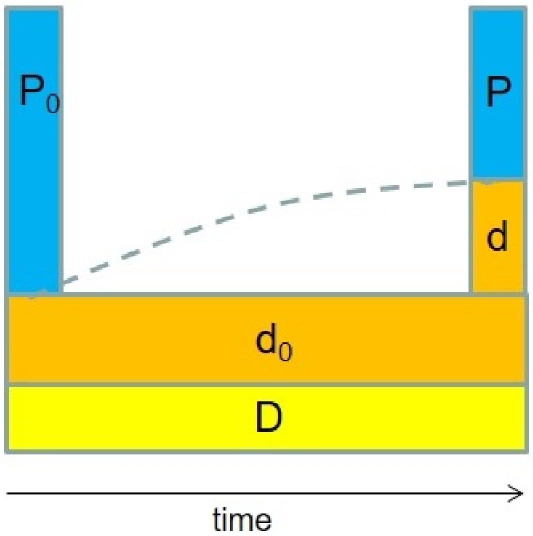
Some radioisotopes in rocks produce a daughter product that has a stable version (another isotope of the daughter element) already in the rock. If the two isotopes of the daughter element are evenly mixed when the rock is formed and the parent element is unevenly distributed then isotope ratio measurements can in fact yield an age estimate.

This means that the original amount of daughter product does not need to be known, i.e. the original parent/daughter isotope ratio is not needed.

Several isotope ratio measurements need to be taken from different parts of the same rock sample. In this way variable amounts of the parent isotope will be present.

Isochron dating is theoretically wonderful but in practice there are problems.

Isochron dating

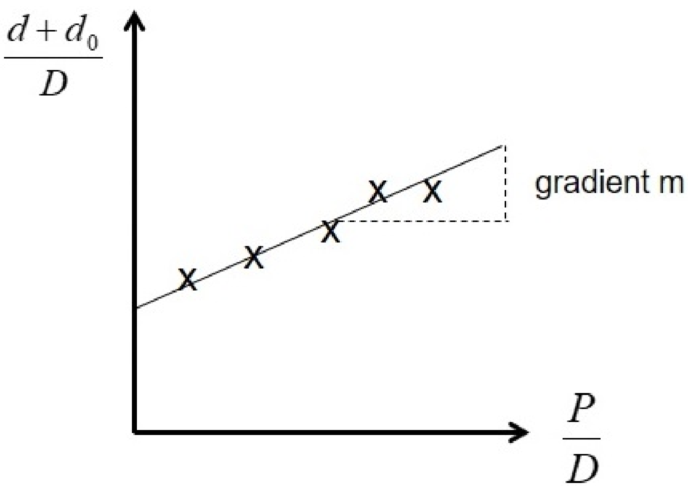


Two isotope ratios need to be measured to determine an isochron date. These are:

$$\frac{P}{D} \text{ and } \frac{d+d_0}{D}$$

A graph is plotted of $\frac{d+d_0}{D}$ against $\frac{P}{D}$

The gradient of this line gives the age of the rock via a simple formula



$$t = ln\left[\frac{m+1}{\lambda}\right]$$

λ = decay constant

Isochron theory

$$P = P_0 e^{-\lambda t} \qquad P_0 = P e^{\lambda t}$$

exponential decay

$$P_0 = P + d$$

number of atoms constant

$$P e^{\lambda t} = P + d \quad \Rightarrow \quad P\left(e^{\lambda t} + 1\right) = d$$

$$d = P\left(e^{\lambda t} + 1\right) \quad \Rightarrow \quad d + d_0 = P\left(e^{\lambda t} + 1\right) + d_0$$

$$\frac{\left(d + d_0\right)}{D} = \frac{P}{D}\left(e^{\lambda t} + 1\right) + \frac{d_0}{D}$$

measured isotope ratios

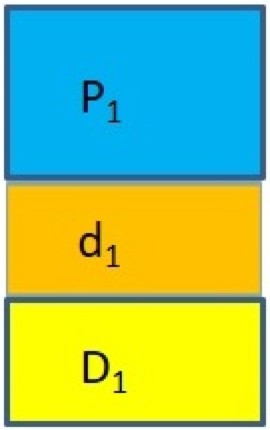
constant at a given time

constant at all times

If a graph is plotted of the isotope ratios, a straight line is obtained whose gradient enables the time to be found.

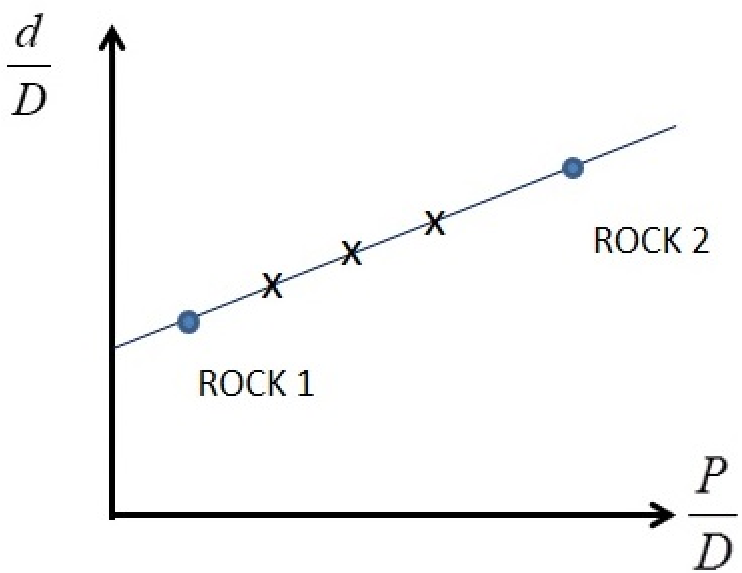
Isochron dating promises mathematical perfection, but …

