Reduced Transition Probabilities

The reduced transition probability $B(^E_M\lambda)$ is related to the nuclear matrix element by the formula

$$B(^E_M\lambda, I_i \to I_f) = \sum_{\mu M_f} | < I_f M_f | M(^E_M\lambda, \mu) | I_i M_i > |^2 \quad (1)$$

According to the Wigner-Eckart theorem a matrix element of an operator $M(\lambda, \mu)$ can be factorized

$$< I_f M_f | M(^E_M\lambda, \mu) | I_i M_i > = (I_i \lambda M_i \mu | I_f M_f) < I_f || M(^E_M\lambda) || I_i > \quad (2)$$

where $(I_i \lambda M_i \mu | I_f M_f)$ is a Clebsch-Gordan coefficient. Then, with use of the orthonormality of the Clebsch-Gordan coefficients, we have another expression for $B(^E_M\lambda)$:

$$B(^E_M\lambda, I_i \to I_f) = \frac{1}{2I_i + 1} | < I_f || M(^E_M\lambda) || I_i > |^2 \quad (3)$$

This expression assures us that the lifetime of a state does not depend on its orientation (rotational invariance). The reduced matrix elements

$< I_f || M(^E_M\lambda) || I_i >$ contain the information about the the nuclear wave functions. The relation of the reduced transition probability between the excitation $B(^E_M\lambda) \uparrow$ and the decay $B(^E_M\lambda) \downarrow$ of the nuclear state is given by

$$B(^E_M\lambda, I_f \to I_i) = \frac{2I_i + 1}{2I_f + 1} B(^E_M\lambda, I_i \to I_f) \quad (4)$$

since the absolute value of the reduced matrix element is invariant under the interchange of I_i and I_f.

In case of an E2 transition between ground state 0^+_{gs} and the first excited state 2^+_1, we obtain

$$B(E2, 0^+_{gs} \to 2^+_1) = 5 * B(E2, 2^+_1 \to 0^+_{gs}) \quad (5)$$