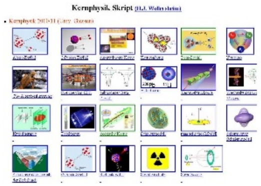
Outline: Transfer Reactions

Lecturer: Hans-Jürgen Wollersheim

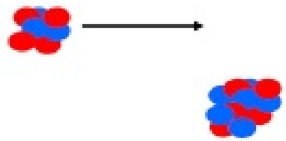
e-mail: h.j.wollersheim@gsi.de

web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. direct reactions
2. Q-values
3. theories: DWBA, Fresco, Faddeev
4. radioactive beams
5. Knock-out reactions (^{11}Li)

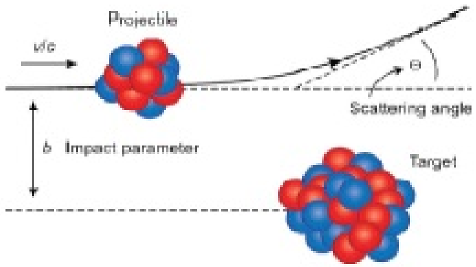
Why reactions?

Elastic:



Traditionally used to extract optical potentials, rms radii, density distributions

Inlastic:



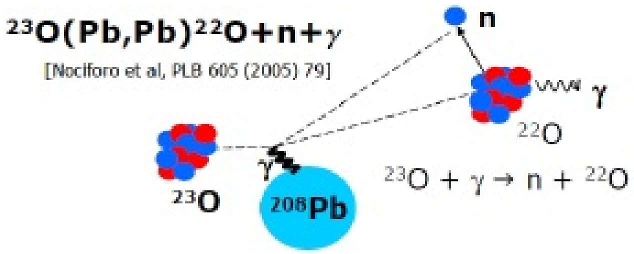
Traditionally used to extract electromagnetic transitions or nuclear deformations.

Transfer:

Traditionally used to extract spin, parity, spectroscopic factors
example: $^{132}Sn(d,p)^{133}Sn$

Traditionally used to study two-nucleon correlations and pairing
example: $^{11}Li(p,t)^{9}Li$

Breakup:



Reactions types

If we consider nuclear reactions where both projectile and target are nuclei, many subcases arise. In the following table we show them by means of examples:

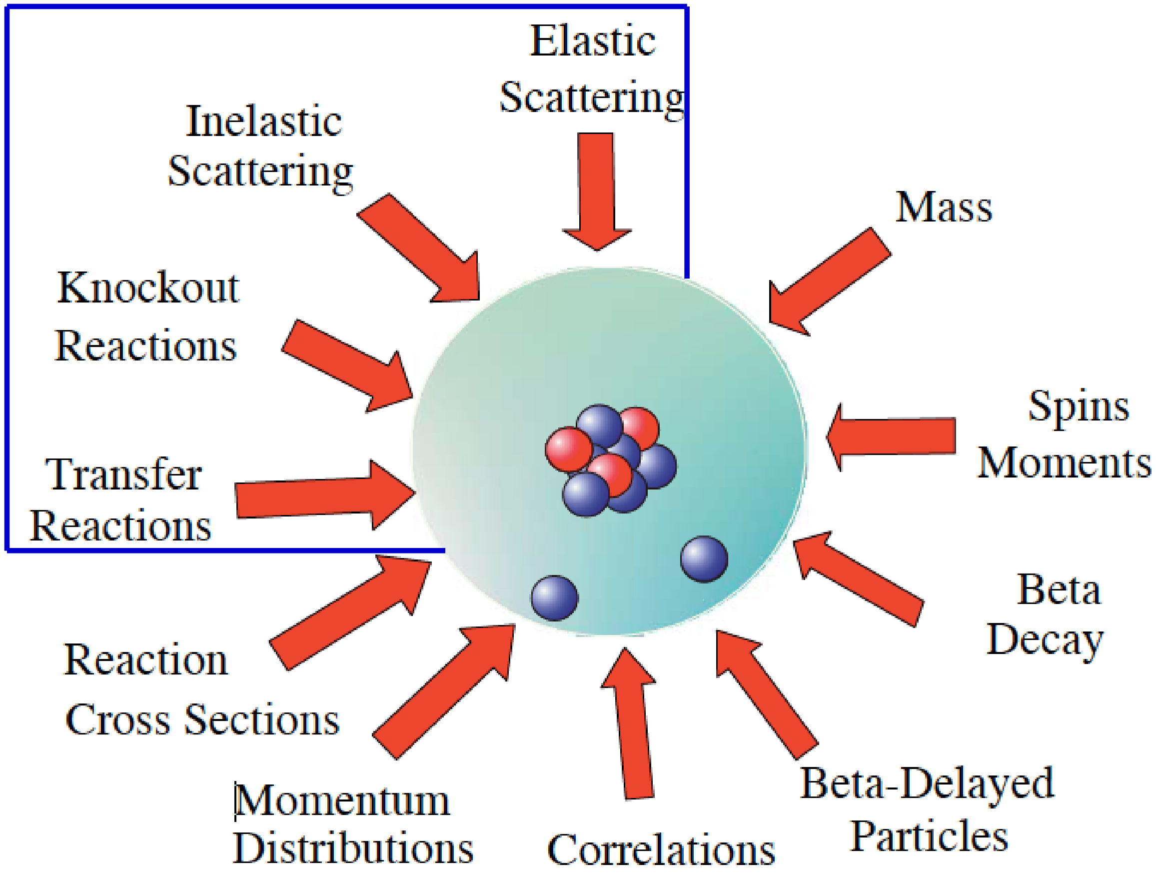
Reaction	Type
^{59}Co(d,d)^{59}Co	Elastic
^{59}Co(d,d')^{59}Co*	Inelastic
^{59}Co(d,γ)^{61}Ni	Radiative Capture
^{59}Co(d,p)^{60}Co	Stripping
^{59}Co(d,n)^{60}Ni	Stripping
^{59}Co(d,^{3}He)^{58}Fe	Pickup
^{59}Co(d,α)^{57}Fe	Pickup

Radiative Capture → The projectile is captured with a gamma ray emission.

Stripping → One (or more) nucleon(s) is(are) stripped from the projectile.

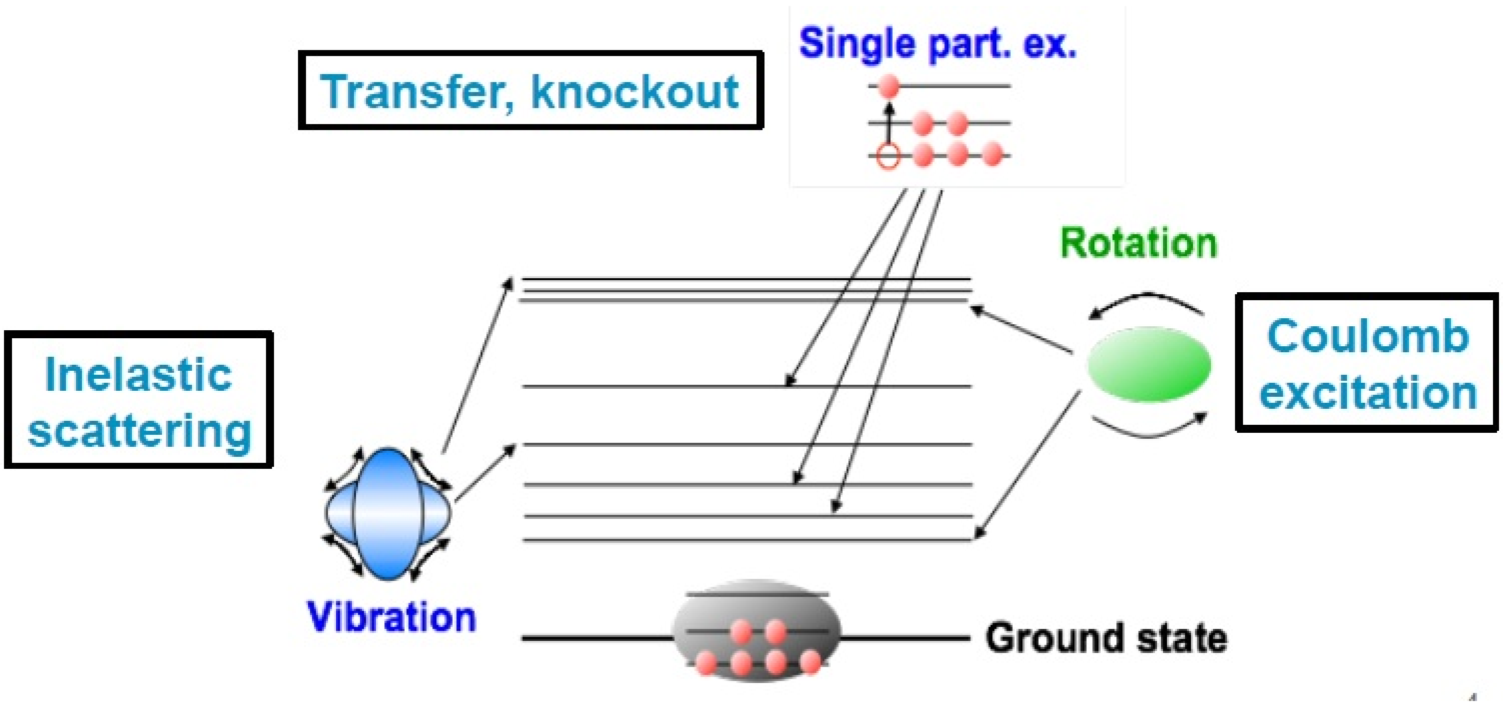
Pickup → One (ore more) nucleon(s) is(are) stripped from the target.