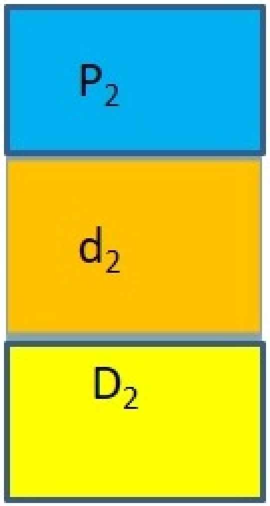
Mixing of rocks can produce fictitious isochron plots



rock-1

If these two rocks are mixed together but not perfectly, and various samples are taken and analyzed to produce an isochron plot, then a straight line graph is obtained whose slope is meaningless.





rock-2

Radioactive dating: Rb-Sr method

Rubidium and strontium are trace elements in natural rocks. Rb can replace K or Na in the crystal lattice, Sr can replace Ca. Rubidium has a radioactive isotope that decays into a strontium isotope by β-decay.

$${}^{87}_{37}Rb \rightarrow {}^{87}_{38}Sr + \beta^- + \bar{\nu} \qquad \text{half life} = 48.8\ \text{Ga} \qquad \lambda = ln2/t_{1/2}$$

Over geological time t after the formation of a rock, the concentration of ^{87}Rb decreases and that that of ^{87}Sr increases

$$[^{87}Rb]_t = [^{87}Rb]_0 \cdot e^{-\lambda \cdot t} \qquad [^{87}Sr]_t = [^{87}Sr]_0 + [^{87}Rb]_0 \cdot \left(1 - e^{-\lambda \cdot t}\right) = [^{87}Sr]_0 + [^{87}Rb]_t \cdot \left(e^{\lambda \cdot t} - 1\right)$$

Because isotope ratios can be measured much more precisely than absolute abundance, it is useful to normalize all concentrations with that of a reference isotope, ^{86}Sr, which is stable and not produced by decay, so that it does not change with time:

$$\frac{[^{87}Sr]_t}{[^{86}Sr]} = \frac{[^{87}Sr]_0}{[^{86}Sr]} + \left(e^{\lambda \cdot t} - 1\right) \cdot \frac{[^{87}Rb]_t}{[^{86}Sr]}$$

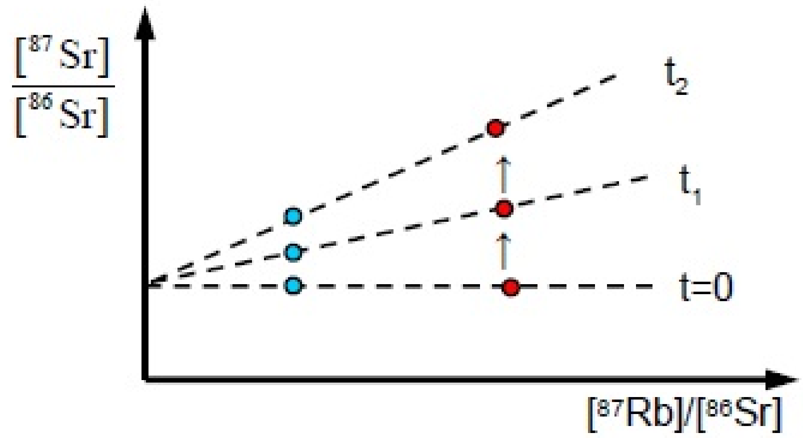
$$y \quad = \quad y_0 \quad + \quad const \quad \cdot \quad x$$

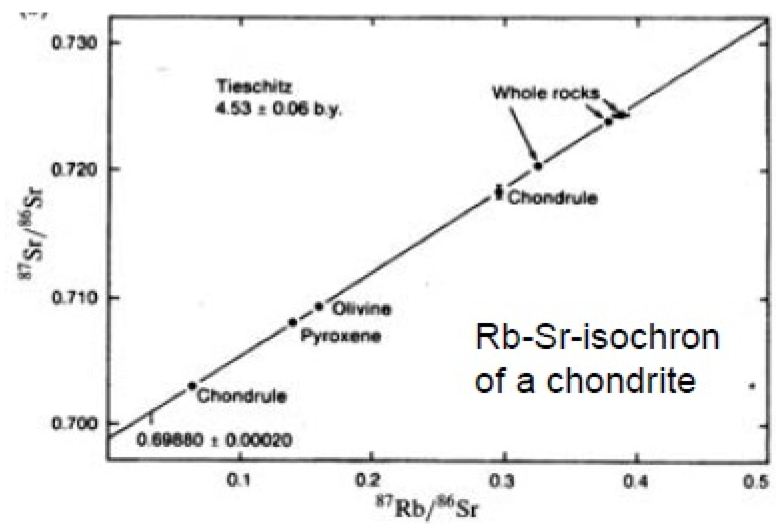
When a rock forms from magma (or solid bodies from the protoplanetary nebula), the source material is well mixed, but during this process it becomes differentiated. The absolute and relative concentrations of Rb and Sr will be different in different mineral grains, in different batches of magma erupted from a magma chamber at different times or in different protoplanets formed from the nebula. The different minerals in a piece of rock, different lava flows coming from the same magma source, or different protoplanets, from a suite of samples with a common origin.

Rb-Sr method

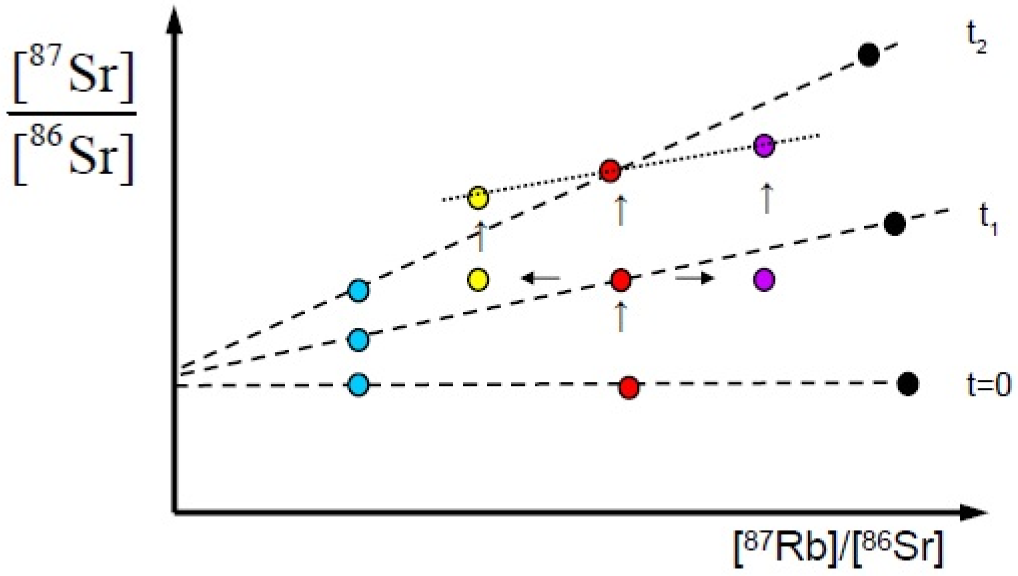
Because the different isotopes of an element behave chemically almost identically, different samples of a suite may have different concentrations of Sr and Rb, but their isotope ratios are initially the same. As time progresses, the ^{87}Sr/^{86}Sr-ratio will grow strongly in a sample with a high Rb/Sr-ratio and weakly in a sample with a low Rb/Sr-ratio.

