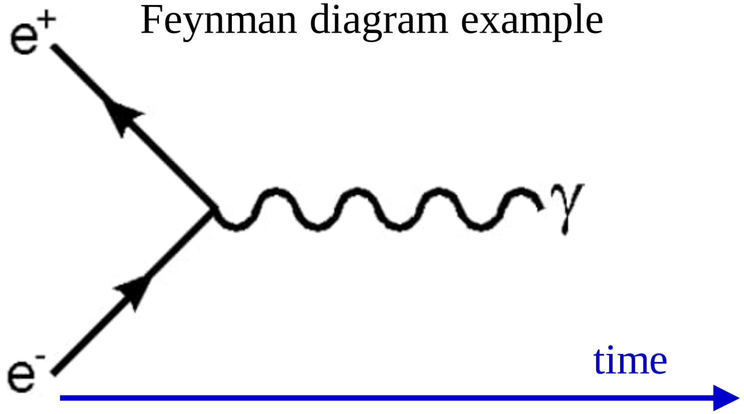
Feynman diagrams

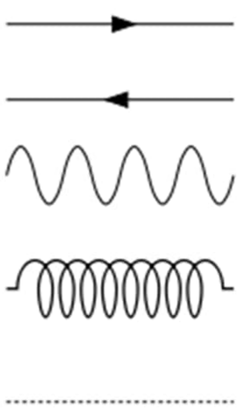
In 1940s, R. Feynman developed a diagram technique to describe particle interactions in space-time.

Richard Feynman

Feynman diagram example



❖ Particles are represented by lines



Fermion f

Antifermion $\bar{f}$

γ, Z, W

Gluon

Boson

- Particles go forward in time
- Antiparticles go backwards in time

Feynman diagrams

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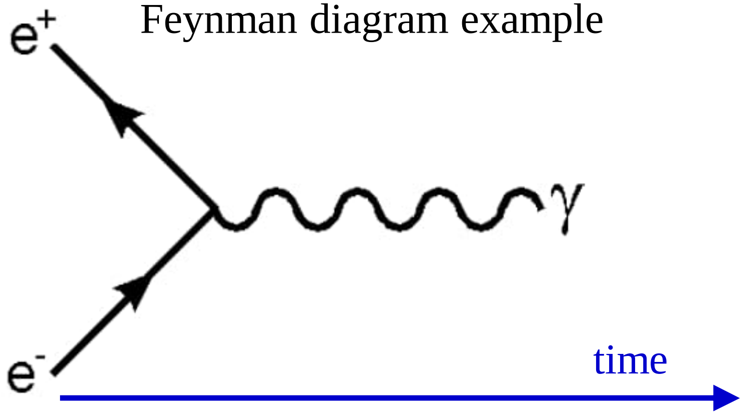


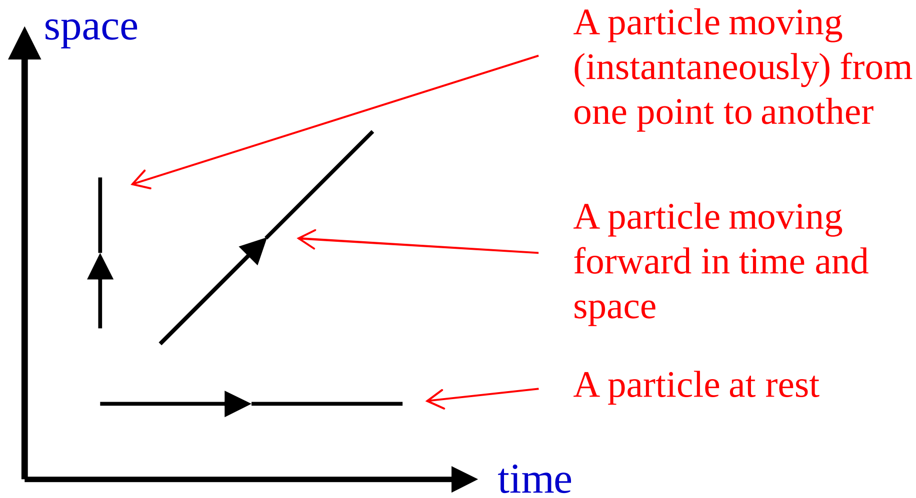
Richard Feynman

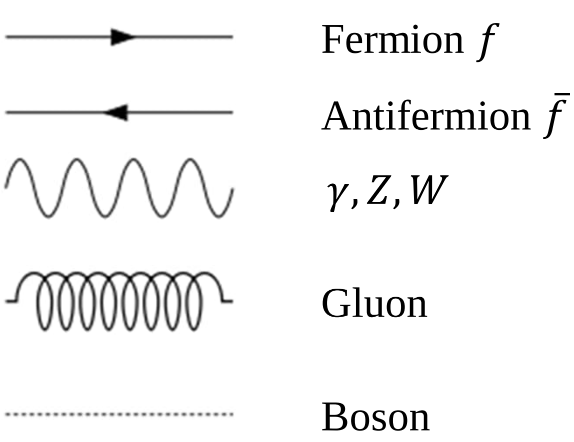
Main assumptions and requirements:

- Time runs from left to right (convention)
- Particles are usually denoted with **solid lines**, and gauge bosons - with **helices** or **dashed lines**
- **Arrow** directed towards the right indicates a particle, otherwise – antiparticle
- *Points at which 3 or more particles meet are called* ***vertices***
- *At any vertex, momentum, angular momentum and charge are conserved (but not energy)*

Feynman diagrams are like circuit diagrams – they show what is connected to, but length and angle of momentum vectors are not relevant.