Reactions: tool to excite and probe nuclear states



B. Jonson, Phys. Report 389 (2004)

Selectivity of direct reactions

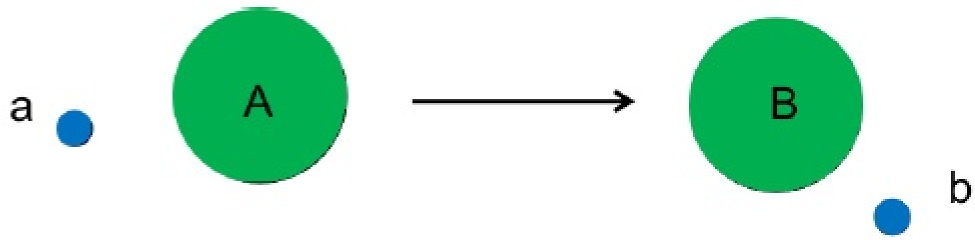


direct reactions with nuclei

- Elastic & inelastic scattering
- Few-particle transfer (stripping, pick-up)
- Charge exchange
- Knockout

Transfer reactions are direct reactions which link entrance channel a, A and exit channel b, B

$$a + A \rightarrow B + b$$



target nucleus ⟶ A(a,b)B ⟵ residual nucleus

projectile ejectile

- direct reactions: short time scale $(10^{-22} \leq \Delta t \leq 10^{-20})$
- cross section related to transition amplitude / matrix element $\sigma \propto \left|\langle i \| \hat{O} \| f \rangle\right|^2$

Q-value = masses (before) – masses (after)
= $M_a + M_A - M_B - M_b$ (in energy units)*

Q-value > **0** : exothermic (exoergic)
Q-value < **0** : endothermic (endoergic)
Q-value = **0** : elastic scattering

Transfer reactions

- determine Q-value by measuring the beam energy T_a, the kinetic energy of the light particle T_b, and the scattering angle θ

$$Q = T_b \cdot \left(1 + \frac{m_b}{m_B}\right) - T_a \cdot \left(1 + \frac{m_a}{m_B}\right) - 2\sqrt{\frac{m_a \cdot m_b}{m_B^2} T_a T_b} \cdot cos\theta$$

- and from this the excitation energy of the ejectile, missing mass technique

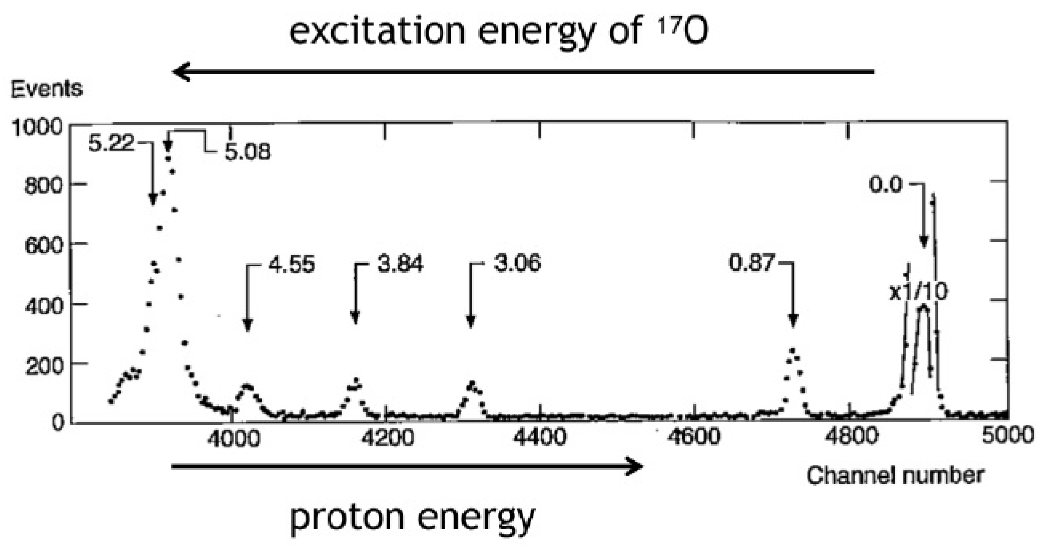
- traditionally done at 10-20 MeV

- measurement of θ and T_b with magnetic spectrometer

- example $^{16}O(d,p)^{17}O$

- done extensively in the past with deuteron beams from tandem accelerators

- tritium beams for two-neutron transfer reactions



Measuring transfer reactions – good kinematics

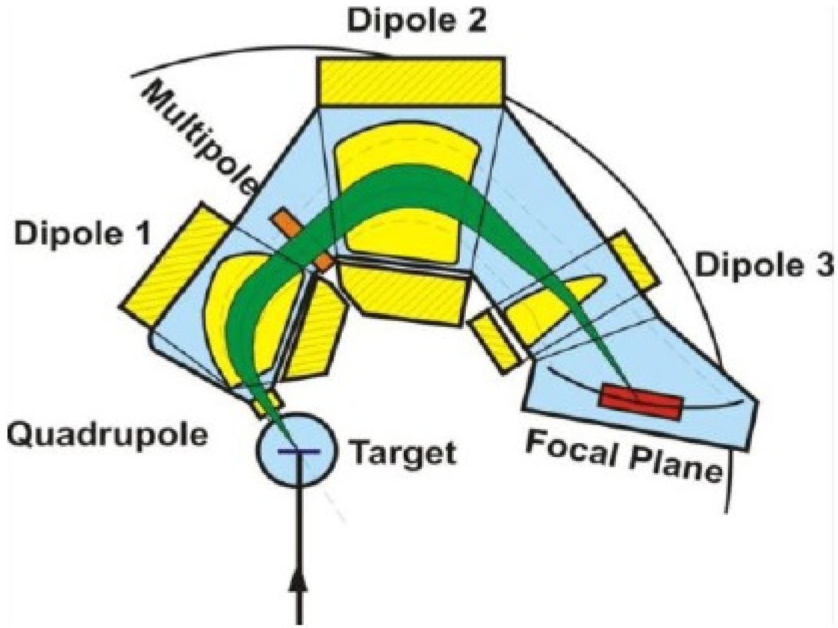
Transfer reactions in normal/forward kinematics (light beam on heavy target) is using a magnetic spectrometer

Momentum-analyze the reaction products

Measure the position at which the particles hit the focal plane → tell us how easily bent in the magnetic field gives the rigidity (momentum/charge)

Reconstruct excitation energy with two-body kinematics

Can also do this with solid-state detectors e.g. silicon arrays

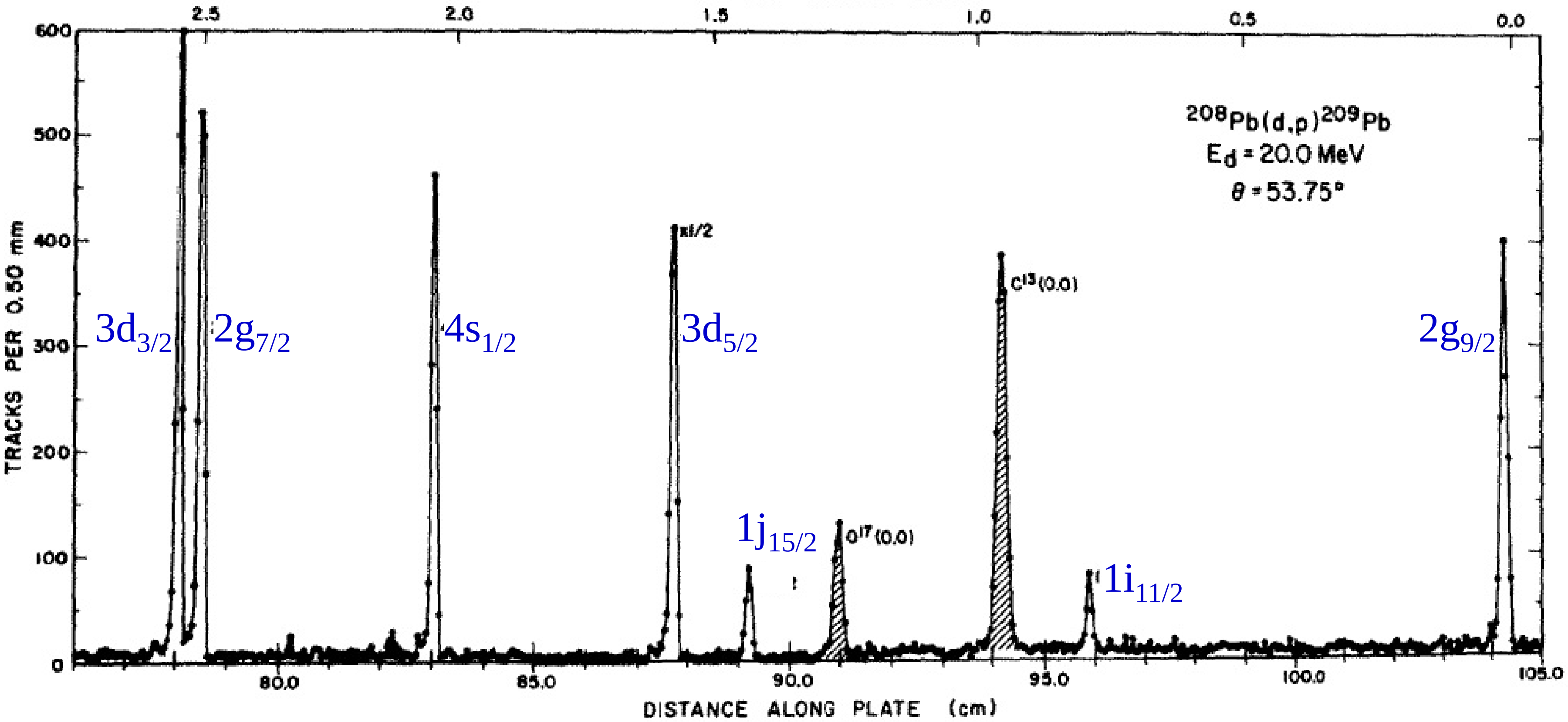


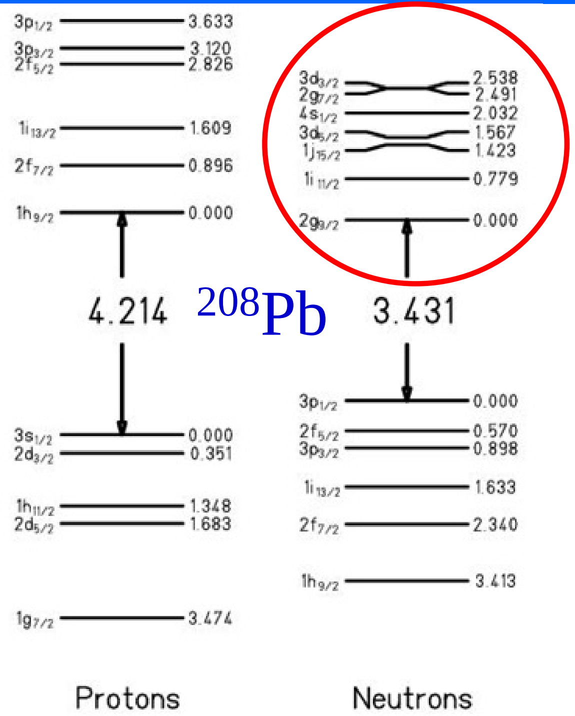
Q_3D spectrometer (formerly) at Munich. The edges of the pole pieces and the multiple correct the kinematic aberrations