The age is obtained by measuring the isotope ratios of several samples of a suite and by calculating the best-fitting linear regression. With the known value of λ the age is obtained from the constant of probability (the slope of the regression line, called isochron). An important condition is that the different samples formed in closed systems, i.e. there was no chemical exchange with the environment after the formation.





Dating a second step of differentiation

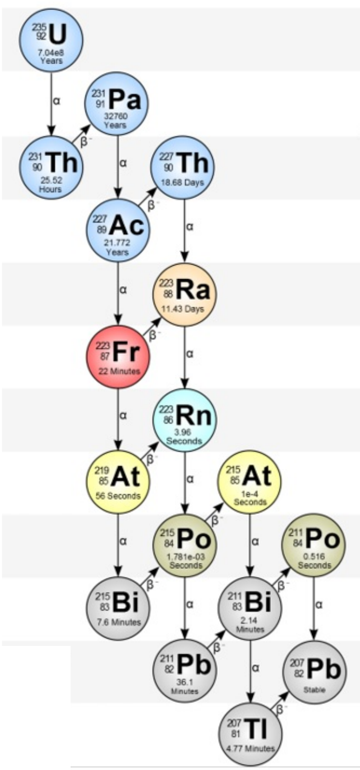


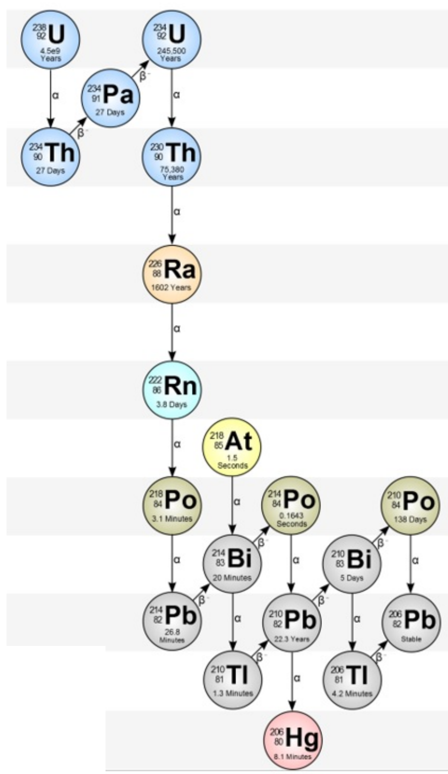
At t=0 a reservoir (e.g. protosolar nebula) splits up into several bodies (planets). At a later time t_1 the red one differentiates into different sub-samples (yellow, red and pink) with different Rb/Sr-ratios. Their $^{87}Sr/^{86}Sr$ ratio is the same at t_1, because they are all drawn from the same reservoir. However, subsequently it will evolve differently because of the different Rb concentrations. The slope connecting these three samples at t_2 indicates the time lapse between t_1 and t_2, i.e., the age of the second differention event. When we want to use the samples from the "red planet" in order to date the first event, we must "remix" them and use them together with data from the other planets (blue and black). If we use at t_2 the blue, yellow and black points, they do not fall on a straight line.

Age of the Earth Pb –Pb method

The Pb-Pb method of dating makes use of two decay series, both starting at an isotope of uranium and ending at a lead isotope

parent	half-life [10^9 a]	daughter	material dated
^{235}U	0.704	^{207}Pb	zircon, uraninite, pitchblende
^{238}U	4.468	^{206}Pb	Zircon, uraninite, pitchblende





Age of the Earth Pb –Pb method

rock dating:

^{235}U decays into ^{207}Pb with $T_{1/2} = 7.038 \cdot 10^8$ y. ^{238}U decays into ^{206}Pb with $T_{1/2} = 4.47 \cdot 10^9$ a

^{204}Pb is a **stable** isotope ($^{232}Th \rightarrow {}^{208}Pb$)

$$^{235}U_0 = {}^{235}U(t_0) = {}^{235}U_{in} \cdot e^{-\Delta t/T_{235}}$$

$$^{207}Pb = 235U_{in} - {}^{235}U_0$$

$$^{207}Pb = 235U_0 \cdot \left(e^{\Delta t/T_{235}} - 1\right)$$

$$R_1 \equiv \frac{^{207}Pb_{in}}{^{204}Pb} + \frac{^{235}U_0 \cdot \left(e^{\Delta t/T_{235}} - 1\right)}{^{204}Pb}$$

and

$$R_2 \equiv \frac{^{206}Pb_{in}}{^{204}Pb} + \frac{^{238}U_0 \cdot \left(e^{\Delta t/T_{238}} - 1\right)}{^{204}Pb}$$

$$\frac{R_1(A) - R_1(B)}{R_2(A) - R_2(B)} = \frac{^{235}U_0}{^{238}U_0} \cdot \frac{e^{\Delta t_A/T_{235}} - e^{\Delta t_B/T_{235}}}{e^{\Delta t_A/T_{238}} - e^{\Delta t_B/T_{238}}}$$

$$\Delta t_A = \frac{ln(T_{235}/T_{238}) + ln\frac{^{238}U_0}{^{235}U_0} + ln\left(\frac{R_1(A) - R_1(B)}{R_2(A) - R_2(B)}\right)}{1/T_{235} - 1/T_{238}}$$

