Leptons

- Leptons are ***spin ½ fermions***, not subject to strong interaction

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

$$\boldsymbol{M_e}(0.511\,MeV/c^2) < \boldsymbol{M_\mu}(105.7\,MeV/c^2) < \boldsymbol{M_\tau}\,(1777.1\,MeV/c^2)$$

  - Electron e^-, muon μ^- and tau τ^- have corresponding neutrinos ν_e, ν_μ, ν_τ.
  - Electron, muon and tau have ***electric charge of –e***. Neutrinos are neutral.
  - Neutrinos possibly have zero or very small mass.
  - For neutrinos, only weak interactions have been observed so far.

- Antileptons are positron e^+, positive muon μ^+ and positive tau τ^+ and antineutrinos:

$$\begin{pmatrix} e^+ \\ \overline{\nu_e} \end{pmatrix} \quad \begin{pmatrix} \mu^+ \\ \overline{\nu_\mu} \end{pmatrix} \quad \begin{pmatrix} \tau^+ \\ \overline{\nu_\tau} \end{pmatrix}$$

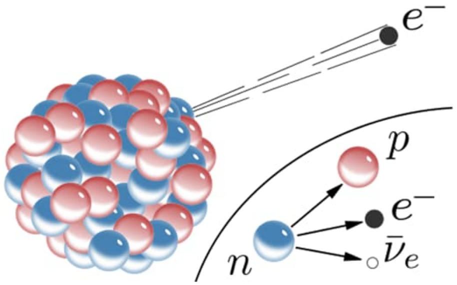
- Neutrinos and antineutrinos differ by the ***lepton number***. Leptons posses lepton number $L_\alpha = 1$ (α stands for e, μ, or τ, and antileptons have $L_\alpha = -1$.
- Lepton numbers are conserved in any interaction.

Leptons

Neutrinos can not be registered by any detector, there are only indirect indications of their quantities.

- First indication of neutrino existence came from β-decays of a nucleus N:

$$N(Z,A) \rightarrow N(Z+1,A) + e^{-} + \bar{\nu}_e$$



β-decay is nothing but a neutron decay

$$n(\mathbf{udd}) \rightarrow p(\mathbf{uud}) + e^{-} + \bar{\nu}_e$$

- Necessity of a neutrino existence comes from the apparent energy and angular momentum non-conservation in observed reactions.

- Note that for the sake of the lepton number conservation, electron must be accompanied by an antineutrino and not neutrino!

Mass limit for $\bar{\nu}_e$ can be estimated from the precise measurements of the β-decay:

$$m_e \leq E_e \leq \Delta M_N - m_{\bar{\nu}_e}$$

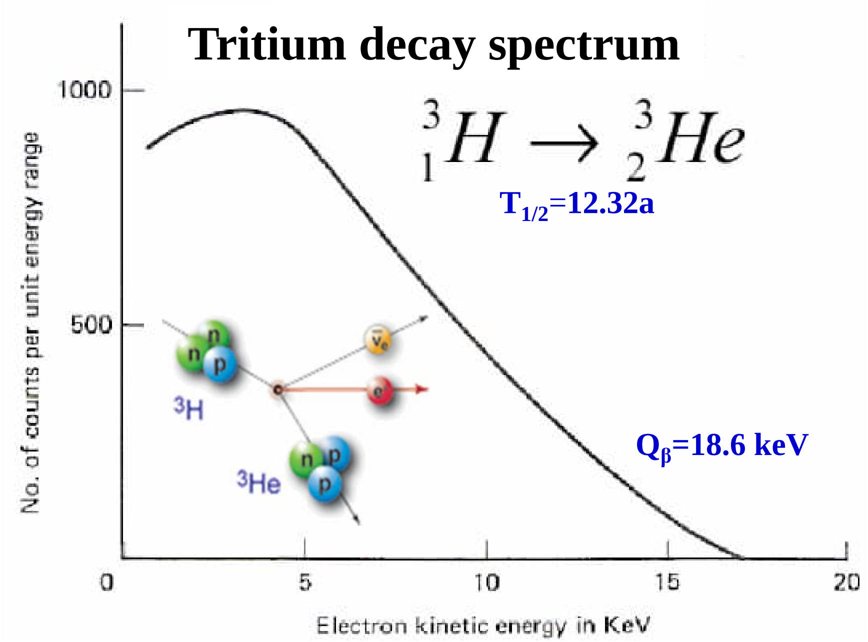
The best results are obtained from the tritium decay:

$$^{3}H \rightarrow \ ^{3}He + e^{-} + \bar{\nu}_e \qquad \boxed{m_{\bar{\nu}_e} \leq 15 \ eV/c^2}$$

Tritium decay

The β^- decay energy is given by the mass difference between mother and daughter nucleus.

This energy will be distributed as kinetic energy on the emitting particles, the electron and the anti-neutrino.
Hence, the electron spectrum is continuous.
It starts at zero energy and ends at the maximum possible energy $E_{max} = E_0 - m_\nu \cdot c^2$ (= Q_β).