^{208}Pb (d,p) ^{209}Pb stripping reaction

population of neutron states

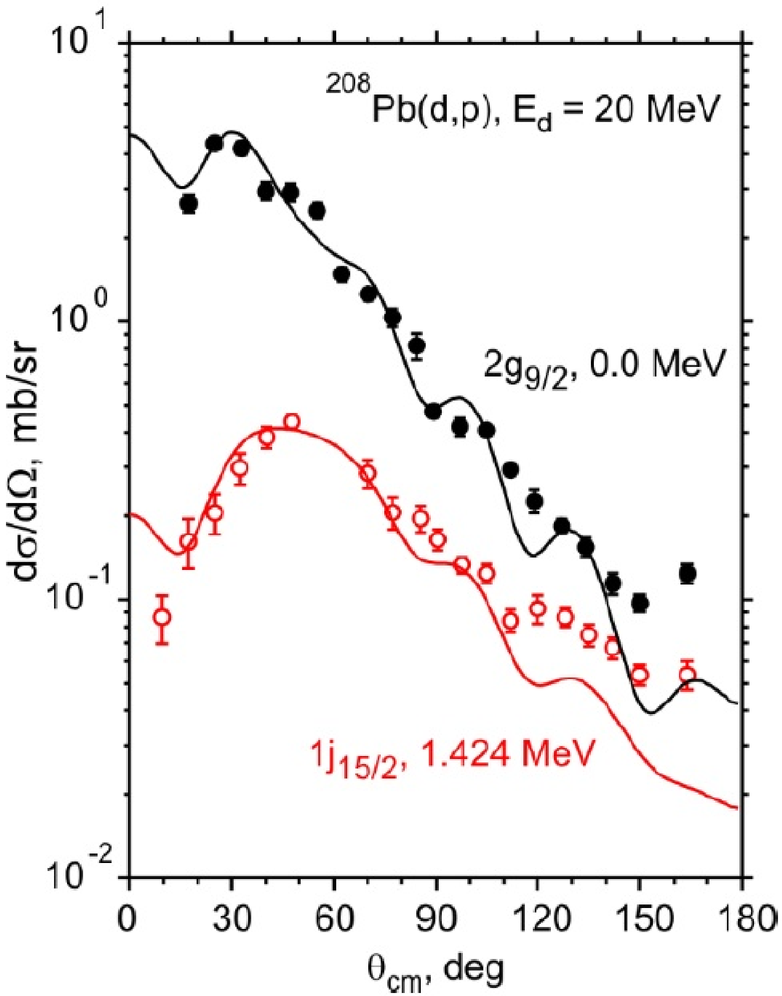
excitation energy (MeV)





^{208}Pb (d,p) ^{209}Pb stripping reaction

population of neutron states

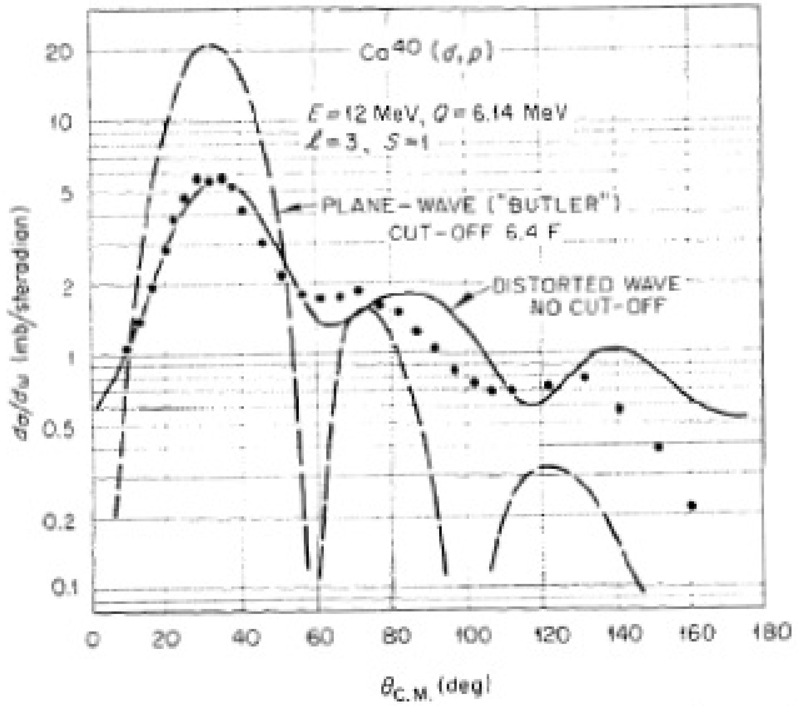


Angular distributions of different reaction channels look different

Transfer reactions: DWBA distorted wave Born approximation

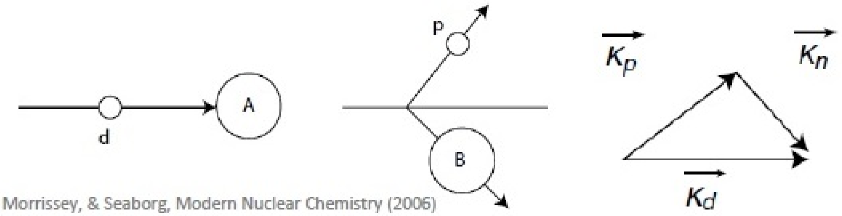
Transfer cross section in DWBA

$$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \sum_{n\ell j} S_{n\ell j} \frac{d\sigma_{\alpha\beta}}{d\Omega}\Big|_{n\ell j}$$



Analysis of experiments:

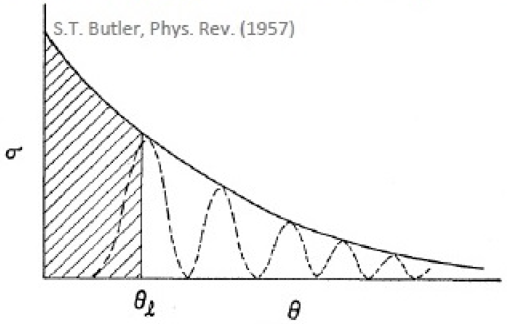
1) measure $d\sigma/d\Omega$
2) calculate $d\sigma/d\Omega$ single particle
3) extract $S_{n\ell j}$ by normalization of theo. vs exp.
4) compare to $S_{n\ell j}$ from theoretical structure model
   or use in the Baranger sum rule for ESPEs



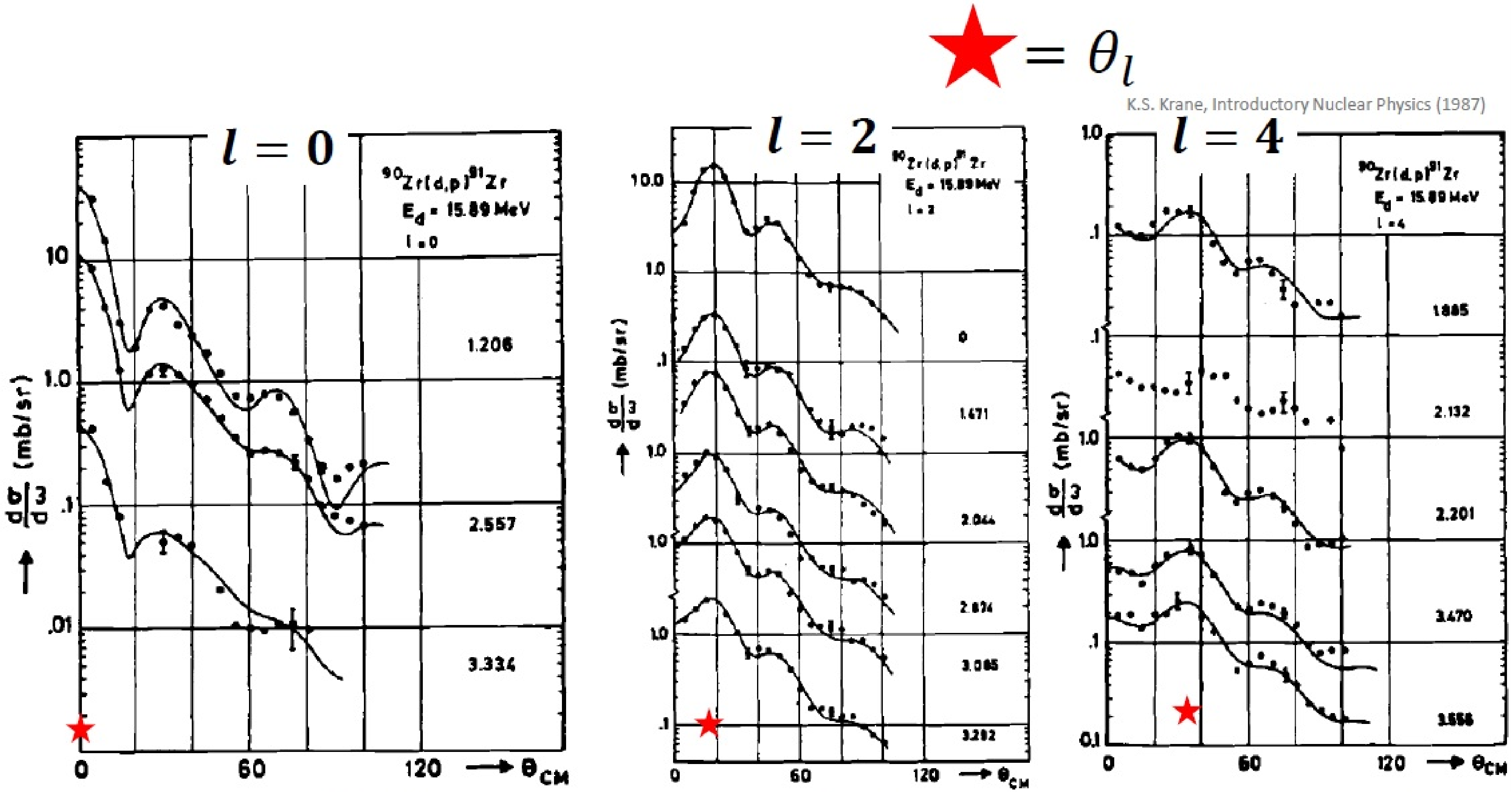
Loveland, Morrissey, & Seaborg, Modern Nuclear Chemistry (2006)