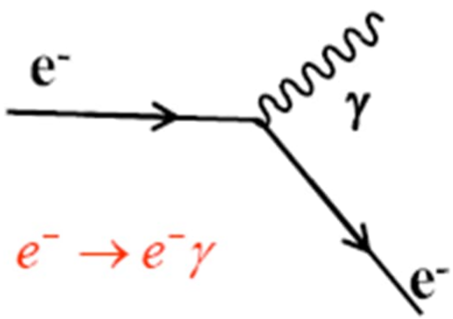
Richard Feynman

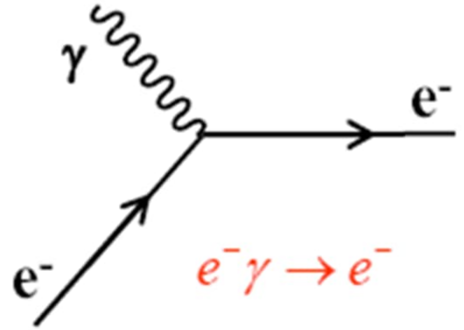




Vertices

- Lines connect into vertices, which are the building blocks of Feynman diagrams

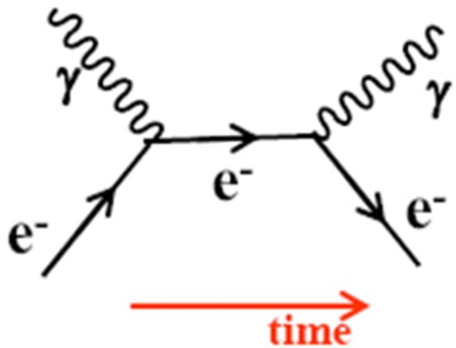




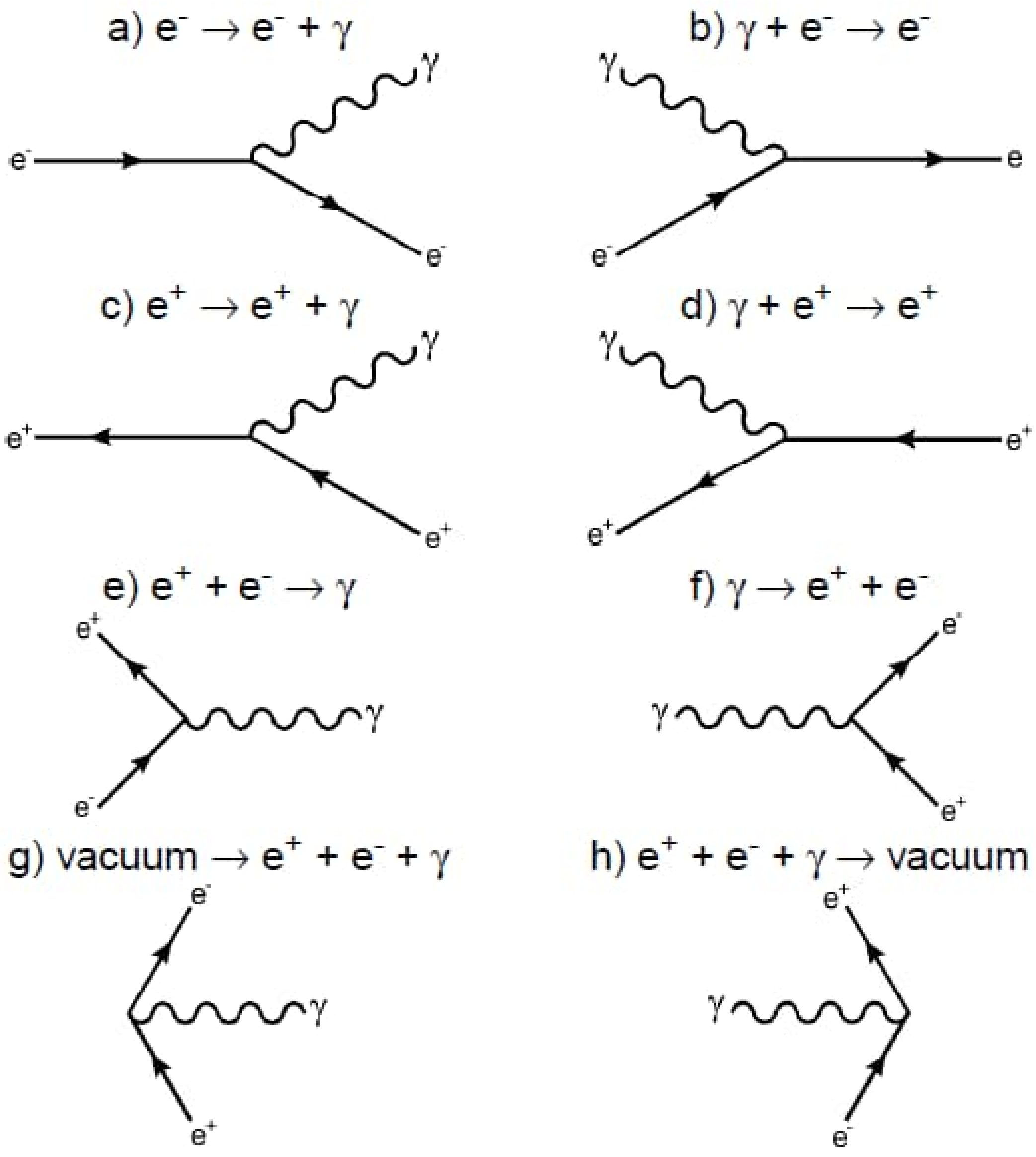
- Charge, lepton number and baryon number as well as momentum are always conserved at a vertex.

Compton scattering

A photon scatters from an
electron producing a photon
and an electron in the final state



Lowest order diagram has two vertices



Feynman diagrams for basic processes involving electron, positron and photon

Feynman Diagrams

Each Feynman diagram represents an Amplitude (**M**)

Fermi's Golden Rule: $transition\ rate = \frac{2\pi}{\hbar}|M|^2 \times (phase\ space)$

In lowest order perturbation theory **M** is the Fourier transformation of the potential. "Born Approximation"

Differential cross section for two body scattering (e.g. pp → pp) in the CM system:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{q_f^2}{v_i v_f}|M|^2$$

q_f = final state momentum
v_f = speed of final state particle
v_i = speed of initial state particle

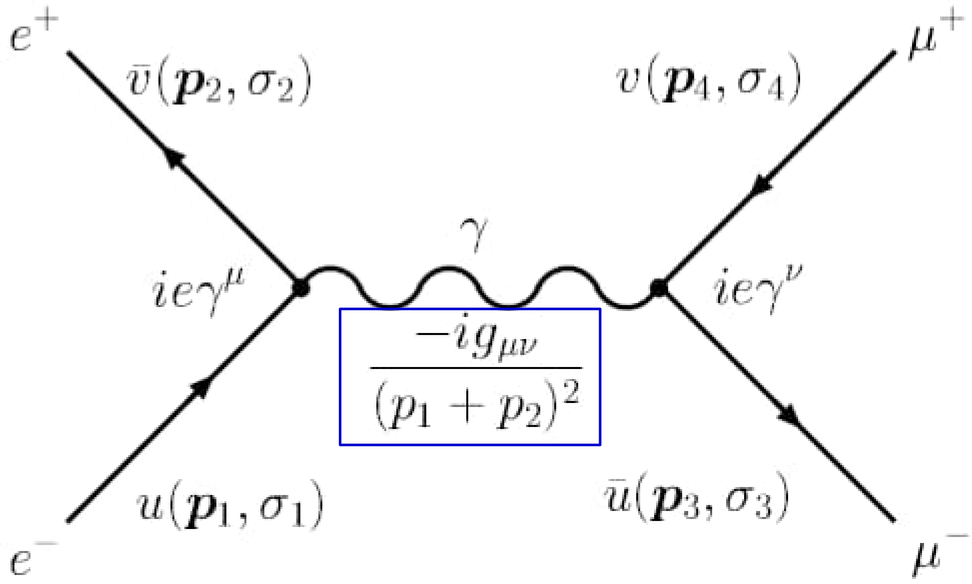
The decay rate (Γ) for a two body decay (e.g. $K^0 \rightarrow \pi^+\pi^-$) in the CM system:

$$\Gamma = \frac{S \cdot |\vec{p}|}{8\pi\hbar m^2 c}|M|^2$$

m = mass of parent
p = momentum of decay particle
S = statistical factor (fermions/bosons)

In most cases $|\boldsymbol{M}|^2$ cannot be calculated exactly.
Often ***M*** is expanded in a power series.
Feynman diagrams represent terms in the series expansion of ***M***.