Tritium decay spectrum

$${}^3_1H \rightarrow {}^3_2He$$

$T_{1/2}$=12.32a

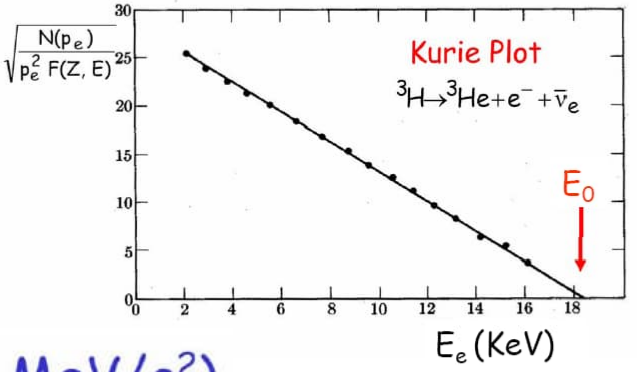
Q_β=18.6 keV

β decays have a long lifetime and a small decay probability, the related interaction is small compared to other interactions in the nucleus, therefore time dependent perturbation theory is a good approximation.

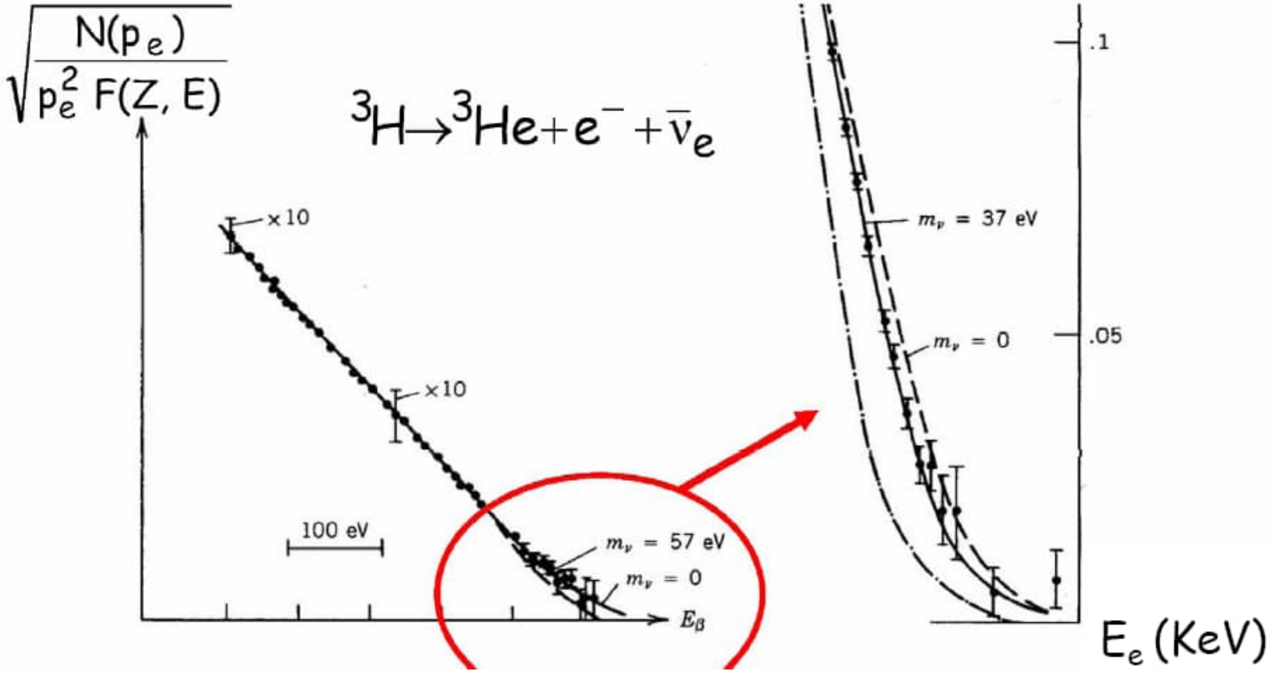
Tritium decay

Kurie plot:

$$\sqrt{\frac{N(p_e)}{F(Z,\varepsilon)\cdot p_e^2}} = const\cdot(\varepsilon_0-\varepsilon)$$



$m_\nu < 3$ eV/c² (c.f. $m_e = 0.511$ MeV/c²)



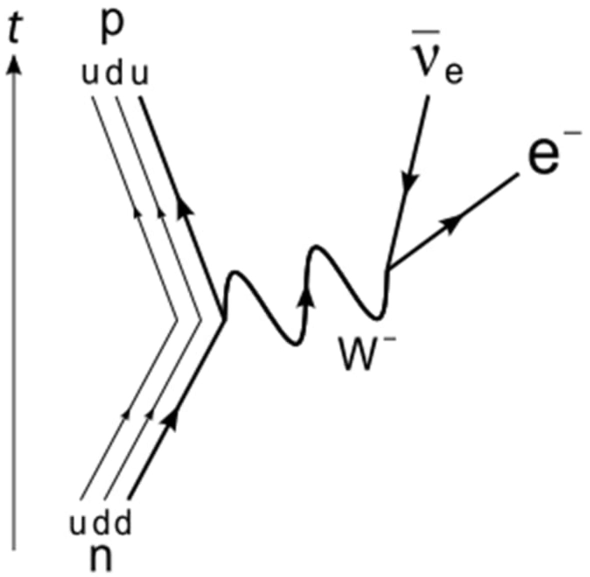
Weak Interaction

- Consider a neutron ***n (udd) β-decay***

Although the neutron is heavier than its sister proton ***p (uud),*** it cannot decay to proton without changing the flavor of one of its ***down quarks d***.