- Consider the deuteron stripping reaction $^{90}Zr(d,p)$ for a 5MeV deuteron
  - $p_d = \sqrt{2m_d E_d} \approx 140MeV$
  - The reaction Q-value and excitation energy of the recoil nucleus are much less than the incoming deuteron energy, so , so $p_p \approx p_d \approx 140MeV$
  - Note that $p^2 = p_a^2 + p_b^2 - 2p_a p_b \cos(\theta) = (p_a - p_b)^2 + 2p_a p_b(1 - \cos(\theta))$
  - So, $p \approx \sqrt{2p_a p_b(1 - \cos(\theta))}$ and it's still true that $p = l\hbar/R$
  - Meaning, $l \approx \frac{c}{\hbar c} R\sqrt{2p_a p_b(1 - \cos(\theta))}$
  - For this case $l = \frac{c}{197MeVfm} r_0 90^{1/3}\sqrt{2(140MeV/c)(140MeV/c)(1 - \cos(\theta))} \approx 8\sin\left(\frac{\theta}{2}\right)$
  - I.e. $l = 0$ at $0°$, $l = 1$ at $14°$, etc.
- This of course is a classical estimate, what it really tells us is the angle θ_l at which the angular distribution for a given l transfer will peak

S.T. Butler, Phys. Rev. (1957)



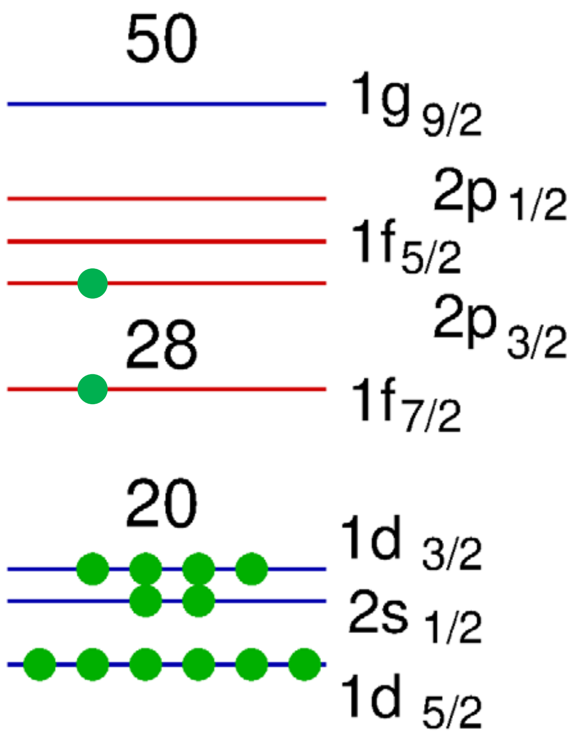
Angular distribution examples

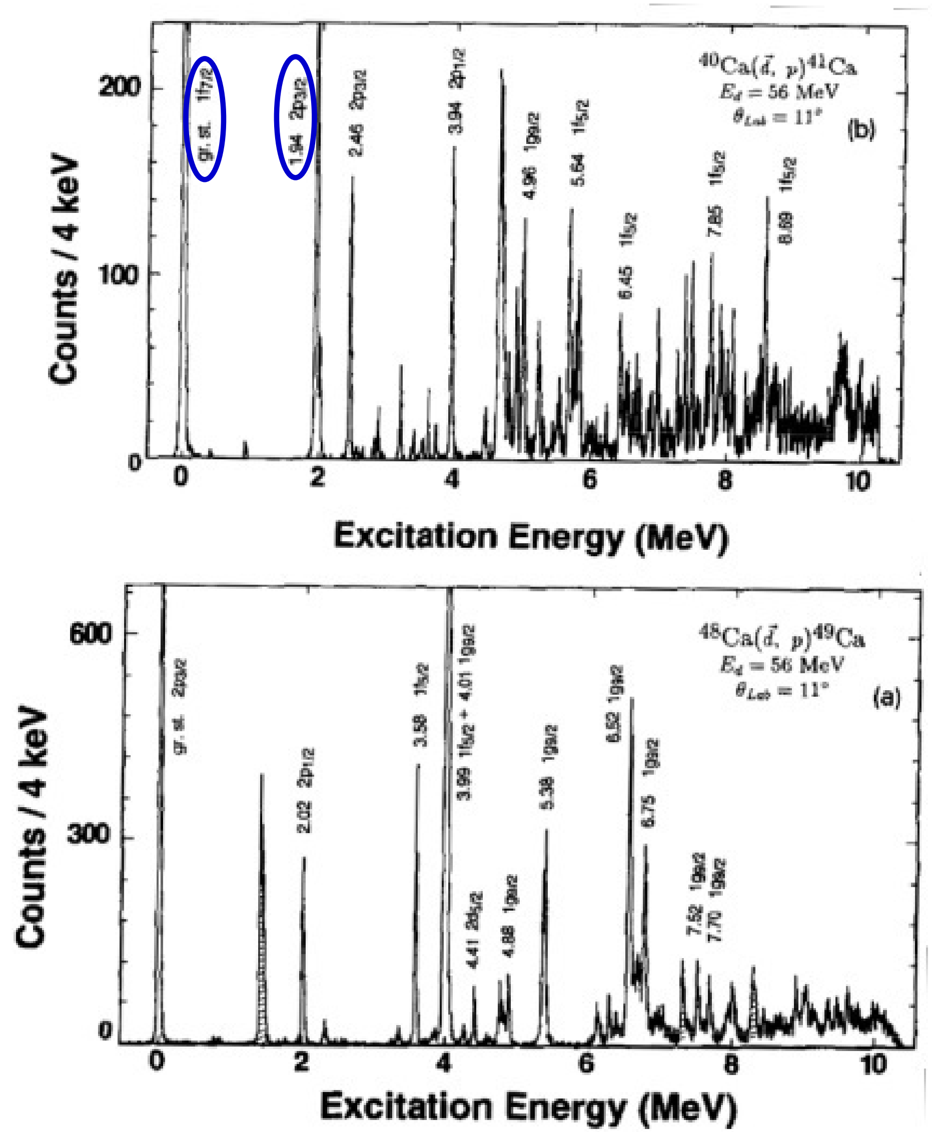


*These are plots of θ_{cm} and we did our calculations for θ_{lab}, but for a light low-energy beam on a heavy target, $\theta_{cm} \approx \theta_{lab}$

Probing occupation of levels

- measure energy of levels and determine their occupation
- (d,p) transfer reaction to add a neutron