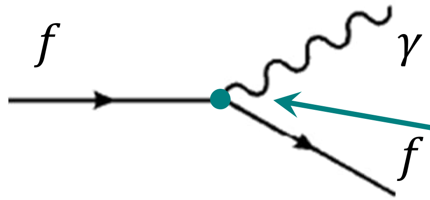
Feynman Diagrams



$$-iM = [\bar{u}(p_3, \sigma_3)(ie\gamma^\nu)v(p_4, \sigma_4)]\,\frac{-ig_{\mu\nu}}{(p_1 + p_2)^2}\,[\bar{v}(p_2, \sigma_2)(ie\gamma^\mu)u(p_1, \sigma_1)]$$

for massive particle: $\dfrac{-i\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right)}{p^2 - m^2}$

- A coupling constant (multiplication factor) is associated with each vertex.
- Value of coupling constant depends on type of interaction



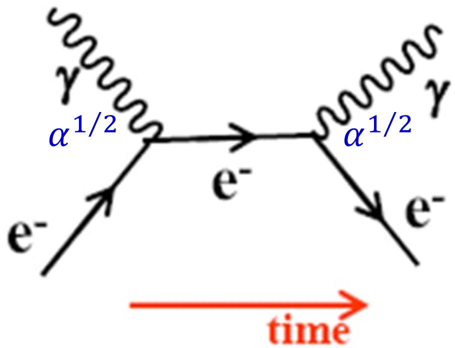
Electromagnetic $f \rightarrow f\gamma$

$Q_f = fermion\ charge$

(in units of electron charge)

$$Q_f \cdot \sqrt{\alpha} = Q_f \cdot \sqrt{\frac{e^2}{4\pi}}$$

- Example: Compton scattering of an electron

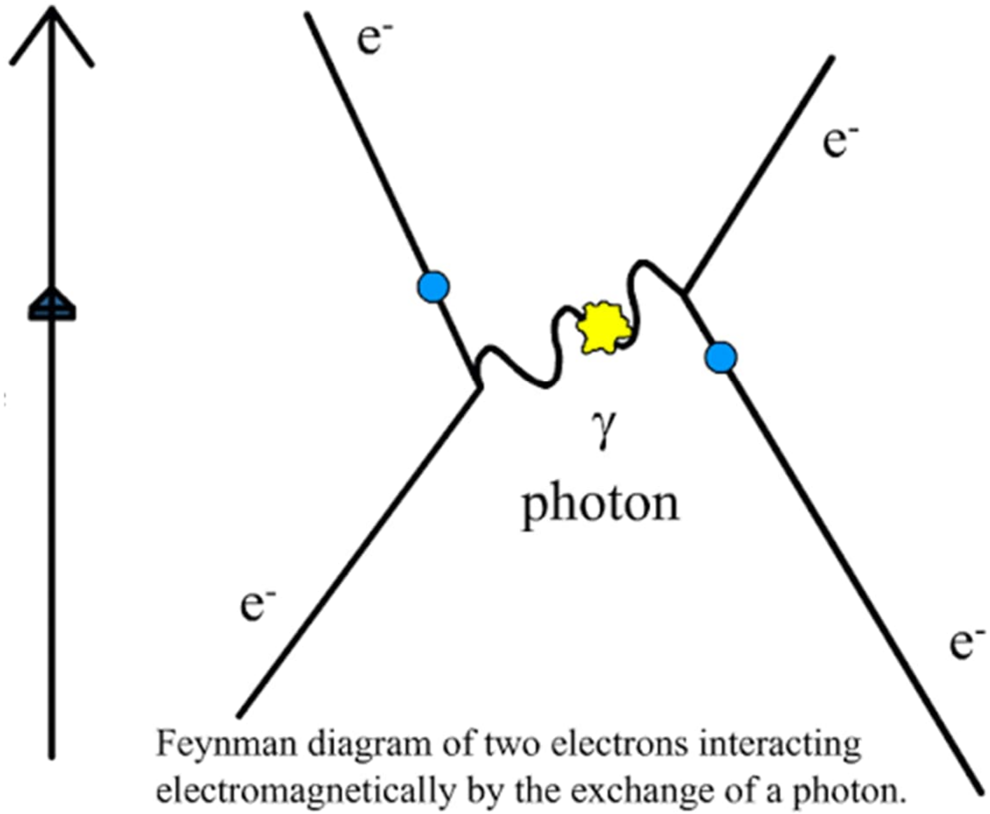


$$Diagram \ \propto (coupling)^2 \propto \alpha$$

$$\sigma \propto |Diagram|^2 \propto \alpha^2 \propto e^4$$

$= {}^1/_{137}$

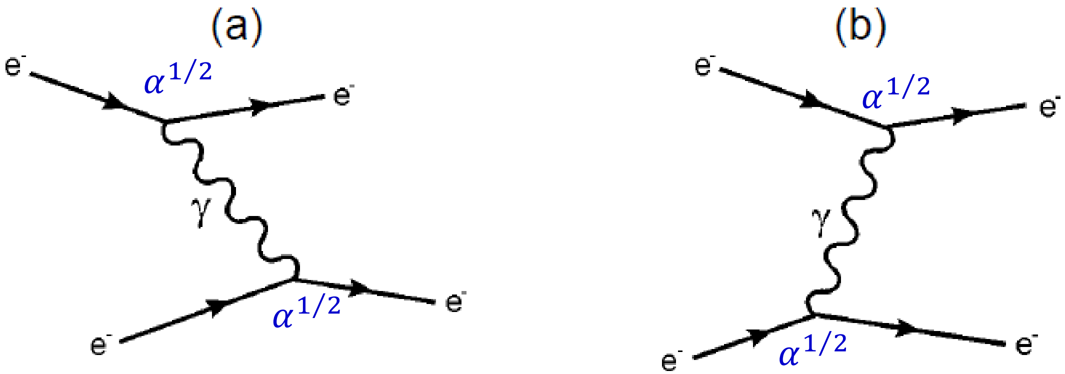
- Total four-momenta conserved at a vertex
- Can move particle from initial to final state by replacing it into its antiparticle $f \rightarrow f\gamma$ *becomes* $f\bar{f} \rightarrow \gamma$



Feynman diagram of two electrons interacting electromagnetically by the exchange of a photon.