- Neither EM nor strong interactions allow to change the flavor. It must proceed through weak interaction.

$$d \rightarrow u + W^- \rightarrow u + e^- + \overline{\nu}_e$$



$$\boxed{n \rightarrow p + e^- + \overline{\nu}_e}$$

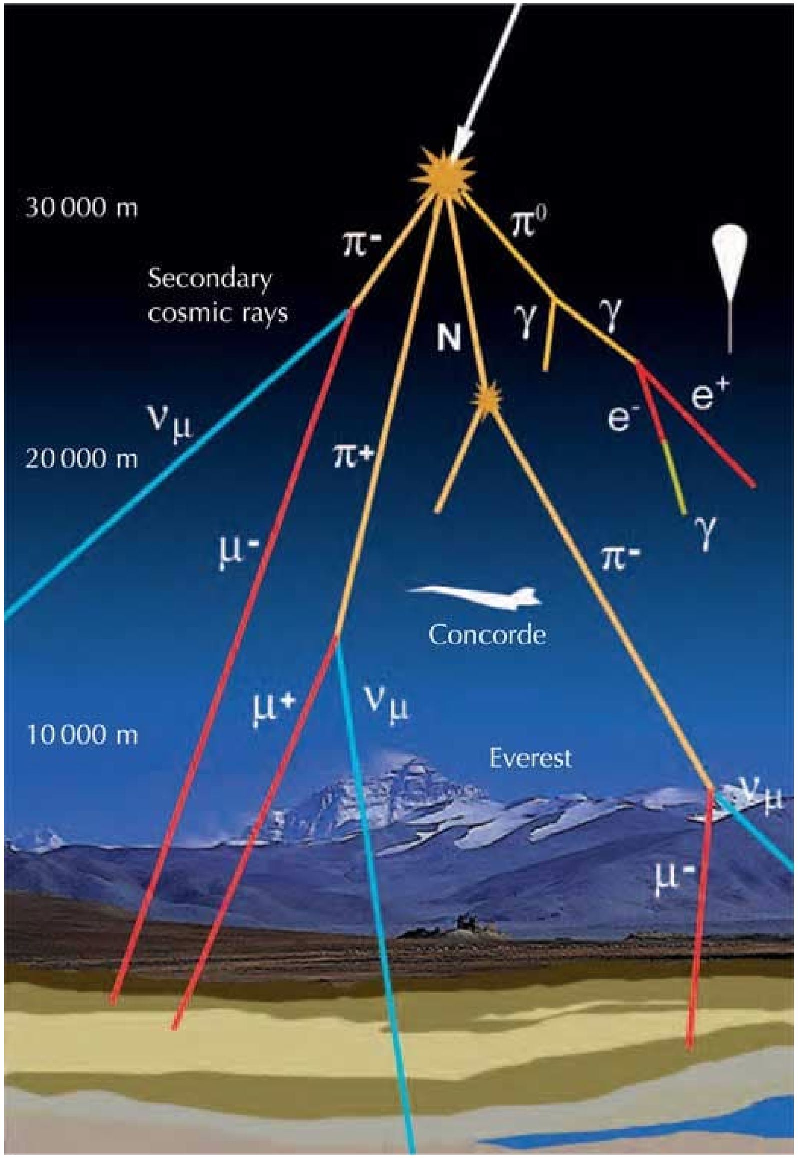
Source of Neutrinos

Nuclear reactions

- Fusion in the sun
- Fission in reactors
- Big bang nucleosynthesis

High energy collisions

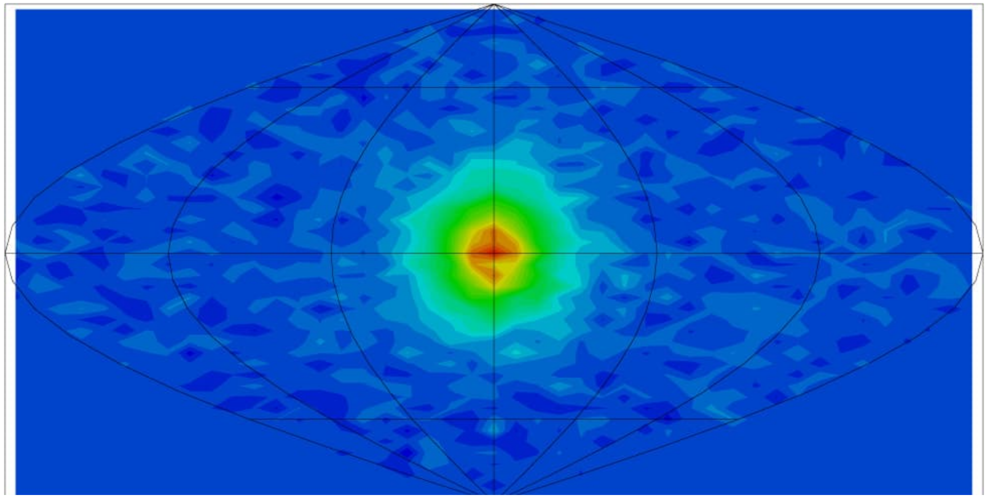
- Particle colliders
- Cosmic ray showers