$^{235}U_0/{}^{238}U_0 = 1/137.9$

assumption for both rocks: same initial conditions of $^{207}Pb/^{204}Pb$ and $^{206}Pb/^{204}Pb$

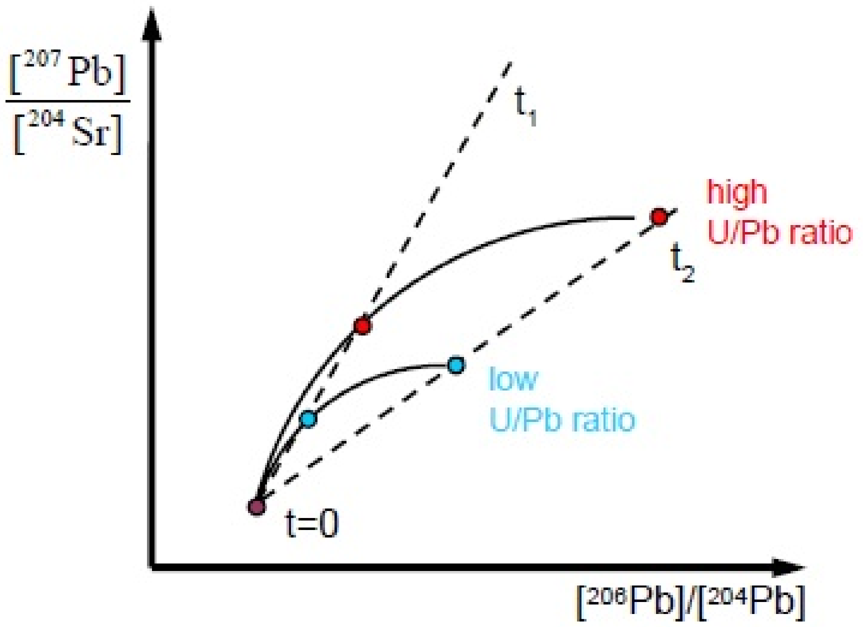
Pb-Pb method

$$\frac{[^{207}Pb]_t}{[^{204}Pb]} = \frac{[^{207}Pb]_0}{[^{204}Pb]} + \frac{e^{\lambda_{235}\cdot t}-1}{e^{\lambda_{238}\cdot t}-1}\cdot\frac{[^{235}U]_t}{[^{238}U]_t}\cdot\left(\frac{[^{206}Pb]_t}{[^{204}Pb]} - \frac{[^{206}Pb]_0}{[^{204}Pb]}\right)$$

$$y \quad = \quad y_o \quad + \quad const \quad \cdot \quad (x - x_0)$$

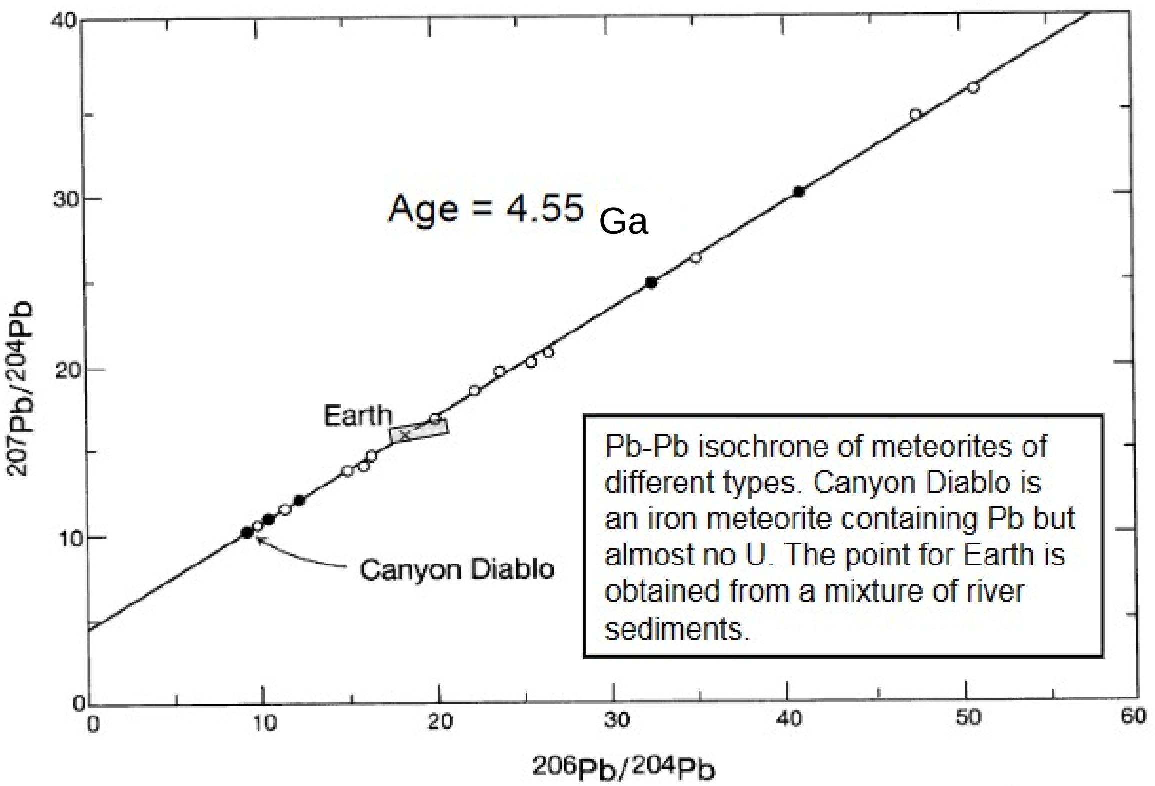
$$R = {}^{[235U]}/_{[238U]} = {}^{1}/_{137.9} \quad \text{(today)}$$

At a given time, R is the same for all samples. Because of the short half-life of ^{235}U, the $^{207}Pb/^{204}Pb$-ratio grew rapidly early on, but grows more slowly in more recent times. Again, samples from a cogenetic suite fall on an isochron, whose slope relates to the age through the above equation. With this method only isotopes ratios of a single element need to be measured.