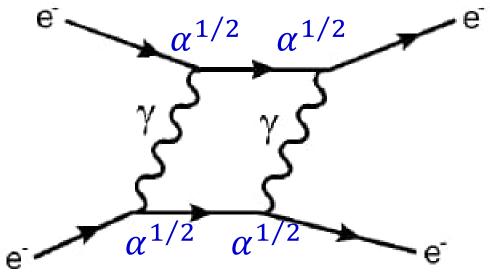
Real processes

For a real process there must be energy conversation → it has to be a combination of virtual processes



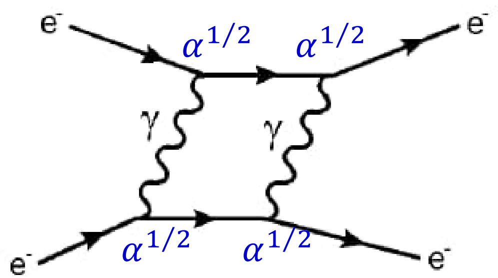
Electron-electron scattering, single photon exchange

Any real process receives contributions from all possible virtual processes



Two-photon exchange contribution

Real processes



Two-photon exchange contribution

- Number of vertices in a diagram is called its ***order***
- Each vertex has an associated probability proportional to a ***coupling constant***, usually denoted as "α". In the electromagnetic processes this constant is

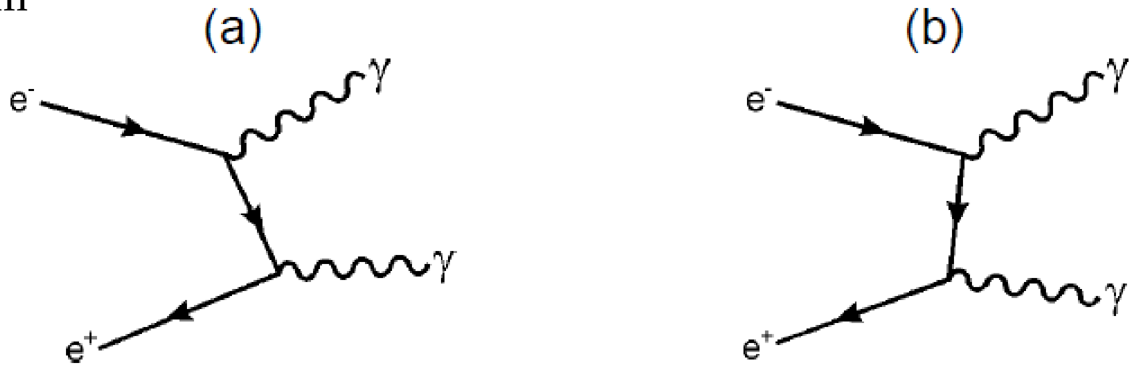
$$\alpha_{em} = \frac{e^2}{4\pi\varepsilon_0} \approx \frac{1}{137}$$

- For the real processes, a diagram of the ***order n*** gives a contribution of order $\boldsymbol{\alpha^n}$

- Provided that α is small enough, higher order contributions to many real processes can be neglected.

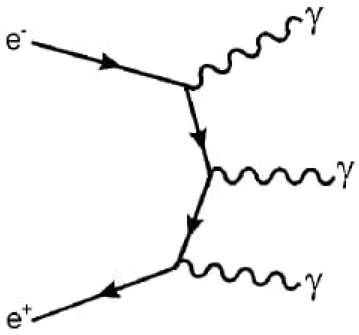
Real processes

Diagrams which differ only by time-ordering are usually implied by drawing only one of them



Lowest order contributions to $e^+e^- \rightarrow \gamma\gamma$

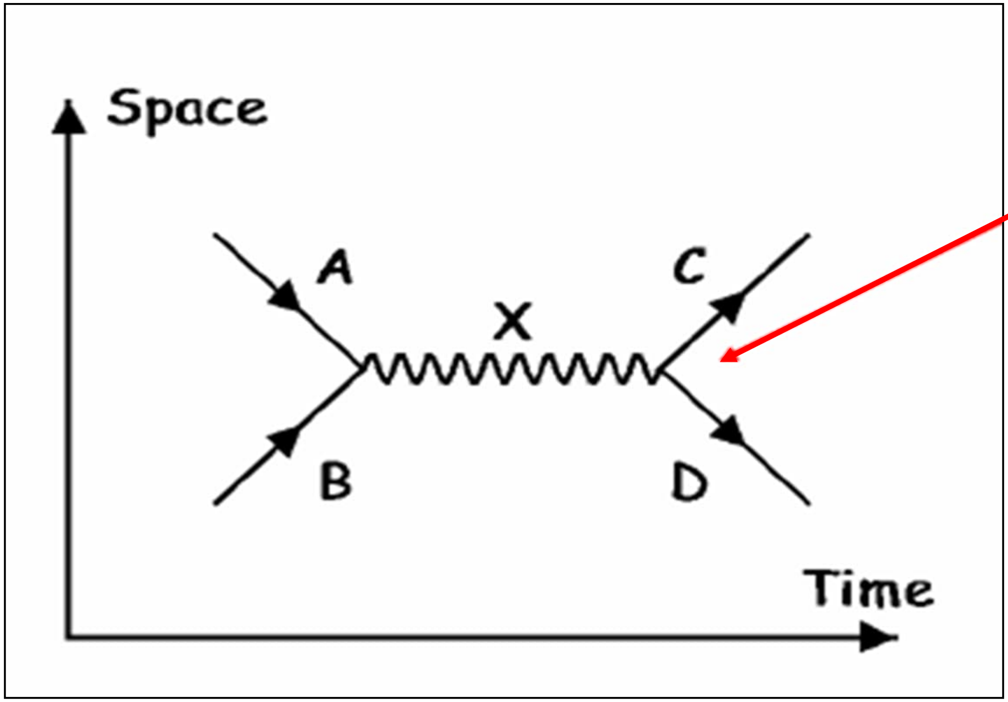
This kind of process implies 3! = 6 different time orderings



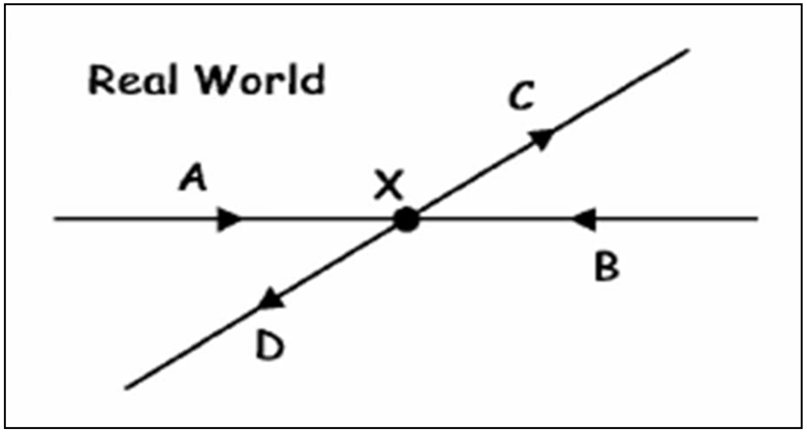
Lowest order contributions to $e^+e^- \rightarrow \gamma\gamma\gamma$

Annihilation diagram

Annihilation / Formation Diagram: Particle A and B collide to form particle X which later decays to C and D