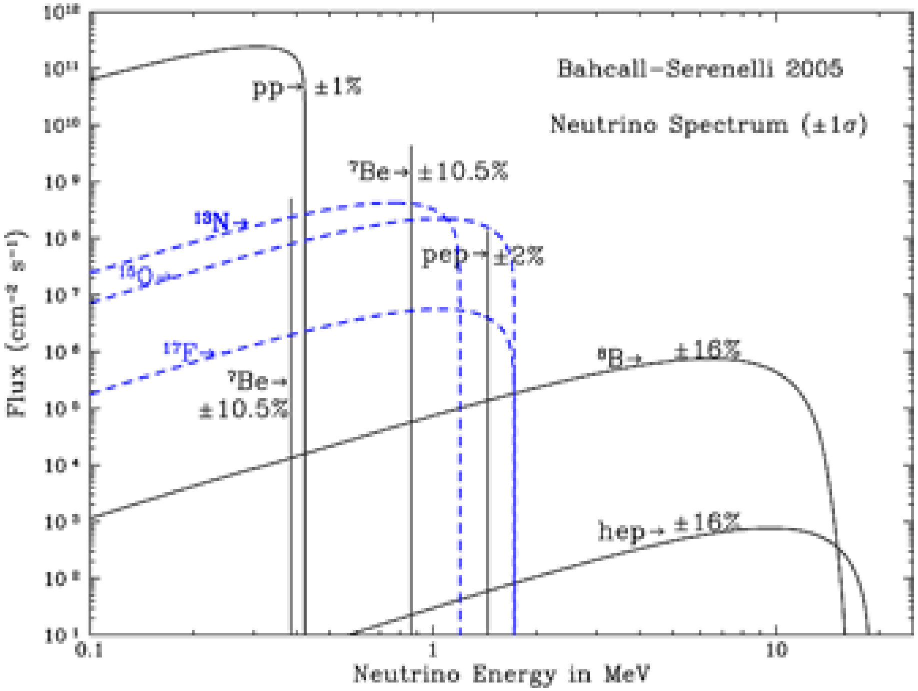


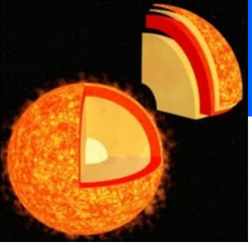
Solar Neutrinos

- Electron neutrinos produced in fusion chain
- 99% of solar neutrinos from pp fusion
- First observation in 2014 by Borexino
- Small fraction from ^{7}Be and ^{8}B
- Extend to high energy, easier to detect

- Bachall predicted the solar neutrino flux in 1964. He refined this with an incredibly precise solar model over the next 50 years.