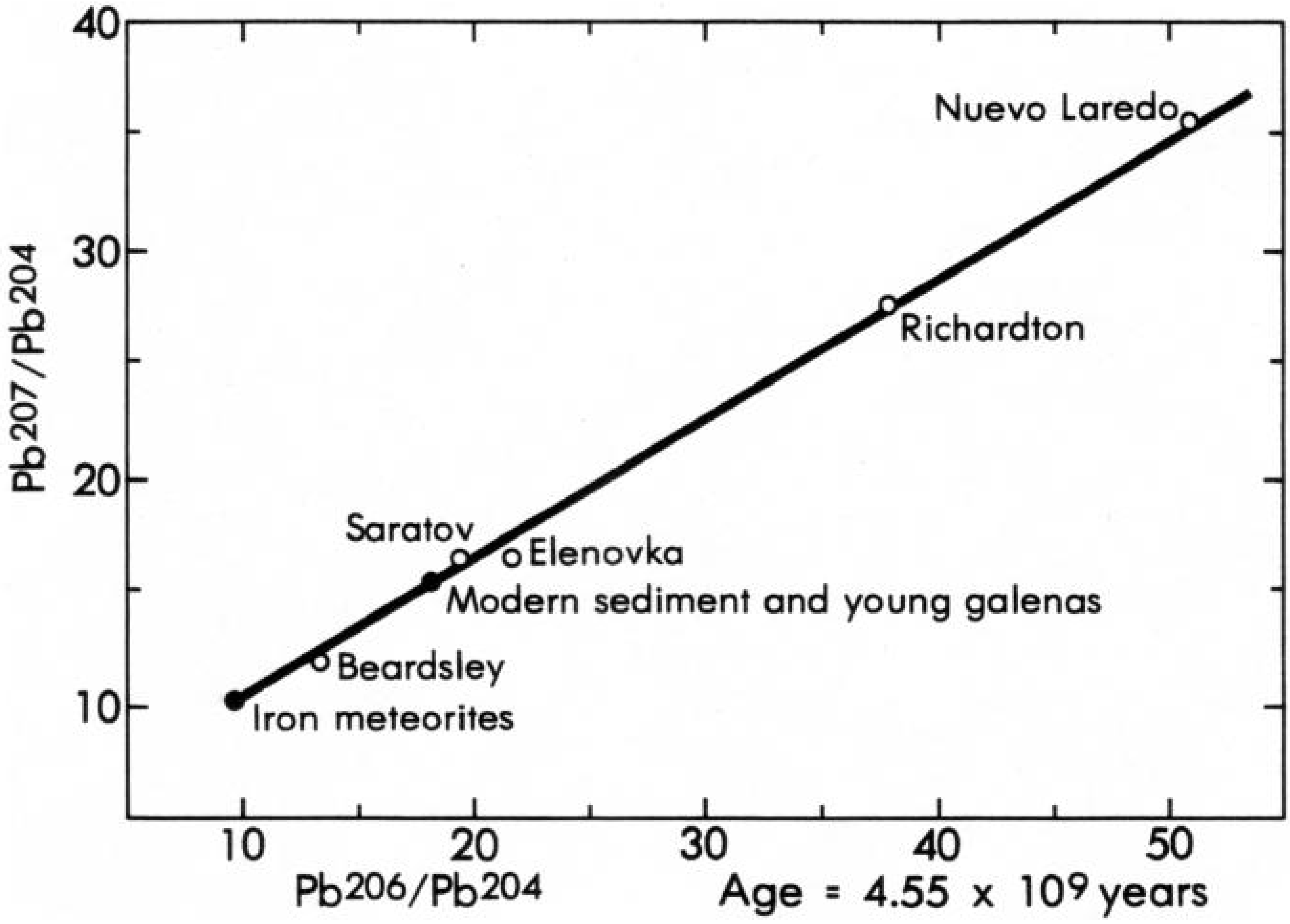


Age of the Earth and the Solar system

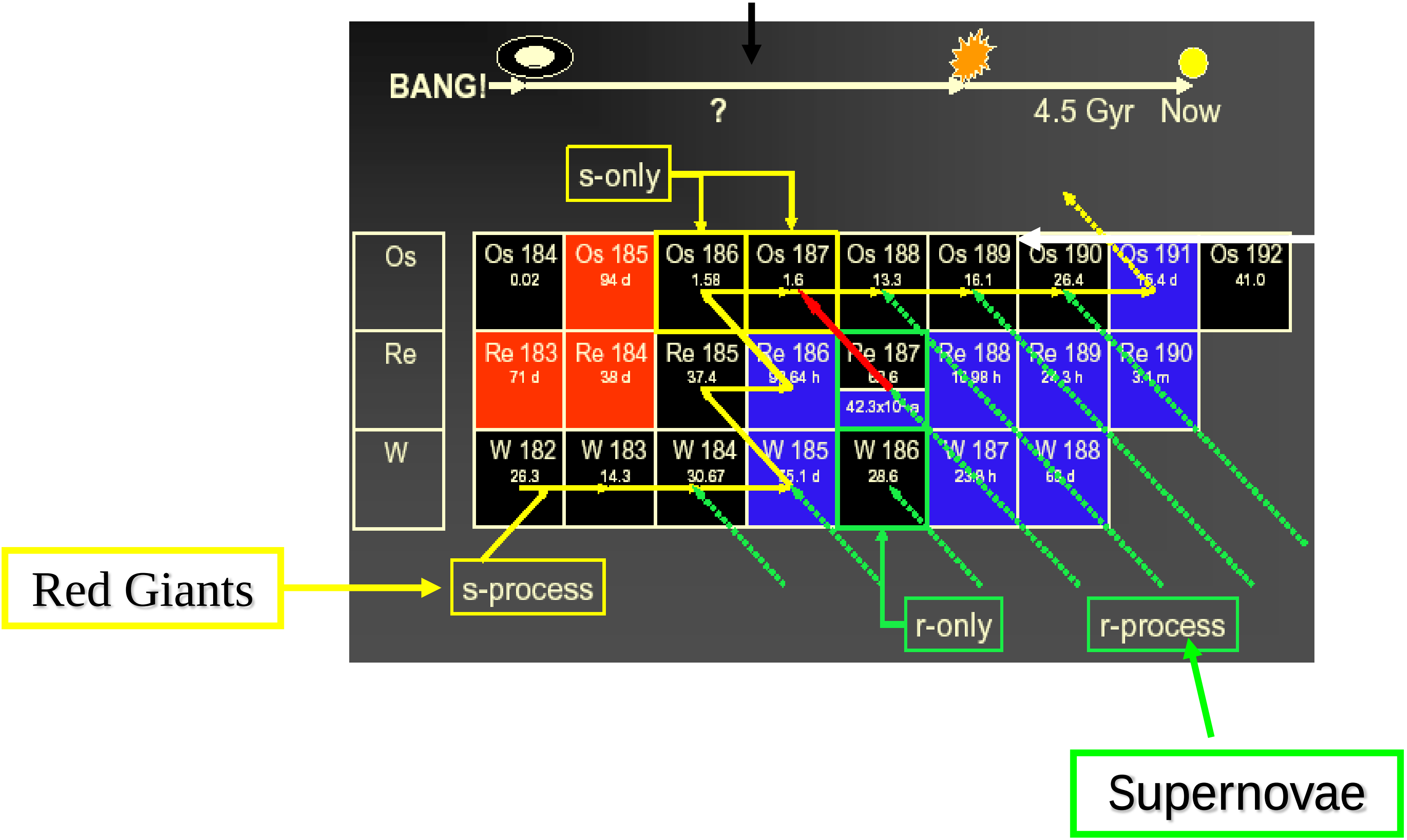
oldest stones on Earth	$3.7 \cdot 10^9$ a
oldest stones on Moon	$(4.5 - 4.6) \cdot 10^9$ a
oldest meteorites	$4.57 \cdot 10^9$ a
age of the solar system	$(4.57 \pm 0.07) \cdot 10^9$ a