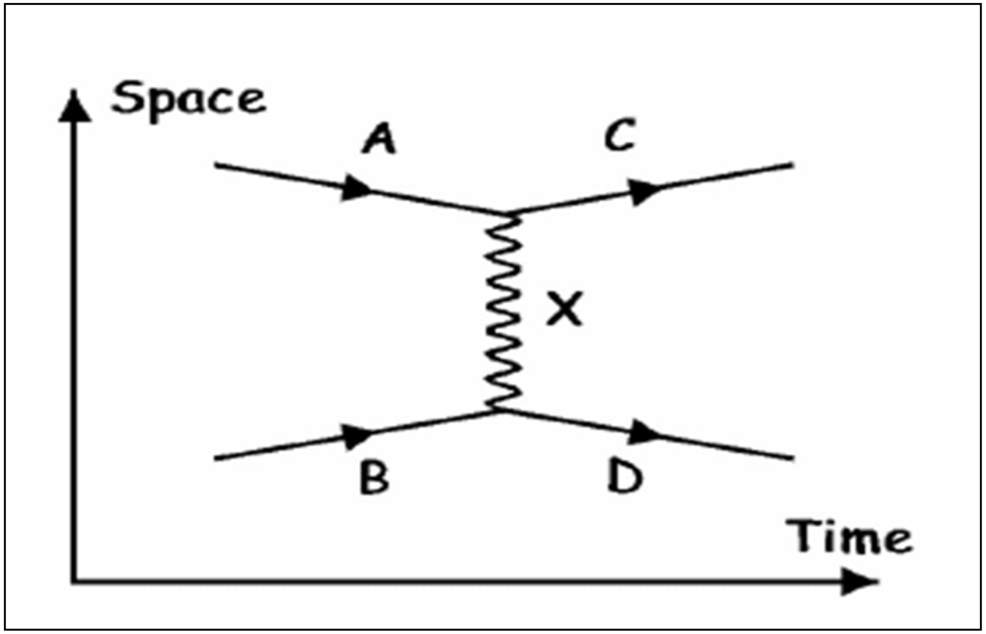


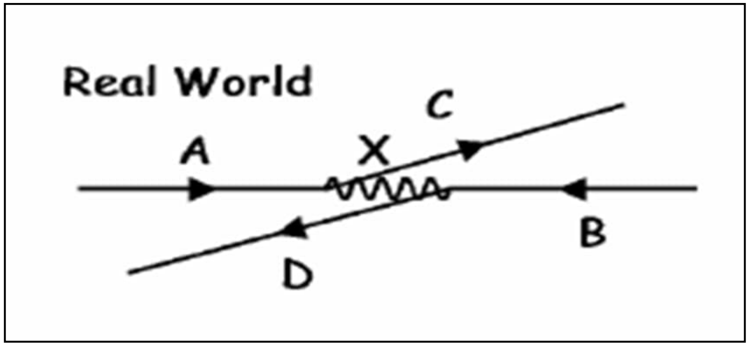
At each vertex, electric charge must be conserved and, except in Weak Interactions, quark or lepton flavours.

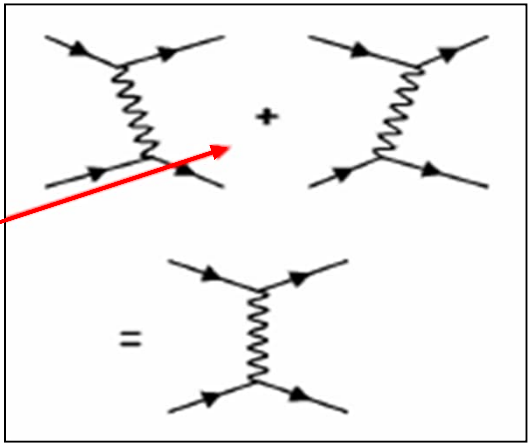


Exchange diagrams

Exchange Diagram: Particles A scatters off particle B by exchanging particle X. Particle A becomes particle C and B becomes D.



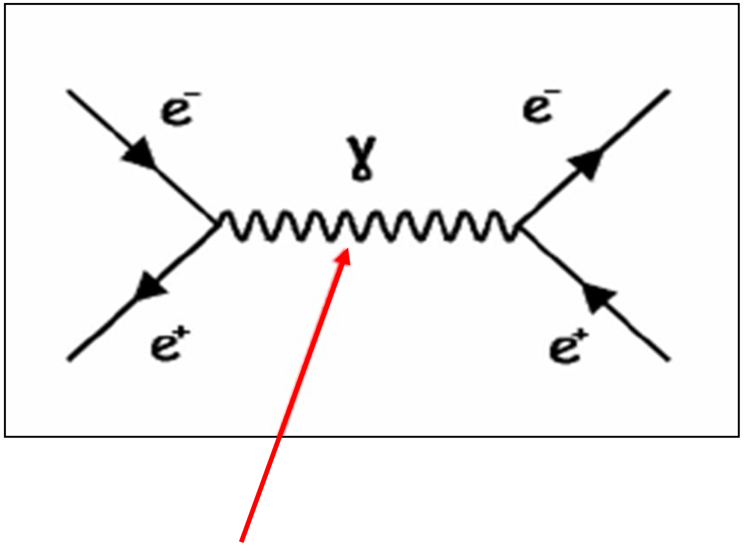


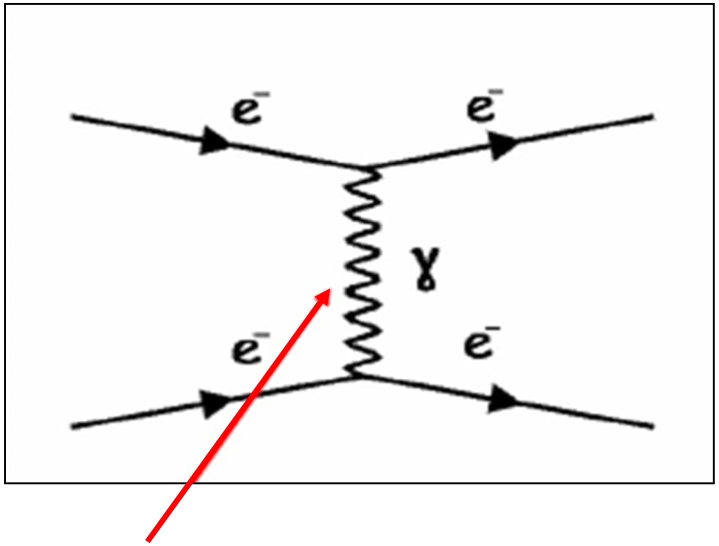


We don't know if A emitted X and B absorbed it or vice versa.

Virtual particles

In both previous cases particle X is ´virtual´ and the time it exists is governed by the uncertainty principle $\Delta E \cdot \Delta t \sim \hbar$. The mass of particle X is usually not its rest mass.





If an electron and positron annihilate, X is a photon (γ) with zero charge, zero momentum and energy $2E_e$ and hence an apparent mass of $2\,E_e/c^2$.