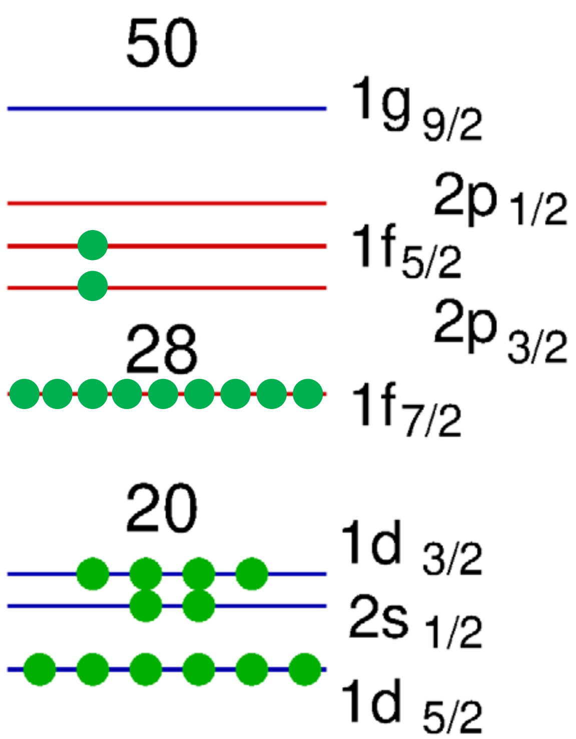


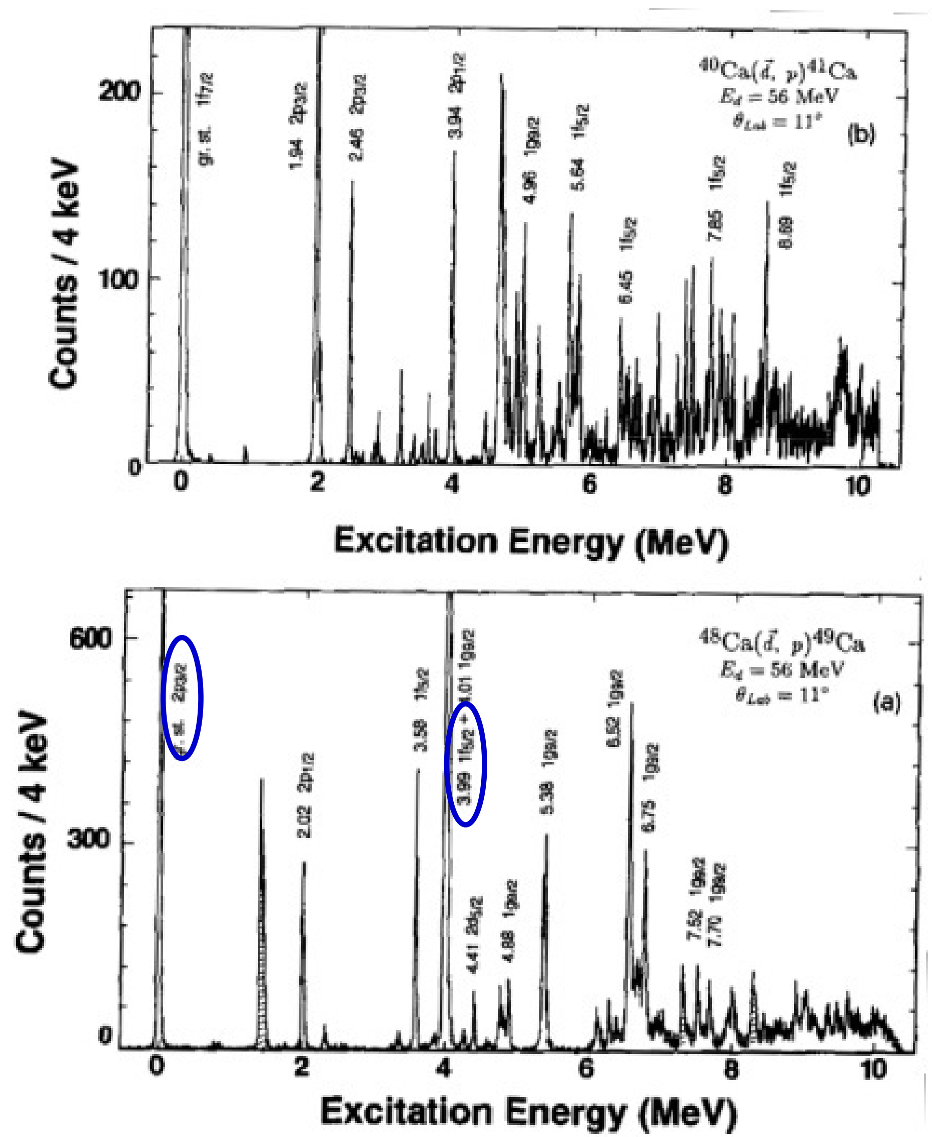


Y. Uozumi et al., Nucl. Phys. A576 (1994) 123

Probing occupation of levels

- measure energy of levels and determine their occupation
- (d,p) transfer reaction to add a neutron
- map valence space above a magic number





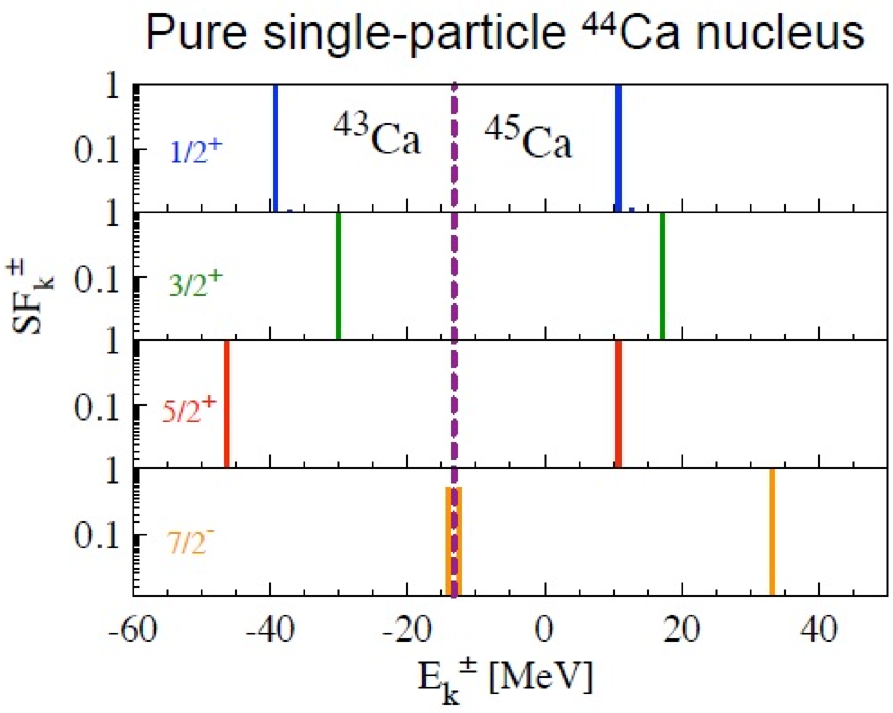
Y. Uozumi et al., Nucl. Phys. A576 (1994) 123

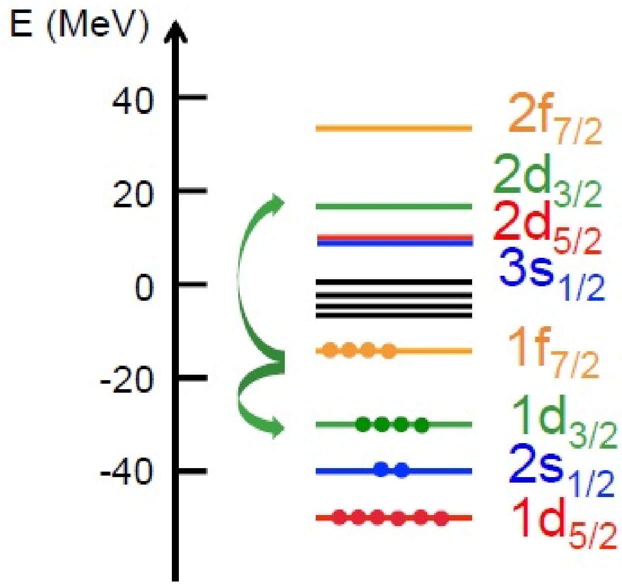
Intuitive view of spectroscopic factors

Spectroscopic factor: the square overlap of a final state with a single particle state

$$S_k^{n\ell j\,+} = \left\langle \Psi_k^{A+1} \middle| a_{n\ell j}^{+} \middle| \Psi_0^{A} \right\rangle^2 \qquad S_k^{n\ell j\,-} = \left\langle \Psi_k^{A+1} \middle| a_{n\ell j} \middle| \Psi_0^{A} \right\rangle^2$$

Pick-up, exp: ^{44}Ca(d,p)^{45}Ca   Stripping, exp: ^{44}Ca(p,d)^{43}Ca





Intuitive view of spectroscopic factors

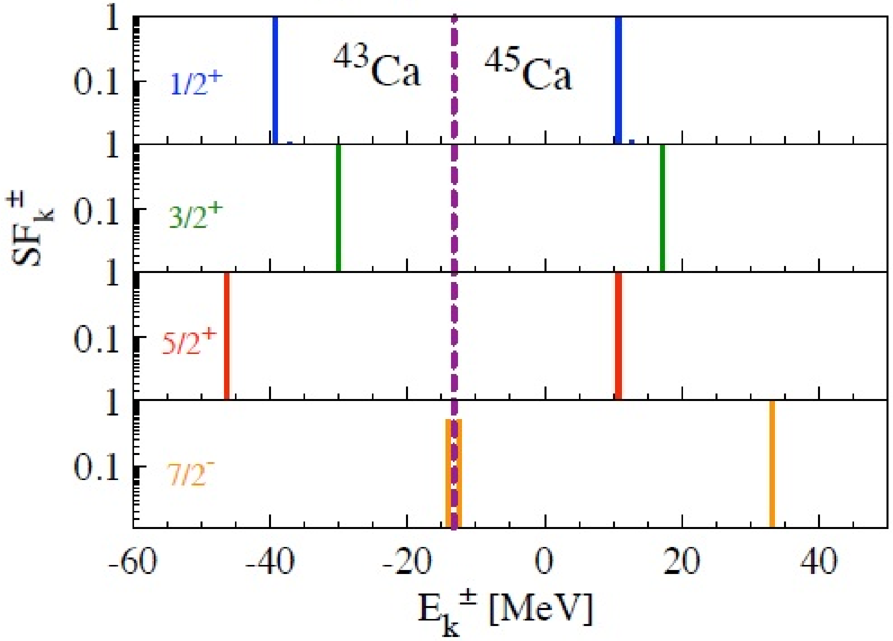
Spectroscopic factor: the square overlap of a final state with a single particle state

$$S_k^{n\ell j\,+} = \left\langle \Psi_k^{A+1} \middle| a_{n\ell j}^{+} \middle| \Psi_0^{A} \right\rangle^2 \qquad\qquad S_k^{n\ell j\,-} = \left\langle \Psi_k^{A+1} \middle| a_{n\ell j} \middle| \Psi_0^{A} \right\rangle^2$$