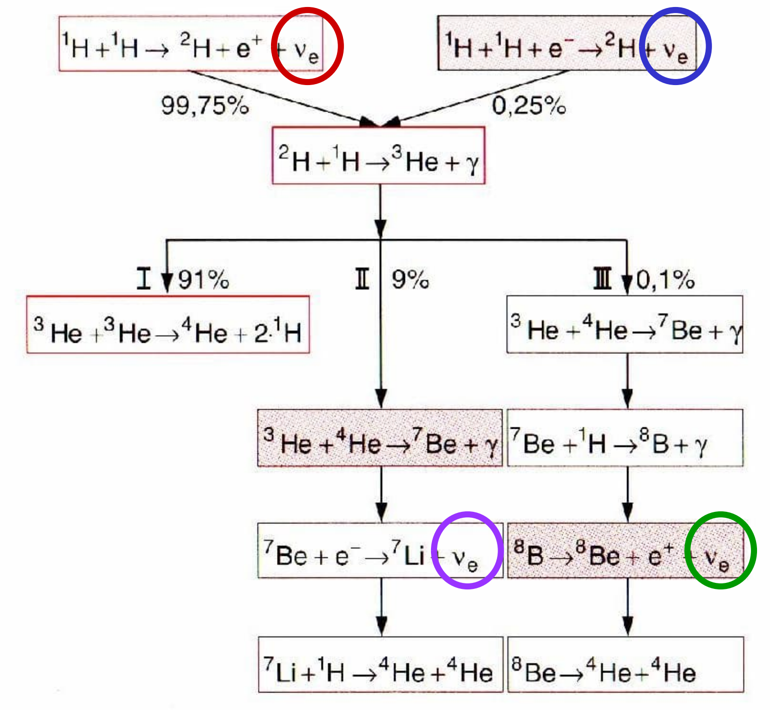
Neutrinos from the Sun

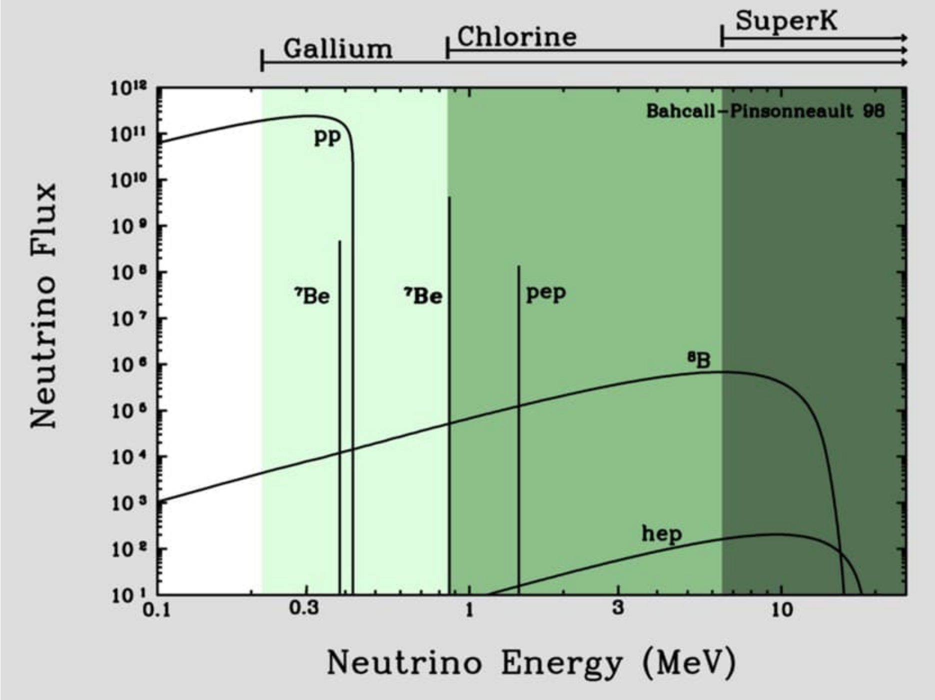
$$4\ \text{Protons} \rightarrow \text{Helium} + 2e^{+} + 2\nu + 26MeV$$

- Known: total irradiated energy
- Known: Energy per fusion process

➢ Number of neutrinos produced [s^{-1}]!
On Earth: 66 billion ν per ($cm^2 \cdot s^1$)

Different fusion pathes





Solar neutrino energy spectrum

Leptons

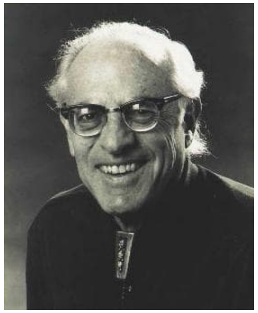
- An inverse β-decay also takes place:

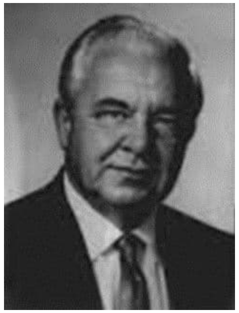
$$\nu_e + n \rightarrow e^- + p$$

or

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

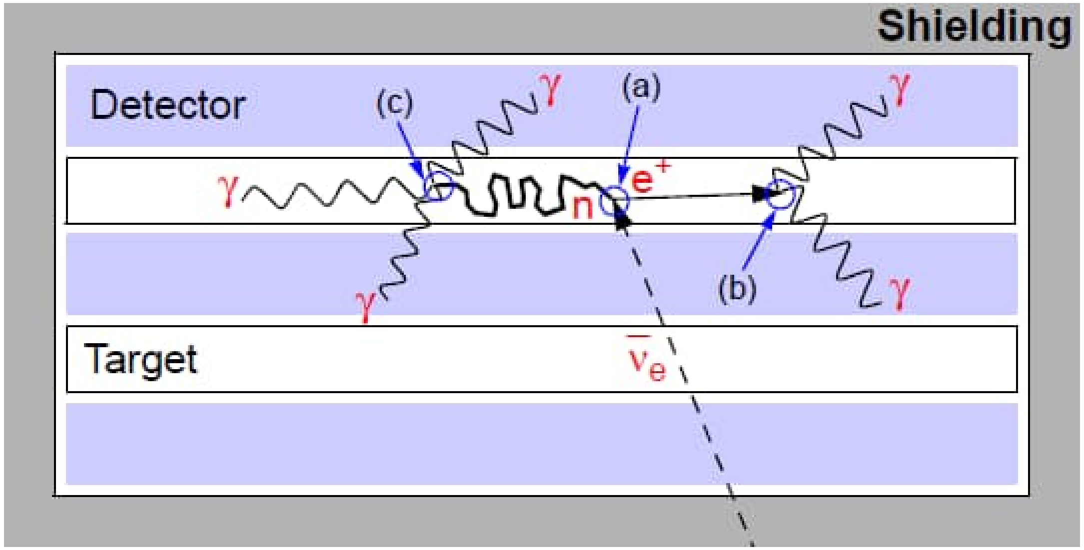
However, the probability of these processes is very low, therefore to register it one needs a very intense flux of neutrinos. Reines and Cowan experiment (1956)


Frederick Reines


Clyde L. Cowan

- Using antineutrinos produced in a nuclear reactor, it is possible to obtain around 2 (10) events per hour.
- Aqueous solution of $CdCl_2$ used as the target (Cd used to capture neutrons).
- To separate the signal from the background, the “delayed coincidence” scheme was used: signal from neutron comes later than one from positron.

Leptons



Schematic representation of the F. Reines and C. Cowan experiment

Main stages:

a) Antineutrino interacts with proton, producing neutron and positron

b) Positron annihilates with an atomic electron, produces fast photon which gives rise to softer photons through the Compton effect.

c) Neutron captured by a Cd nucleus, releasing more photons

Leptons

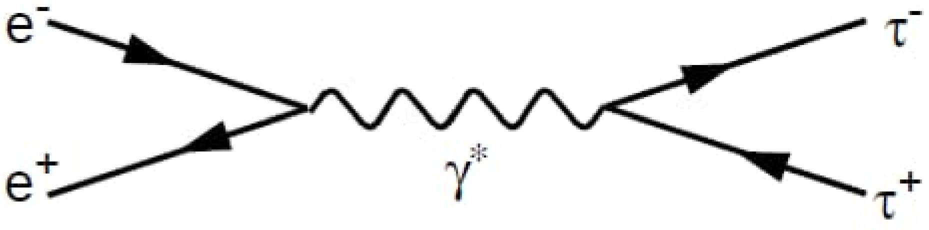
- Muons were first observed in 1936, in ***cosmic rays***