If two electrons scatter, X is a photon (γ) with zero charge, momentum $2p_e$ and zero energy and hence an apparent imaginary mass of $\sqrt{-p_e^2/c^2}$

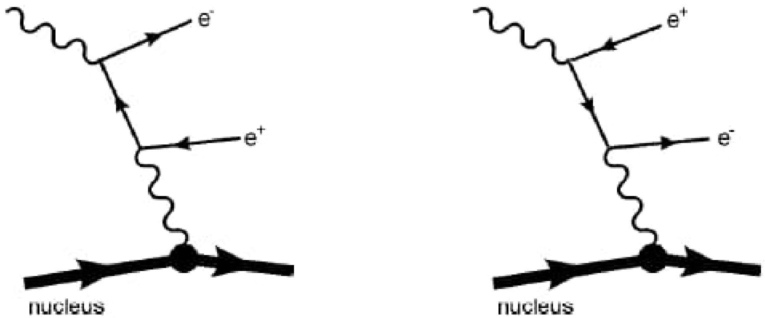
$$E^2 = p^2c^2 + m^2c^4$$

Real processes

- From the order of diagrams one can estimate the ratio of appearance rates of processes:

$$R \equiv \frac{Rate(e^+e^- \rightarrow \gamma\gamma\gamma)}{Rate(e^+e^- \rightarrow \gamma\gamma)} = O(\alpha)$$

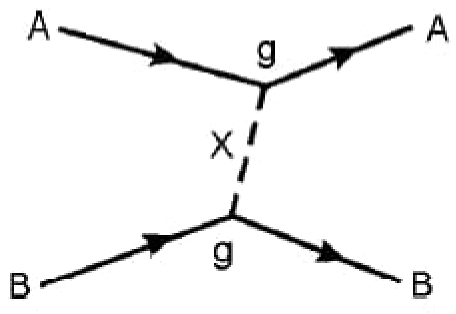
This ratio can be measured experimentally; it appears to be R = $0.9 \cdot 10^{-3}$, which is smaller than $\alpha_{em} = 7 \cdot 10^{-3}$, but the equation above is only a first order prediction.



Diagrams are not related by time ordering

For nucleus, the coupling is proportional to $Z^2\alpha$, hence the rate of this process is of the order of $Z^2\alpha^3$.

Exchange of a massive boson



Exchange of a massive particle X

In the rest frame of particle A:

$$A(E_0, \overrightarrow{p_0}) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$$

where $E_0 = M_A, \quad \overrightarrow{p_0} = (0,0,0)$

$$E_A = \sqrt{p^2 + {M_A}^2}, \qquad E_X = \sqrt{p^2 + {M_X}^2}$$

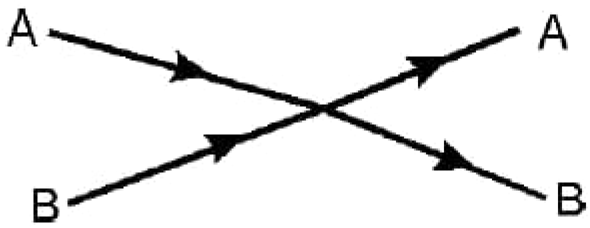
From this one can estimate the maximum distance over which X can propagate before being absorbed: $\Delta E = E_X + E_A - M_A \geq M_X$

This energy violation can exist only for $\Delta t \approx \hbar / \Delta E$, the *interaction range* is

$$r \approx R \equiv \hbar c / M_X$$

Exchange of a massive boson

- For a massless exchanged particle, the interaction has an infinite range (e.g. electromagnetic)
- In case of a very heavy exchanged particle (e.g. a W boson in weak interaction), the interaction can be approximated by a *zero-range*, or *point interaction*



Point interaction as a result of $M_X \to \infty$

$$R_W = \hbar c / M_W = \hbar c / 80.4\ GeV/c^2 = \frac{197.3 \cdot 10^{-18}}{80.4} \approx 2 \cdot 10^{-18} m$$

Considering particle X as an electrostatic potential V(r), the Klein-Gordon equation for it will look like

$$\nabla^2 V(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = M_X^2 \cdot V(r)$$

Yukawa potential (1935)

Integration of the previous equation gives the solution of

$$V(r) = -\frac{g^2}{4\pi r} e^{-r/R}$$

Here ***g*** is an integration constant, and it is interpreted as the coupling strength for particle X to the particle A and B (strong nuclear charge).

- In Yukawa theory, ***g*** is analogous to the electric charge in QED, and the analogue of α_{em} is

$$\alpha_X = \frac{g^2}{4\pi}$$

$\boldsymbol{\alpha_X}$ characterizes the strength of the interaction at distances $r \leq R$

Consider a particle being scattered by the potential (given above), thus receiving a momentum transfer $\vec{q}$

- the above potential has the corresponding amplitude, which is its Fourier-transform (like in optics):

Yukawa potential (1935)

A particle is scattered by this potential, thus receiving a momentum transfer $\vec{q}$

Fourier-transform:

$$f(\vec{q}) = \int V(\vec{x}) e^{1\vec{q}\vec{x}} \; d^3\vec{x}$$

Using polar coordinates, $d^3\vec{x} = r^2 sin\theta \; dr \; d\theta \; d\phi$, and assuming $V(\vec{x}) = V(r)$, the amplitude is

$$f(\vec{q}) = 4\pi g \int_0^\infty V(r) \frac{sin(qr)}{qr} r^2 \, dr = \frac{-g^2}{q^2 + M_X^2}$$

- For the point interaction, $M_X^2 \gg q^2$, hence $f(\vec{q})$ becomes a constant:

$$f(\vec{q}) = -G = \frac{-4\pi\alpha_X}{M_X^2}$$

That means that the point interaction is characterized not only by α_X, but by M_X as well.

Yukawa potential (1935)

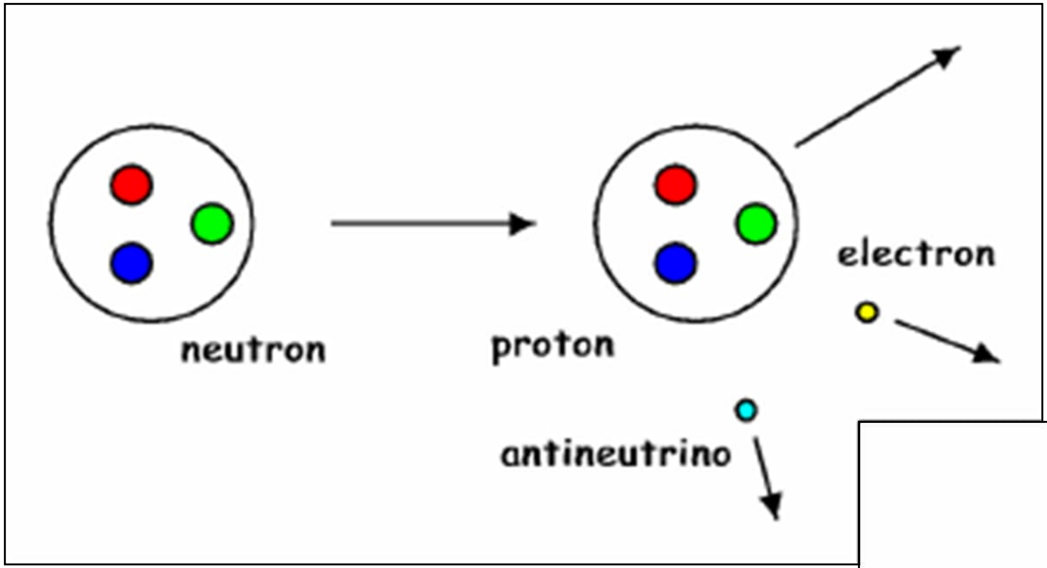
For nuclear forces with a range $R{\sim}10^{-15}m$, Yukawa hypothesis predicted a spinless quantum of mass:

$$Mc^2 = \hbar c/R \approx 100\ MeV$$

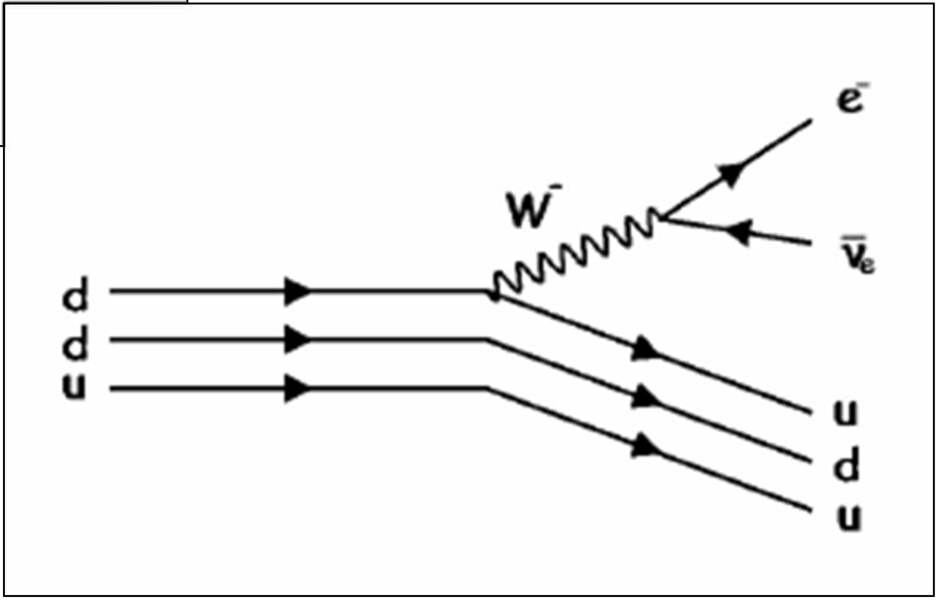
The pion observed in 1947 had $M = 140[MeV/c^2]$, $spin = 0$ and strong nuclear interactions.

Nowadays: pion exchange still accounted for the longer-range part of nuclear potential.

However, full details of interaction are more complicated.

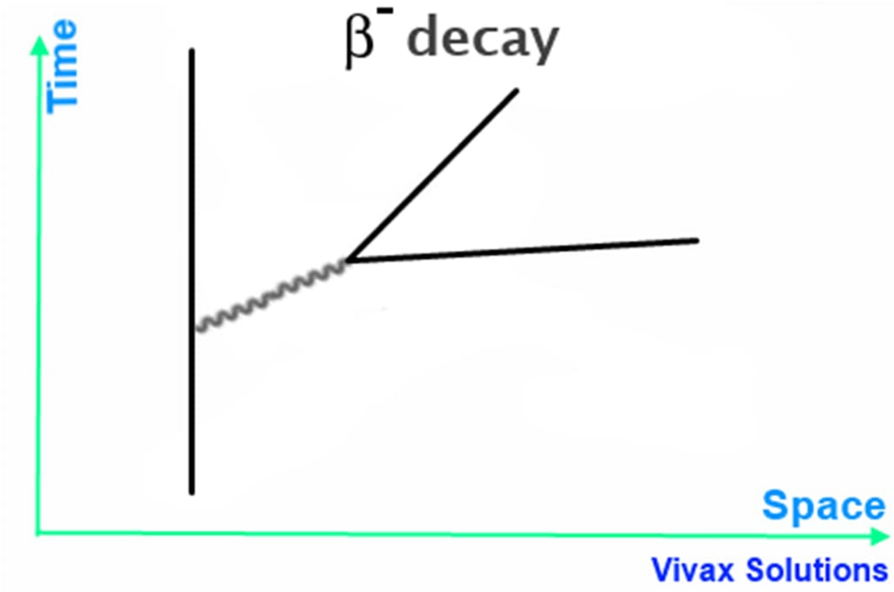


Beta decay: $n \rightarrow p + e^{-} + \overline{\nu_e}$

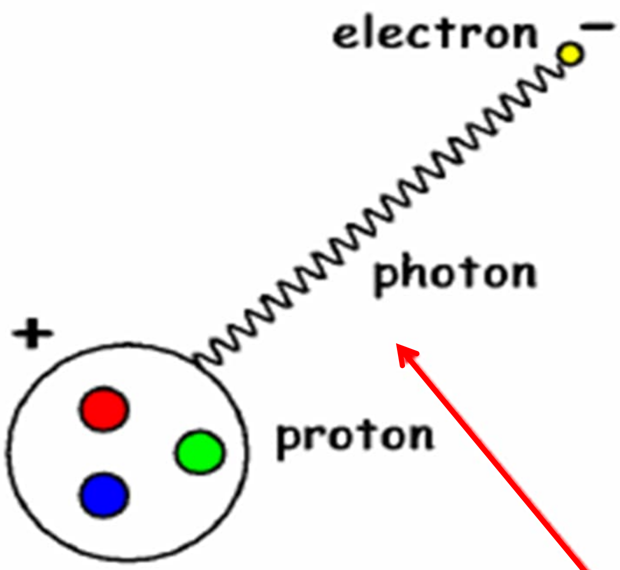


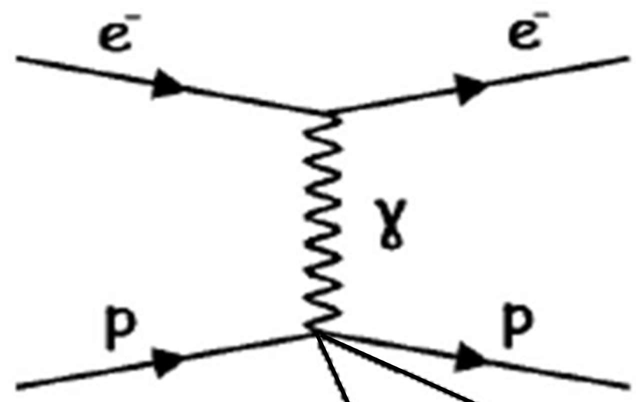
Mediated by charged W exchange:

The charge that goes into the vertex ***must*** equal the charge that comes out of it.