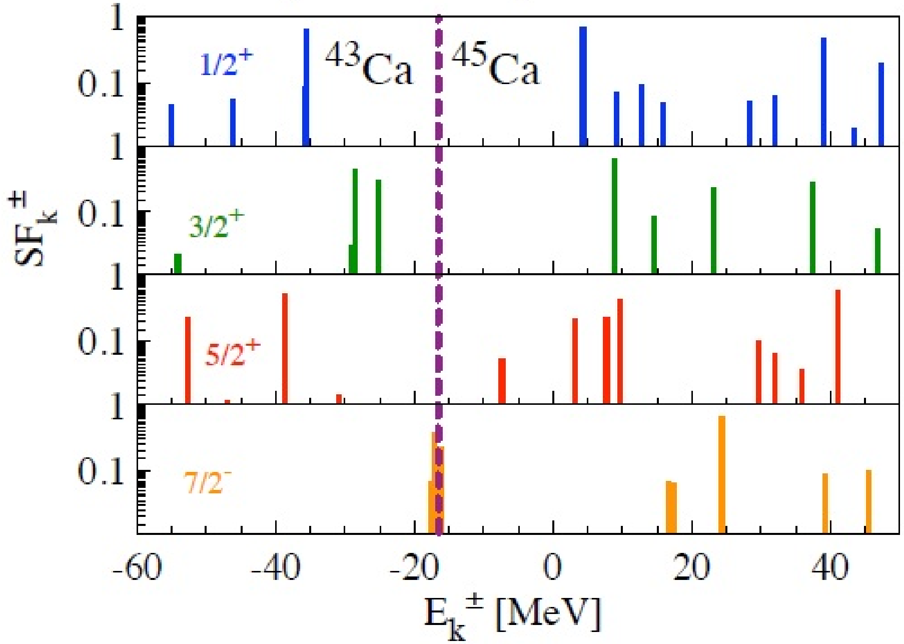
Pick-up, exp: ^{44}Ca(d,p)^{45}Ca

Stripping, exp: ^{44}Ca(p,d)^{43}Ca

Pure single-particle ^{44}Ca nucleus

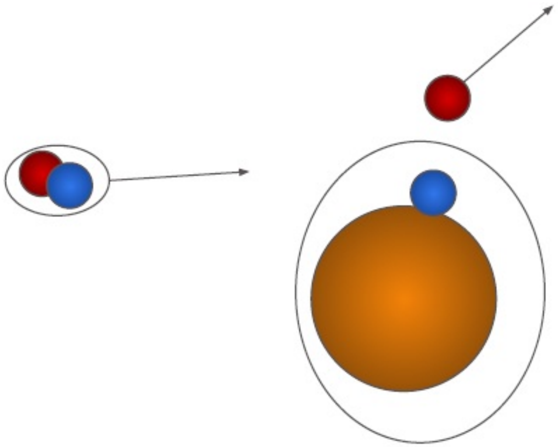


Real (correlated) ^{44}Ca nucleus



In reality: 0 < SF < 1

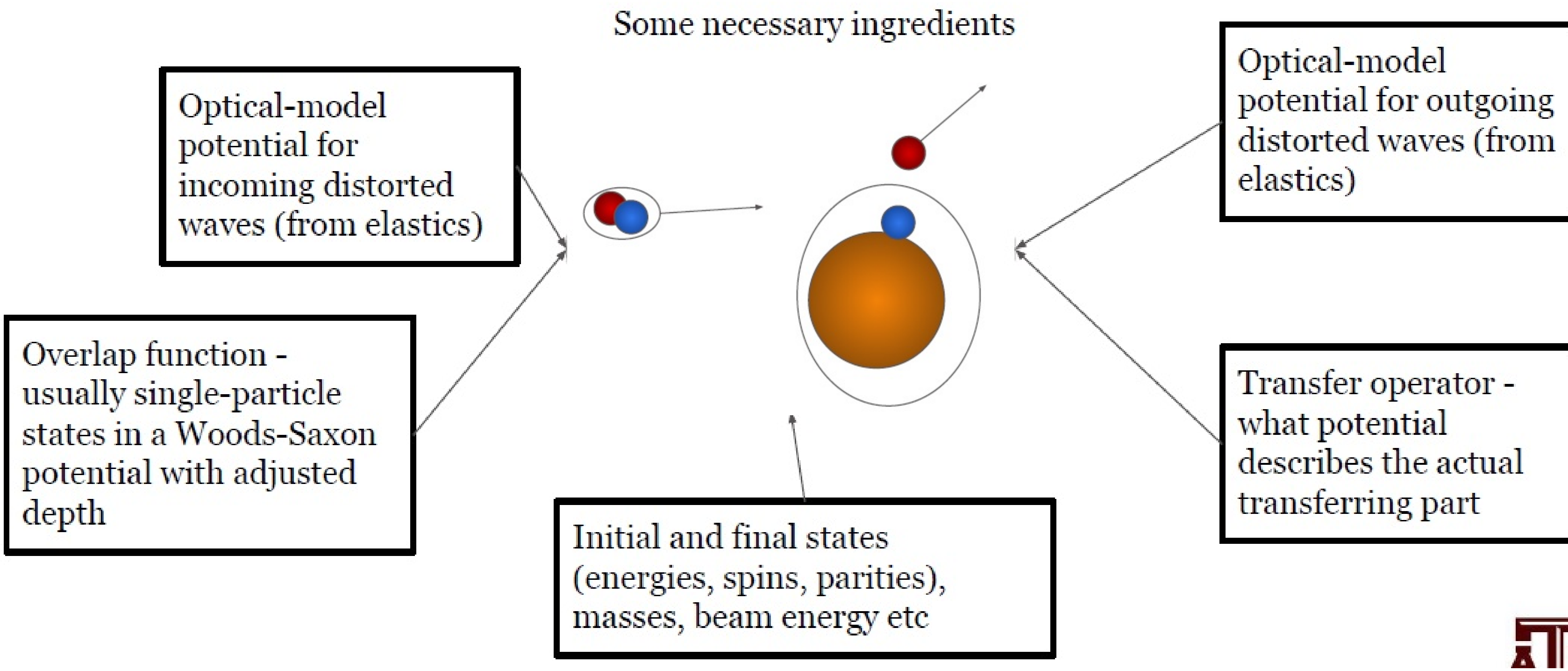
Theory of transfer reactions



Assume the following:

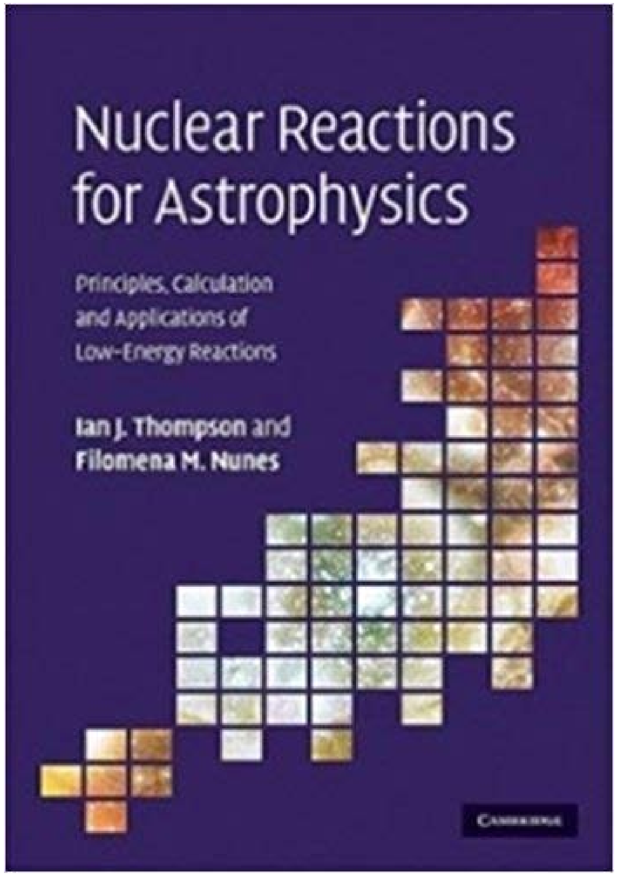
- Entrance and exit channels dominated by elastic scattering
- Transfer is weak - treat as **first-order perturbation**
- Transfer proceeds directly between two channels
- Direct transfer into the final state with no other rearrangement of the core

Theory of transfer reactions



Calculations

This book



Fresco

Coupled Reaction Channels Calculations
www.fresco.org.uk

Home

Documentation
Download
Input examples
Reactions book
Related Programs
Special Functions
Contact
Site Information

Fresco Input Examples

If a simple click on these files does not download them, *then* right-click and select 'download' or 'save as' or 'save link as'.

From Appendix B of the book:

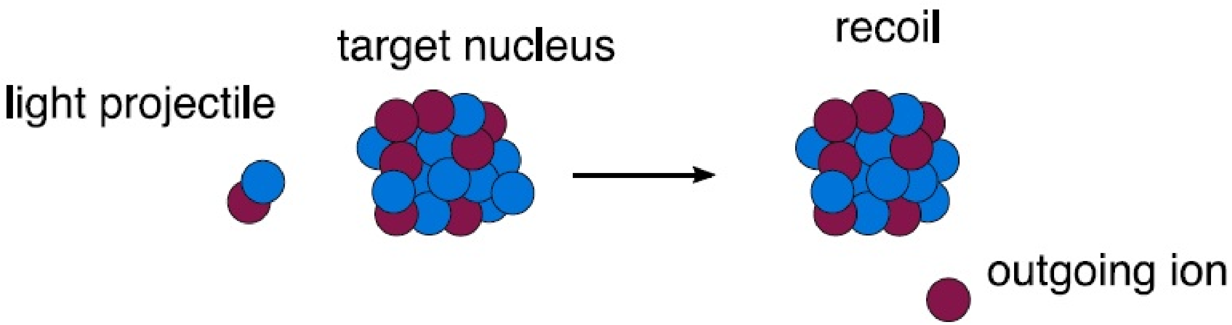
		Input Files	Expected Output Files
B1	Elastic Scattering	B1-example-el.in	B1-example-el.out
B2	Inelastic Scattering	B2-example-inel2.in	B2-example-inel2.out
B3	Breakup (long form)	B3-example-br-long.in	B3-example-br-long.out
B4	Breakup (short form)	B4-example-br-short.in	B4-example-br-short.out
B5	Transfer	B5-example-tr.in	B5-example-tr.out
B6	Capture	B6-example-capture.in	B6-example-capture.out
B7 B8 B9	Parameter search	B7-p-cd.frin B8-p-cd.search B9-p-cd.min	B9-p-cd.out B9-p-cd-init.plot B9-p-cd-fit.plot

Note that there are some misprints in listing the inputs in the book. The above input files are those which do work: they give the output files.

Computer Program FRESCO

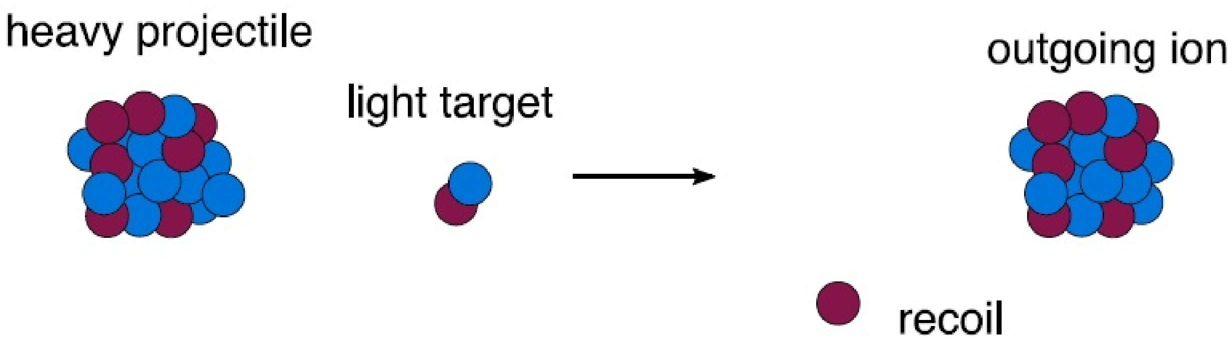
has all the gory details and, tells you how to use the code FRESCO, which is mainly for coupled-channels, but can do DWBA as well