Cosmic rays have two components:

1. ***primaries***, which are high-energy particles coming from the outer space, mostly hydrogen nuclei
2. ***secondaries***, the particles which are produced in collisions of primaries with nuclei in the Earth atmosphere; muons belong to this component

- Muons are 200 times heavier than electrons and are very penetrating particles.
- Electromagnetic properties of muon are identical to those of electron (upon the proper account of the mass difference)

Tau is the heaviest of leptons, was discovered in e^+e^- annihilation experiments in 1975



τ pair production in e^+e^- annihilation

Lepton Type Conservation

- Electron is a stable particle, while muon and tau have a finite lifetime:

$$\tau_\mu = 2.2 \cdot 10^{-6}\ s \qquad \text{and} \qquad \tau_\tau = 2.9 \cdot 10^{-13}\ s$$

Muon decays in a purely leptonic mode:

$$\mu^- \rightarrow \nu_\mu + e^- + \overline{\nu_e}$$

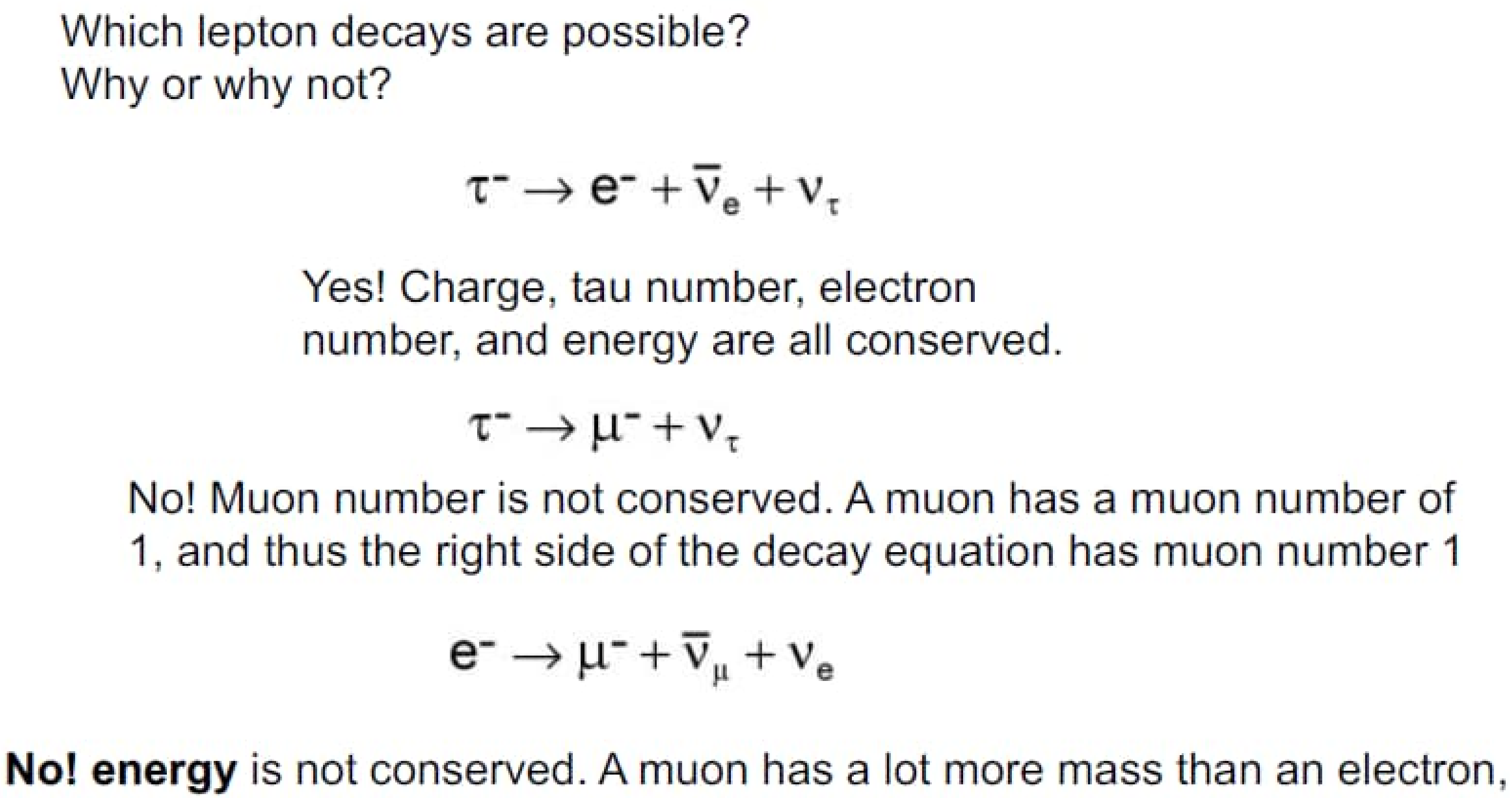
electron number 0 = 0 + 1 + -1

muon number 1 = 1 + 0 + 0

tau number 0 = 0 + 0 + 0

Electrons and their neutrinos have electron number +1
Positrons and their antineutrinos have electron number -1

