Surface Oscillations around a Spherical Shape

In the simplest version of the vibrational model one assumes that the nucleus has spherical symmetry in its ground state and that the excited states are due to harmonic oscillations of the nuclear surface. The spectra of the harmonic quadrupole ($\lambda = 2$) and octupole ($\lambda = 3$) oscillator is shown in fig. 1.

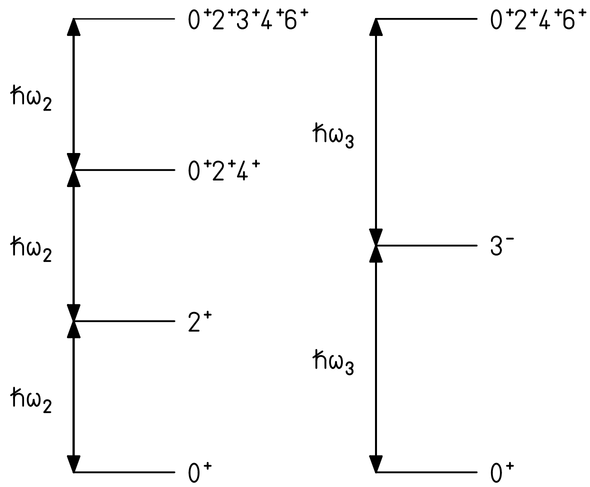

Figure 1: Harmonic energy spectra for the quadrupole (left) and octupole (right) surface oscillations.

In the harmonic approximation, the phonon states E(n_λ) are energetically degenerated in spin I and are equally spaced with respect to n_λ

$$E(n_{\lambda+1}) - E(n_\lambda) = \hbar\omega_\lambda \tag{1}$$

with the frequencies

$$\omega_\lambda = \left(\frac{C_\lambda}{B_\lambda}\right)^{1/2} \tag{2}$$

where B_λ is the inertia parameter and C_λ is the stiffness parameter. Both parameters can be calculated if the collective motion of nucleons in nuclei can be described as the irrotational flow of a fluid. In this classical model the two parameters are given by

$$B_\lambda = \frac{3AMR_0^2}{4\pi\lambda} \tag{3}$$

$$C_\lambda = (\lambda - 1)[(\lambda + 2)R_0^2\sigma - \frac{3Z^2e^2}{2\pi(2\lambda + 1)R_0}] \tag{4}$$

where the surface tension σ=1.1 MeV/fm^2 can be calculated from the semi-empirical mass formula (atomic mass unit M=931.478 MeV/c^2). From the structure of the wave functions and the multipole operators the following mode-dependent selection rules for the reduced transition probabilities and static moments can be deduced: a) the multipolarity of the allowed electric transitions is equal to the phonon multipolarity, b) electric transitions can only occur between states with $n_\lambda - n_{\lambda'} = \pm 1$ and all static electric multipole moments are zero, c) all magnetic dipole transitions are forbidden. Thus the only nonvanishing M1 matrix elements are

$$< n_\lambda I||M(M1)||n_\lambda I >= \sqrt{\frac{3}{4\pi}} g_R \mu_N \sqrt{I(I+1)(2I+1)} \tag{5}$$

The quantity g_R is the effective g-factor for the collective motion; it is expected to be of the order of $g_R \sim Z/A$ ($\mu_N = \frac{e\hbar}{2Mc}$).

For a shape vibration of the order $\lambda = 2$ and a uniform charge distribution, the reduced transition matrix element (Eq. **??**) connecting the 1-phonon state with the ground state is given by

$$< I = 2, n_2 = 1||M(E2)||I = 0, n_2 = 0 >= \sqrt{5}\ Q_{vib}\ e \tag{6}$$

where the quantity Q_{vib} is calculated in the liquid-drop model

$$Q_{vib} = \frac{3ZR^2}{4\pi} \sqrt{\frac{\hbar}{2B_2\omega_2}} \tag{7}$$

Energy-B(E2) Product It should be noted that the vibrational model predicts a strong correlation between the B(E2) value of the first 2^+ state and its energy $E_{2^+} = \hbar\omega_2$.

$$E_{2^+}\ B(E2; 2^+ \rightarrow 0^+) = (\frac{3ZeR^2}{4\pi})^2 \frac{\hbar^2}{2B_2} \tag{8}$$

with

$$B_2 = \frac{3AMR_0^2}{8\pi} \tag{9}$$

Thus, we get for the vibrational energy-B(E2) product,

$$E_{2^+}\ B(E2; 2^+ \rightarrow 0^+) = 1.44\ 10^{-3} \frac{Z^2}{A^{1/3}}\ MeV\ e^2 barn^2 \tag{10}$$

Experimentally, a similar relation has been found for all the nuclei throughout the nuclear table. However, the product of the energy and B(E2)-value reaches only 7-8% of the vibrational limit.

The nonvanishing matrix elements to the 2-phonon quadrupole states are

$$< I = 4, n_2 = 2||M(E2)||I = 2, n_2 = 1 >= \sqrt{18}\ Q_{vib}\ e \tag{11}$$

$$< I = 2, n_2 = 2||M(E2)||I = 2, n_2 = 1 >= \sqrt{10}\ Q_{vib}\ e \tag{12}$$

$$< I=0, n_2=2||M(E2)||I=2, n_2=1 >= \sqrt{2}\ Q_{vib}\ e \qquad (13)$$

For the transitions to the 3-phonon quadrupole states we obtain

$$< I=6, n_2=3||M(E2)||I=4, n_2=2 >= \sqrt{39}\ Q_{vib}\ e \qquad (14)$$

$$< I=4, n_2=3||M(E2)||I=4, n_2=2 >= \sqrt{\frac{90}{7}}\ Q_{vib}\ e \qquad (15)$$

$$< I=4, n_2=3||M(E2)||I=2, n_2=2 >= \sqrt{\frac{99}{7}}\ Q_{vib}\ e \qquad (16)$$

$$< I=3, n_2=3||M(E2)||I=4, n_2=2 >= \sqrt{6}\ Q_{vib}\ e \qquad (17)$$

$$< I=3, n_2=3||M(E2)||I=2, n_2=2 >= -\sqrt{15}\ Q_{vib}\ e \qquad (18)$$

$$< I=2, n_2=3||M(E2)||I=4, n_2=2 >= \sqrt{\frac{36}{7}}\ Q_{vib}\ e \qquad (19)$$

$$< I=2, n_2=3||M(E2)||I=2, n_2=2 >= \sqrt{\frac{20}{7}}\ Q_{vib}\ e \qquad (20)$$

$$< I=2, n_2=3||M(E2)||I=0, n_2=2 >= \sqrt{7}\ Q_{vib}\ e \qquad (21)$$

$$< I=0, n_2=3||M(E2)||I=0, n_2=2 >= \sqrt{3}\ Q_{vib}\ e \qquad (22)$$

For the static moments we obtain

$$Q(n_2, I) = 0 \qquad (23)$$

$$\mu(n_2, I) = g_R I \mu_N \qquad (24)$$