Radioactive beams



- $A(a,b)B$
- measure ejectile
- good resolution (10 keV)

- reactions with exotic nuclei need to be performed in inverse kinematics
- heavy beam of nucleus of interest impinges on a light target



- $a(A,b)B$ at 5-10 AMeV
- HIE-ISOLDE. TRIUMF, GANIL, OEDO
- measure recoil
- limited resolution (100 - 500 keV)

K. Wimmer, Jour. Phys. G45 (2018) 033002

Knockout reactions ^{11}Li – neutron halo nucleus

- fast projectile mass A collides with light target
- mass (A – 1) residues are detected
- light fragments are unobserved, final state tagging by γ-ray if needed
- sudden approximation:

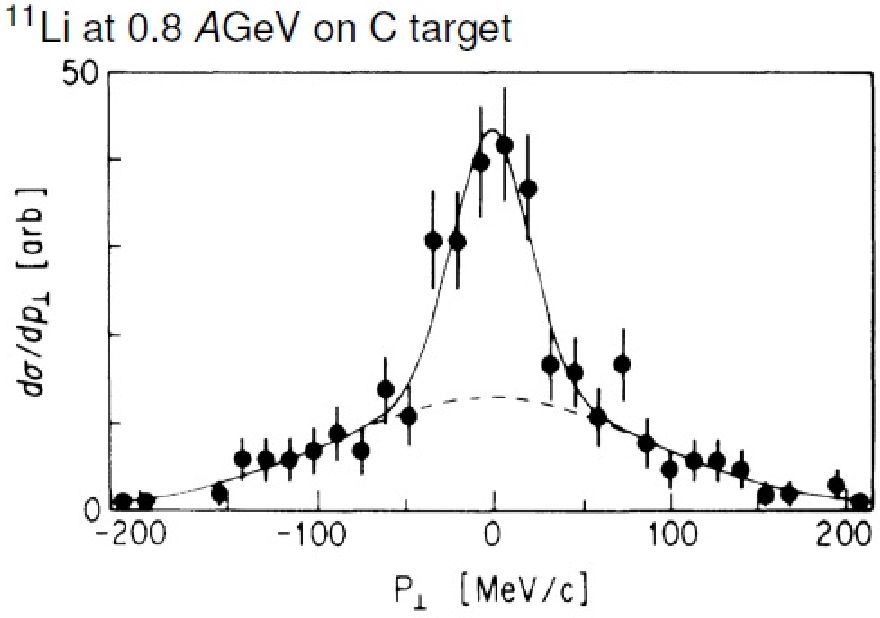
$$\vec{k}_3 = \frac{A-1}{A} \cdot \vec{k}_A - \vec{k}_{A-1}$$

momentum of the stuck nucleon k_3 is related to the residues k_{A-1}

- two components in the transverse momentum distribution of ^{9}Li residues
- broad like for stable nuclei
- very narrow component
  → removal of weakly bound neutrons
- uncertainty relation

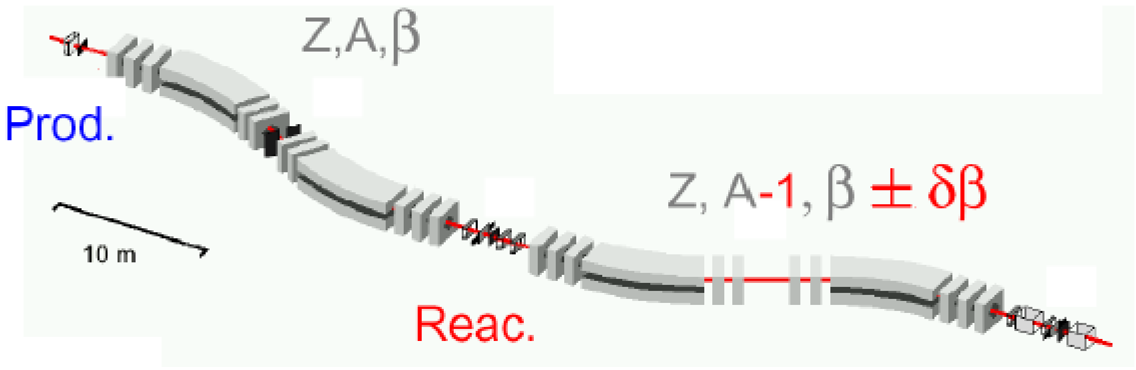
$$\Delta p \cdot \Delta x \geq \hbar$$

small → large



Measurement of the reaction cross section

- ***800 MeV/u ^{11}B primary beam***
- ***Fragmentation***
- ***FRagment Separator FRS***



test of the extended wave function

momentum distribution:

- wider momentum distribution for strongly bound particles

- narrow momentum distribution for weakly bound particles

interpretation:

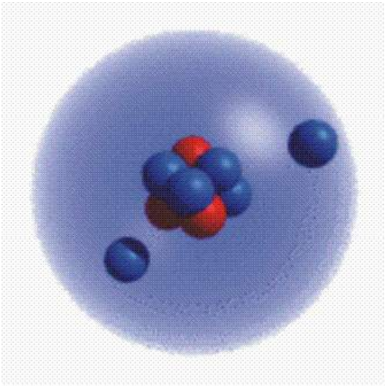
One can simplify ^{11}Li by describing it as a ^{9}Li core plus a di-neutron

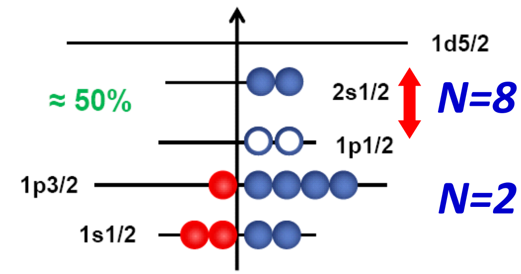
One can use the arguments of an extended wave function with an exponential decline:

S_{2n}=250(80) keV

$$\Psi(r) \propto \frac{e^{-\kappa r}}{r}$$

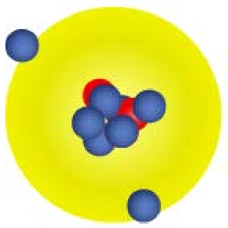
$$\kappa^2 = \frac{2 \cdot \mu_{2n} \cdot S_{2n}}{\hbar^2}$$



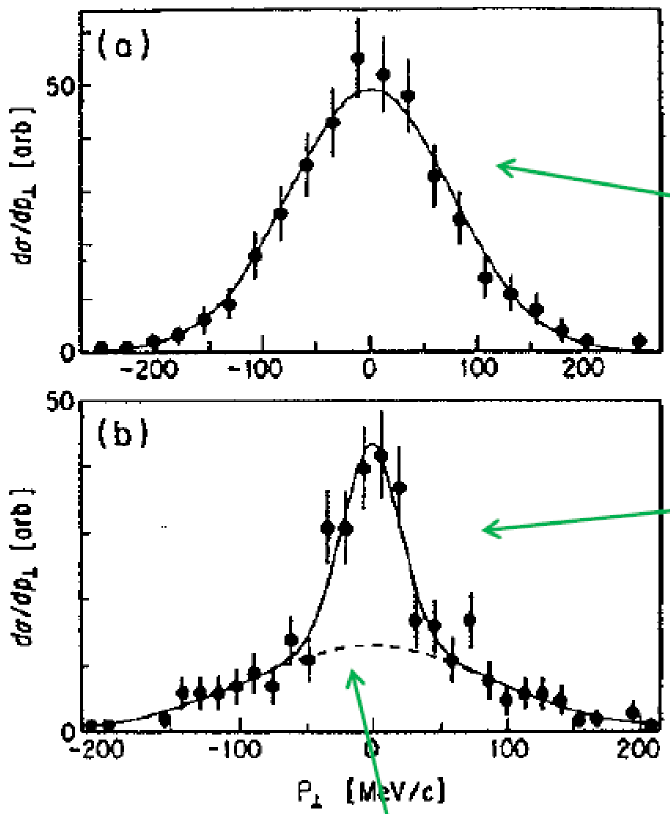


Discovery of halo nuclei

Momentum distribution of ^{11}Li



^{11}Li
(neutron-rich)



^{6}He distribution from ^{8}He

similar to Goldhaber model

^{9}Li distribution from ^{11}Li (**very narrow**!)