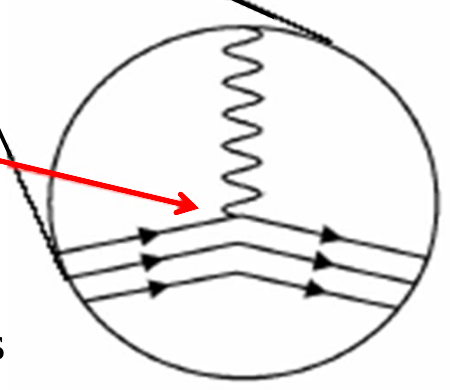

β⁻ decay
Time
Space
Vivax Solutions

Electromagnetism



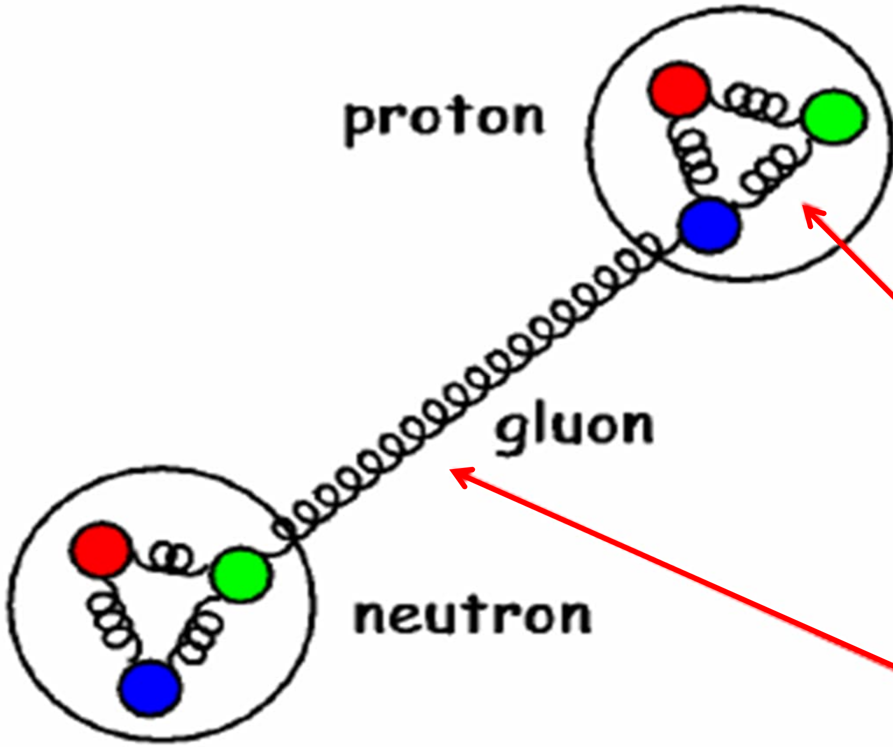


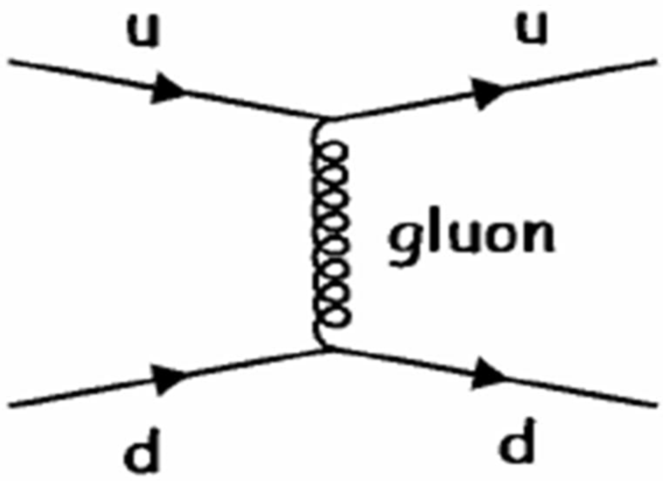
At particle physics level the interaction is with the quarks



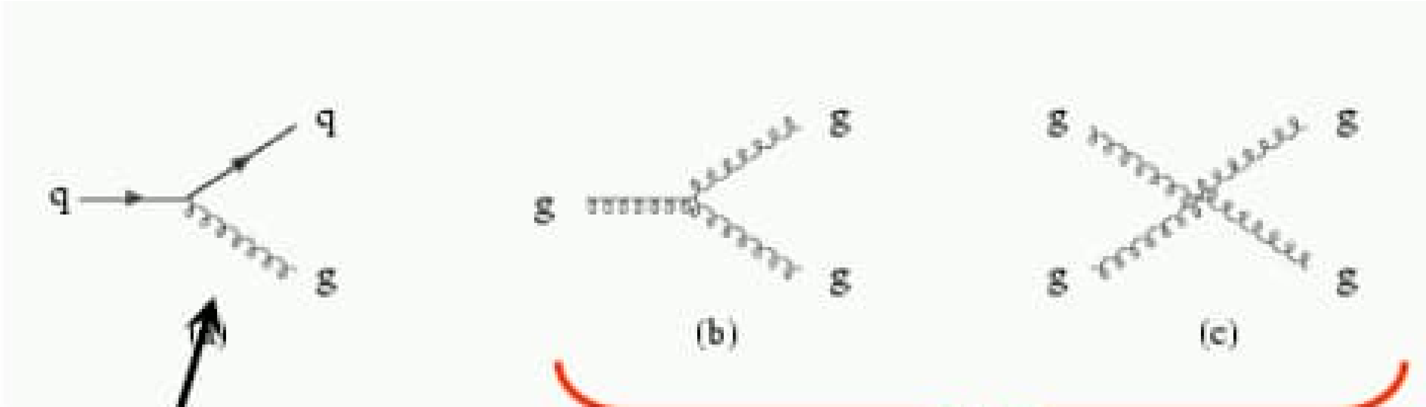
Photons mediate the force between protons and electrons

Strong Interaction





Gluons hold protons and neutron together and are responsible for the Strong force between them



"self-coupling" diagrams

[not present in QED]

analogous to QED

Colour conserved at vertex

Gluons are themselves "coloured"

"colour confinement"

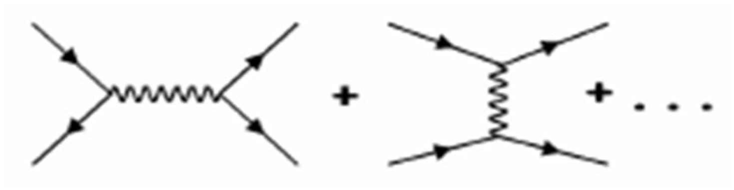
"asymptotic freedom"

$\alpha_S > 1$ → QCD phenomenology very different from QED's

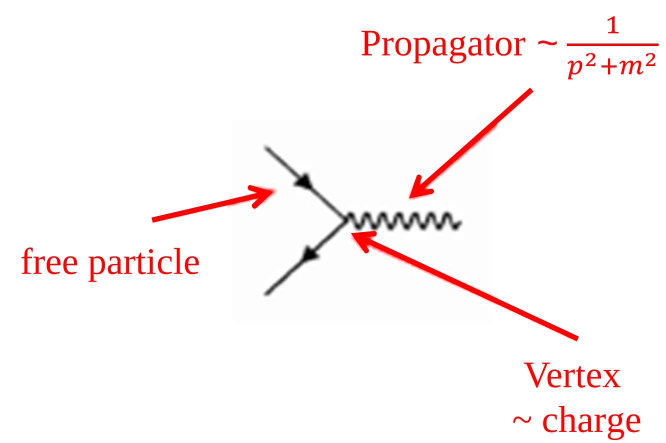
Use of Feynman Diagrams

Although they are used pictorially to show what is going on, Feynman Diagrams are used more seriously to calculate cross sections or decay rates.

- ❖ Draw all possible Feynman Diagrams for the process:



- ❖ Assign values to each part of the diagram:



- ❖ Calculate the amplitude by multiplying together.
- ❖ Add the amplitudes for each diagram (including interference).
- ❖ Square the amplitude to get the intensity/probability (cross section or decay rate).