Quiz



Which lepton decays are possible?
Why or why not?

$$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$$

Yes! Charge, tau number, electron number, and energy are all conserved.

$$\tau^- \rightarrow \mu^- + \nu_\tau$$

No! Muon number is not conserved. A muon has a muon number of 1, and thus the right side of the decay equation has muon number 1

$$e^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_e$$

No! energy is not conserved. A muon has a lot more mass than an electron,

Lepton Decay

- Electron is a stable particle, while muon and tau have a finite lifetime:

$$\tau_\mu = 2.2 \cdot 10^{-6}\, s \qquad \text{and} \qquad \tau_\tau = 2.9 \cdot 10^{-13}\, s$$

- Muon decays in a purely leptonic mode:

$$\mu^- \rightarrow \nu_\mu + e^- + \overline{\nu_e}$$

- Tau has a mass sufficient to produce hadrons, but has leptonic decay modes as well:

$$\tau^- \rightarrow \nu_\tau + e^- + \overline{\nu_e}$$

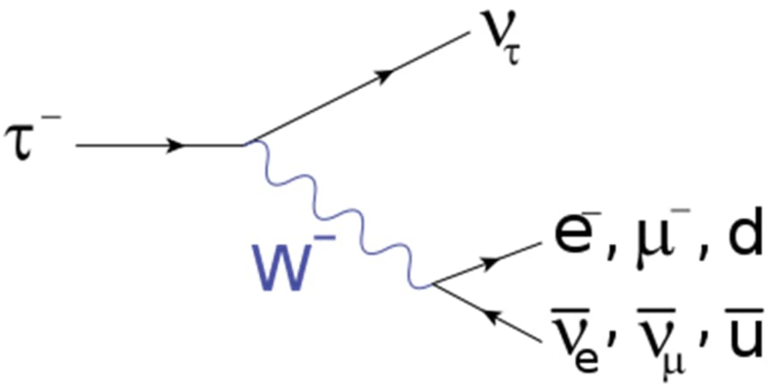
$$\tau^- \rightarrow \nu_\tau + \mu^- + \overline{\nu_\mu}$$

- Fraction of a particular decay mode with respect to all possible decays is called ***branching ratio***.

Branching ratio of both processes are 17.81% and 17.37%, respectively

- Note: lepton numbers are conserved in all reactions ever observed

Tau decay



The branching ratio of the dominant hadronic tau decay are:

- 25.52% for decay into a charged pion, a neutral pion, and a tau neutrino;
- 10.83% for decay into a charged pion and a tau neutrino;
- 9.30% for decay into a charged pion, two neutral pions, and a tau neutrino;
- 8.99% for decay into three charged pions (of which two have the same electrical charge) and a tau neutrino;
- 2.70% for decay into three charged pions (of which two have the same electrical charge), a neutral pion, and a tau neutrino;
- 1.05% for decay into three neutral pions, a charged pion, and a tau neutrino.

The branching ratio of the common purely leptonic tau decays are:

- 17.82% for decay into a tau neutrino, electron and electron antineutrino;
- 17.39% for decay into a tau neutrino, muon and muon antineutrino.

Leptons

Important assumptions:

1) Weak interactions of leptons are identical, just like electromagnetic ones ("interactions universality")
2) One can neglect final state lepton masses for many basic calculations

The ***decay rate*** of a muon is given by the expression:

$$\Gamma(\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu) = \frac{G_F^2\, m_\mu^5}{195\, \pi^3}$$

Here G_F is the ***Fermi constant*** $\left(\boldsymbol{G_F^0 = \frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2 c^4} = 1.1664 \cdot 10^{-5}\ GeV^{-2}}\right)$

Substituting m_μ with m_τ one obtains decay rates of tau leptonic decays, equal for both processes. It explains why branching ratios of these processes have very close values.

Leptons

Using the decay rate, the lifetime of a lepton is:

$$\tau_\ell = \frac{B(\ell^- \to e^-\overline{\nu}_e\nu_\ell)}{\Gamma(\ell^- \to e^-\overline{\nu}_e\nu_\ell)}$$

Here ℓ stands for μ or τ. Since muons have basically only one decay mode, B = 1 in their case. Using experimental values of B and formula for Γ, one obtains the ratio of muon and tau lifetimes:

$$\frac{\tau_\tau}{\tau_\mu} \approx 0.178 \cdot \left(\frac{m_\mu}{m_\tau}\right)^5 \approx 1.3 \cdot 10^{-7}$$