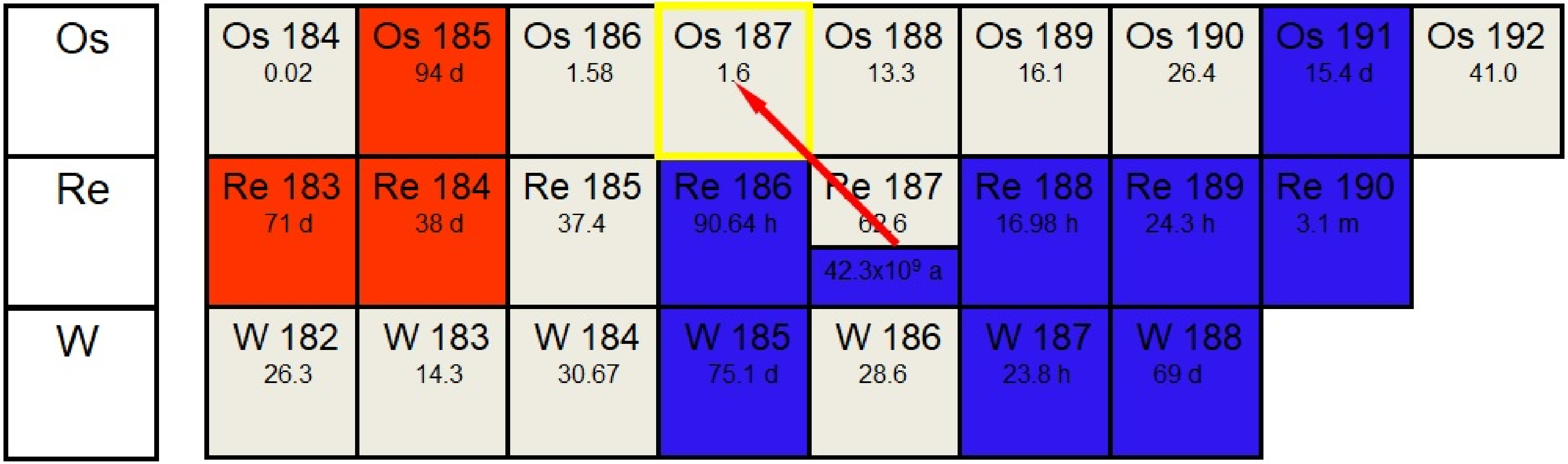
Age of meteorite parent bodies



Re/Os cosmos-chronometer



$^{187}Re/^{187}Os$ clock



The ´best-suited´ eon clock: $^{187}Re/^{187}Os$

$T_N >$ $\tau(^{187}Re)$ x $R\,(^{187}Os/^{187}Re)_d$

61.3 Ga x 0.137

= **8.4 Ga**

but…

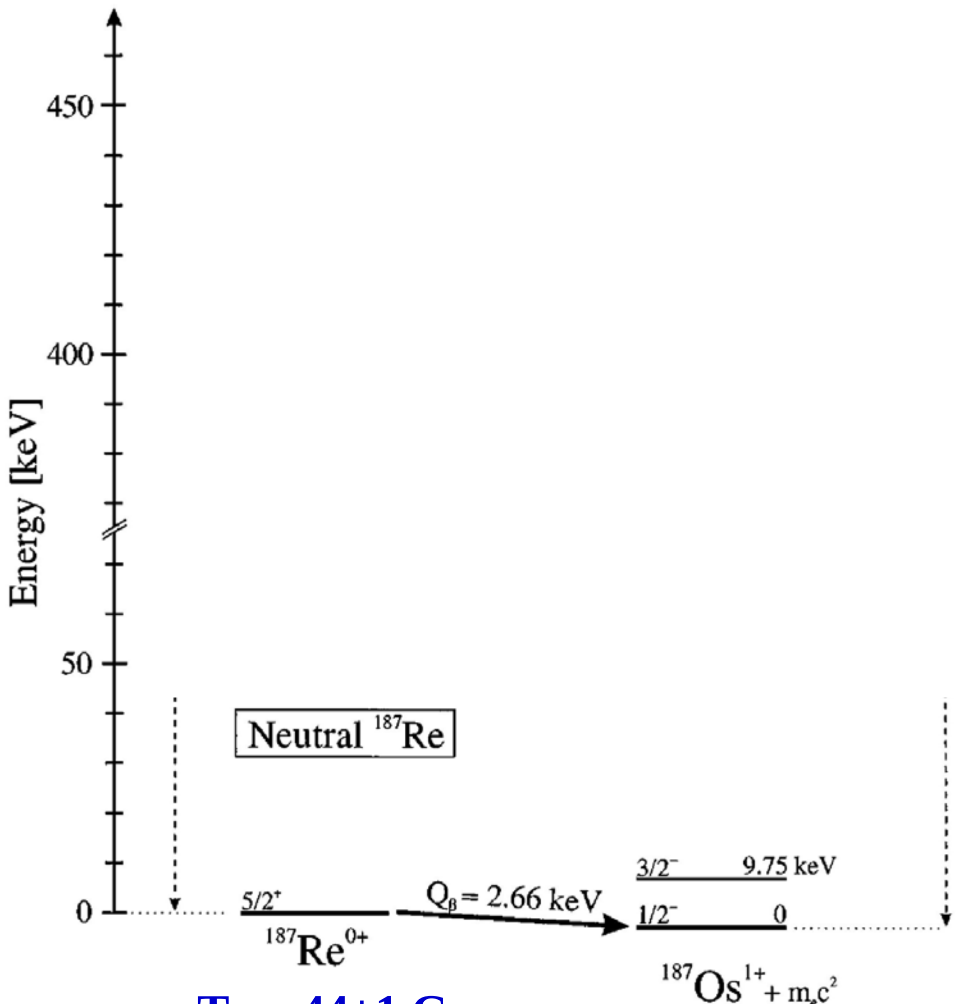
bare and H-like ^{187}Re undergoes β_b decay to the first excited state of ^{187}Os