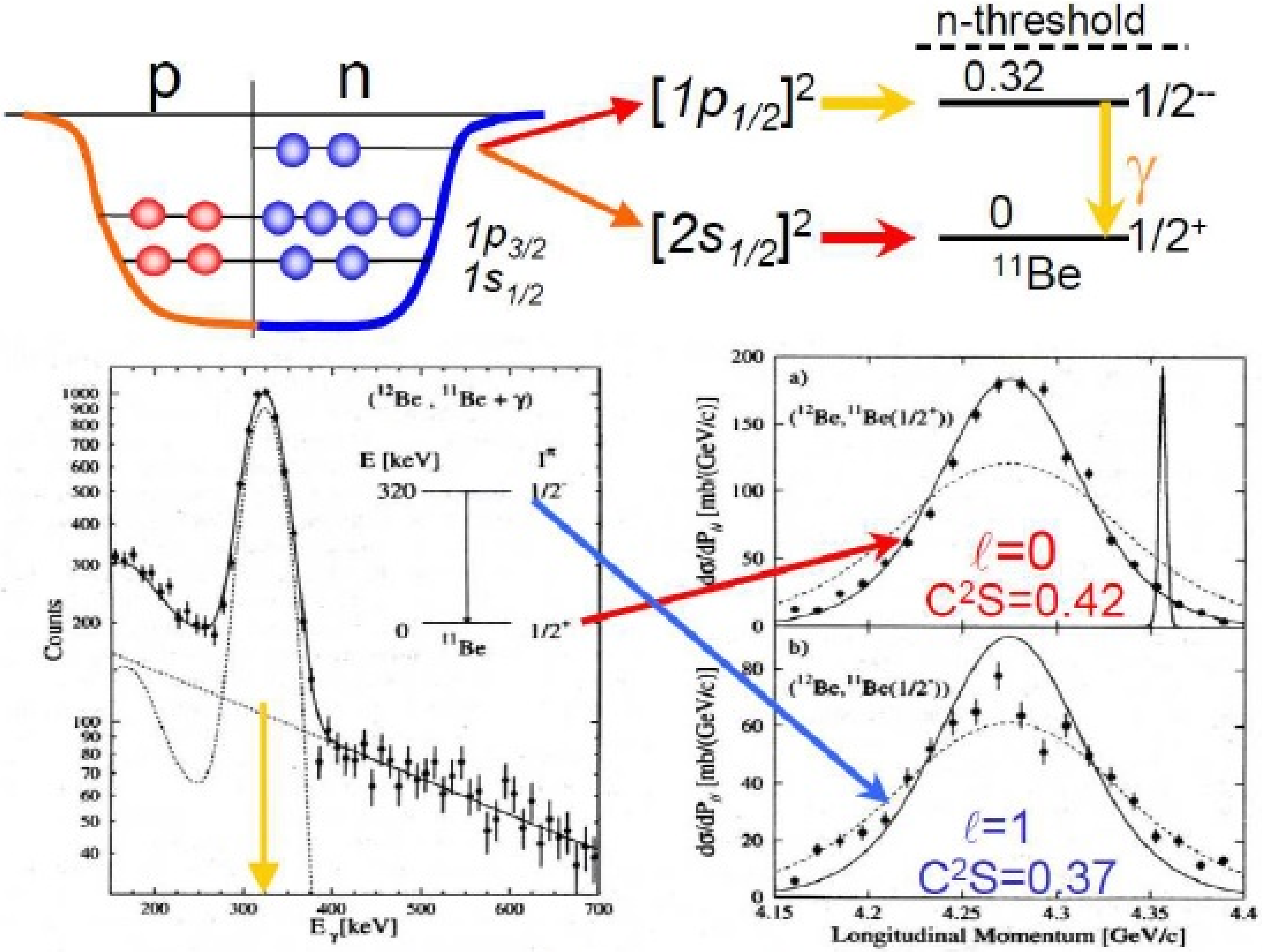
uncertainty principle

$$\Delta p \cdot \Delta x \geq \hbar$$

small → large

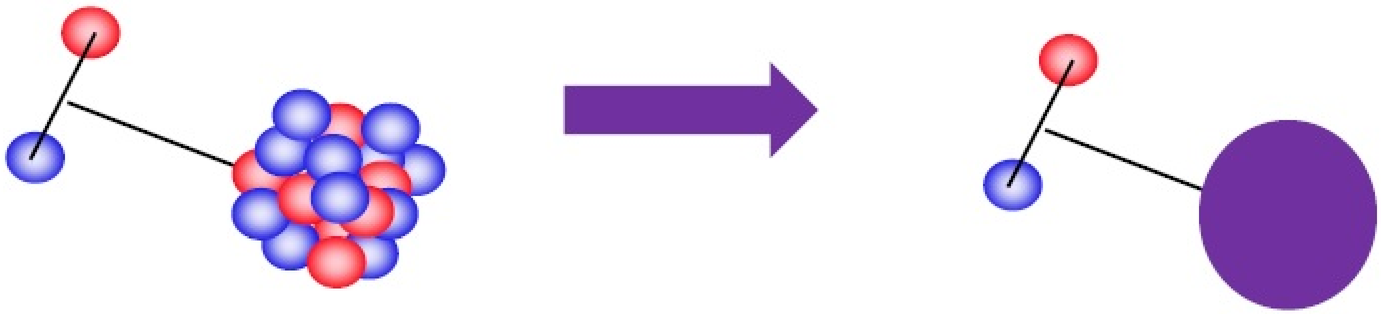
wider distribution is similar to Goldhaber model

Knockout typical result: ^{12}Be



A. Navin *et al.*, Phys. Rev. Lett. 85, 266 (2000)

Reducing the many body to a few body problems

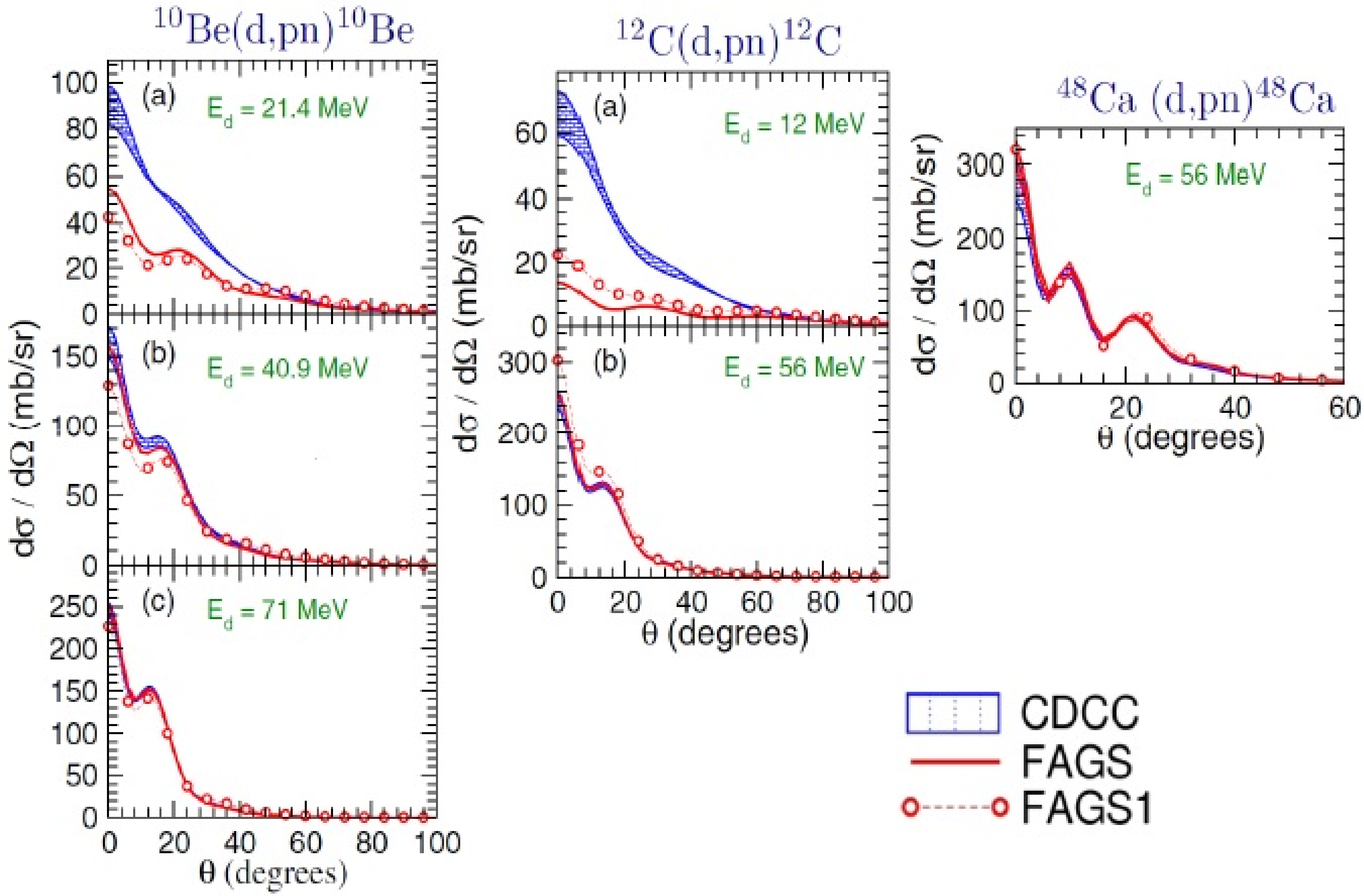


- isolating the important degrees of freedom in a reaction
- keeping track of all relevant channels
- connecting back to the many-body problem

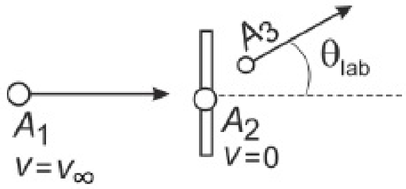
- effective nucleon-nucleus interactions (or nucleus-nucleus)
  (energy dependence/non-local)
- many body input

does not converge for $Z \geq 20$

Reaction methods; comparing CDCC with Faddeev

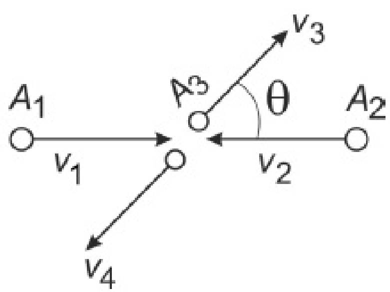


CDCC continuum discretized coupled channels



Laboratory system $A_1 + A_2 \rightarrow A_3 + x$

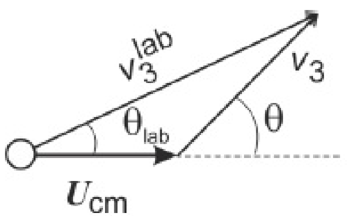
$$E_{lab} = \frac{m_N A_1}{2} v_\infty^2 \qquad E_{c.m.} = \frac{A_2}{A_1 + A_2} E_{lab} = \frac{\mu}{2} v_\infty^2 \qquad \mu = \frac{A_1 A_2}{A_1 + A_2}$$



Center of mass system

$$A_1 v_1 = A_2 v_2 \qquad v_1 = \frac{A_2}{A_1 + A_2} v_\infty \qquad v_2 = \frac{A_1}{A_1 + A_2} v_\infty$$

$$U_{cm} = \frac{A_1 v_1^{lab} + A_2 v_2^{lab}}{A_1 + A_2} = \frac{A_1}{A_1 + A_2} v_\infty$$ – velocity of the center of mass



Laboratory system: $\tan\theta_3^{lab} = \frac{v_3 \sin\theta}{v_3 \cos\theta + U_{cm}} = \frac{\sin\theta}{\cos\theta + \frac{U_{cm}}{v_3}}$

Elastic scattering:

$$v_3 = v_1 = \frac{A_2}{A_1 + A_2} v_\infty; \quad \frac{U_{cm}}{v_3} = \frac{A_1}{A_3}; \quad \tan\theta_3^{lab} = \frac{\sin\theta}{\cos\theta + A_1 / A_2}$$

$$A_1 = A_2 \Rightarrow \tan\theta_3^{lab} = \tfrac{1}{2}\theta$$

CDCC continuum discretized coupled channels