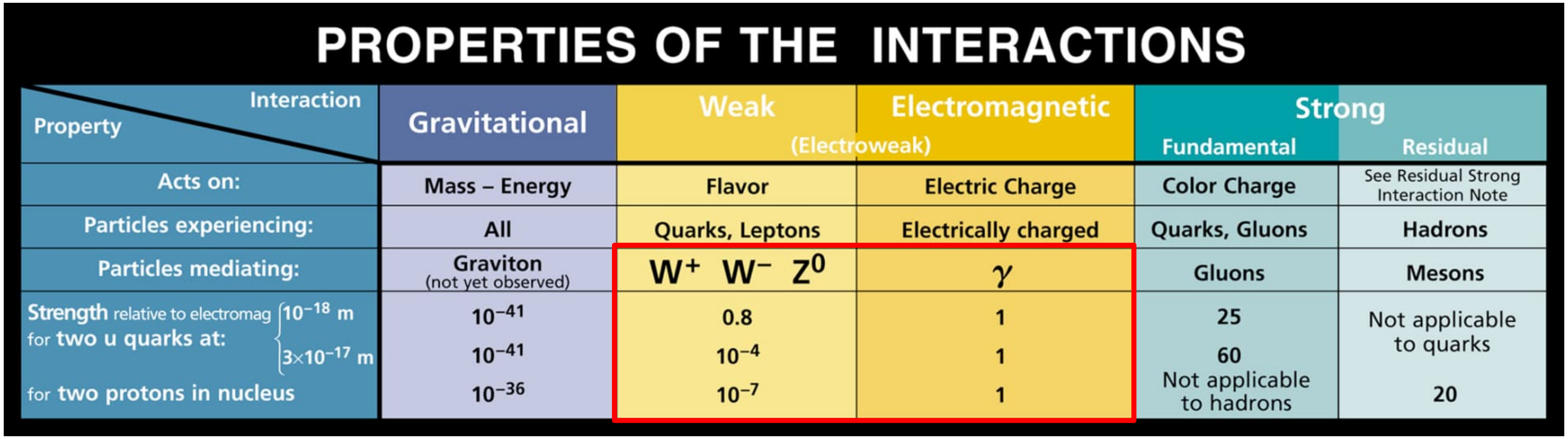
This again is in very good agreement with independent experimental measurements

- ➢ Universality of lepton interactions is provided to big extend. That means that there is basically no difference between lepton generations, ***apart of the mass***.

Electroweak

In the Standard Model the weak and the electromagnetic interactions have been combined into a unified ***electroweak*** theory.

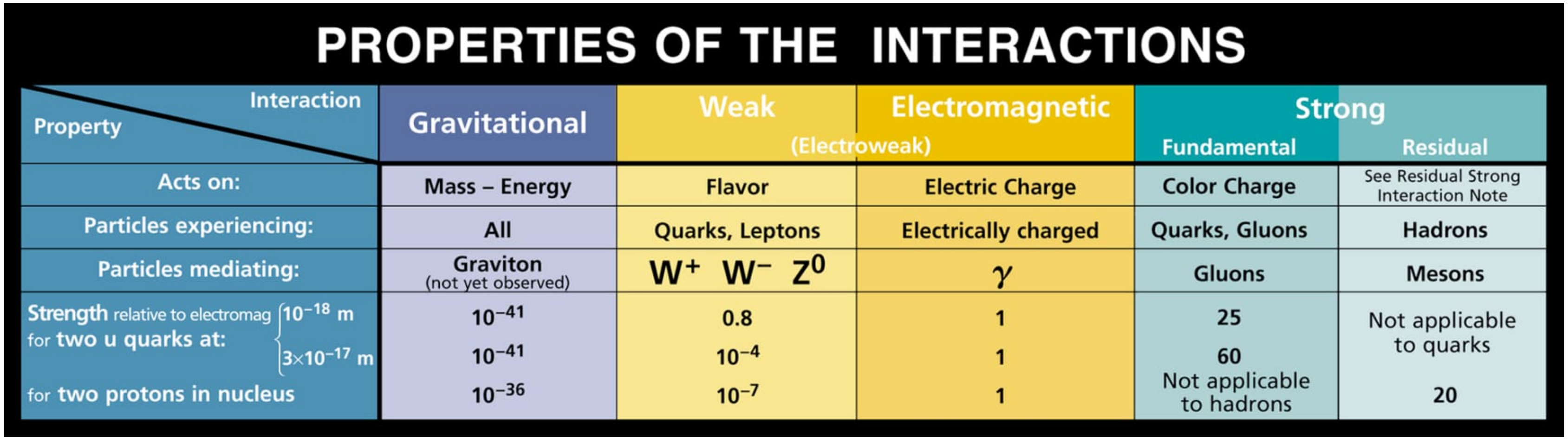
- At very short distances (~ 10^{-18} m) the strength of the weak interaction is comparable to that of the electromagnetic.
- At thirty times that distance ($3 \cdot 10^{-17}$ m) the strength of the weak interaction is 1/10000th than that of the electromagnetic interaction. At distances typical for quarks in a proton or neutron (10^{-15} m) the force is even tinier.



PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak	Electromagnetic	Strong	
		(Electroweak)		Fundamental	Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Electroweak



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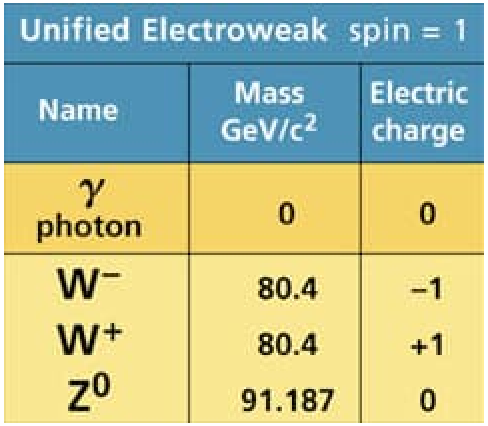
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

- The strength of the interaction depends strongly on both the mass of the force carrier and the distance of the interaction.
- The difference between their observed strengths is due to the huge difference in mass between the $W^{\pm}$ and Z^0 particles, which are very massive, and the photon, which has no mass.



Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0