nuclear matrix element **not** known

measurement of **lifetime τ of bare ^{187}Re** at the ESR gives

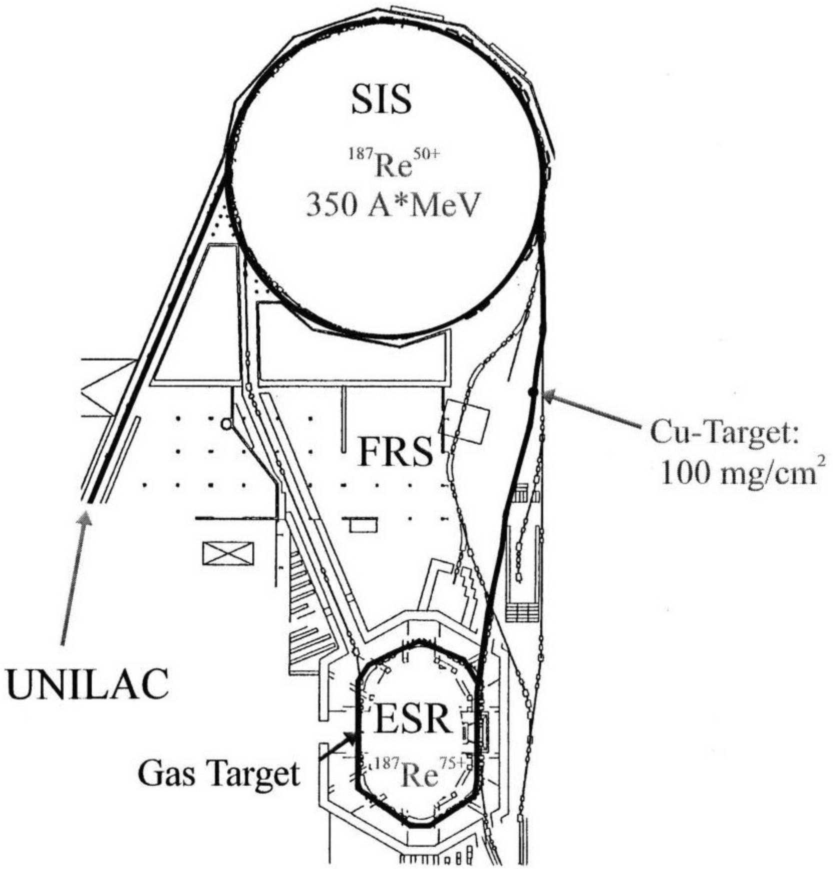
→ **τ(bare ^{187}Re) = (48 ± 3) a**

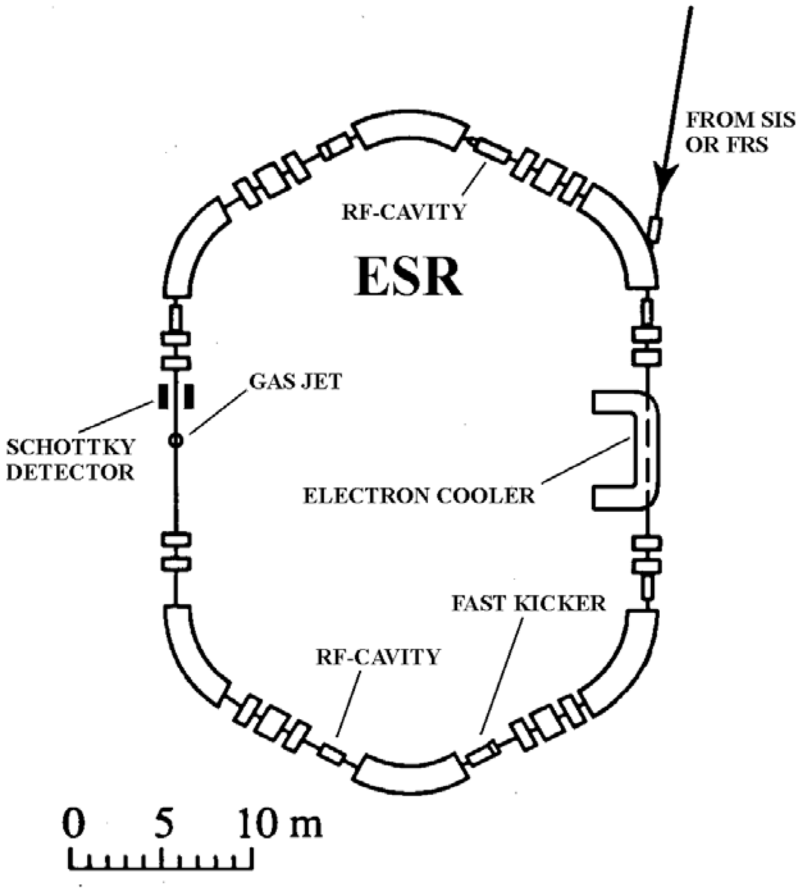
instead of (**61.3 ± 1.9) Ga**



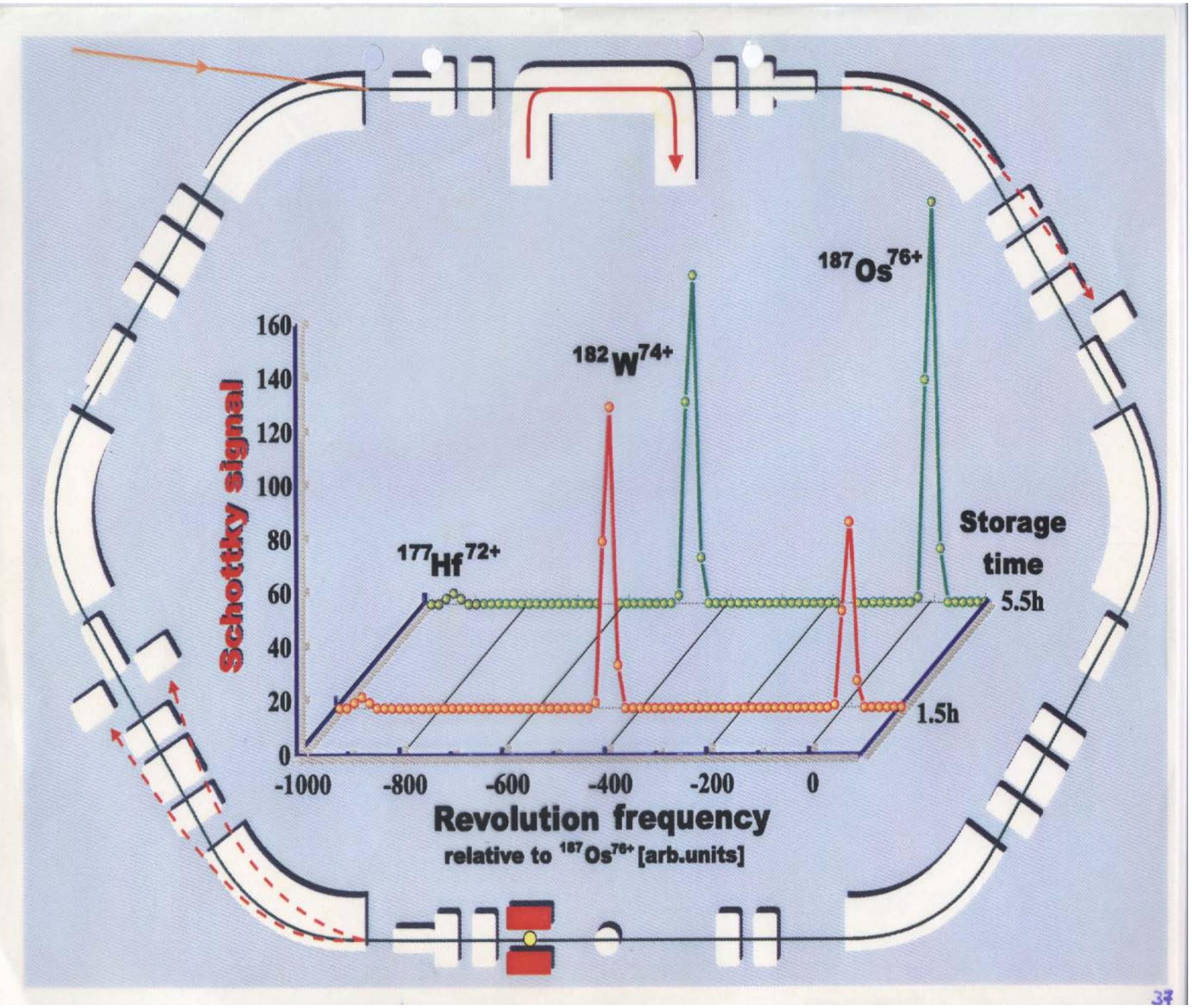
$T_{1/2}$ = 44±1 Ga

The experiment





How to determine a long (33 y) beta half-life?



1. store and cool bare ^{187}Re for various times (hours)

2. the β_b daughters, H-like ^{187}Os, are **not resolved** in Schottky spectrum. Q value only 62 keV at the same atomic charge state **q = 75**$^+$

3. after the (long) storage time **strip the one electron** of ^{187}Os in an intense gas jet, acting for a few minutes only

4. the **bare ^{187}Os** ions are **well-resolved** now, at **q = 76**$^+$

5. the number of nuclear reaction products (Hf, W,..) does **not** depend on storage time

F. Bosch et al., PRL 77 (1996) 5

Summary

Ion storage-cooler rings (and traps) allow to address for the first time
β decays of ions at high, well-defined charge states
which is important in the framework of stellar nucleosynthesis in hot environments.

The temperature of the s-process can be directly determined from β_b decay.

Significant, even dramatic changes of β lifetimes due to β_b decay have been observed.

The Re/Os cosmic clock is strongly affected by the atomic charge state:
nuclear cosmic clocks are not independent on stellar evolution models