HEXADECAPOLE MOMENTS OBTAINED BY SEMICLASSICAL AND QUANTUM MECHANICAL ANALYSES OF COULOMB EXCITATION

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Electric quadrupole and hexadecapole matrix elements of rare-earth nuclei were determined from Coulomb excitation of the 2^+ and 4^+ rotational states in the ground-state band. The E4-matrix elements obtained by second-order perturbation expansion of the quantum mechanical theory and by a coupled channel code agree within 20%.

In recent years, several studies of hexadecapole moments of rare-earth [1–6] and actinide [7] nuclei were performed by using the Coulomb excitation process. In these investigations, α-particles of sufficiently low energy were used to populate the 2^+ and 4^+ rotational levels of the ground-state band. The analysis of such experiments is usually based on the calculation of the 2^+ and 4^+ excitation probability as a function of the relevant E2 and E4 matrix elements. By comparison with the measured excitation probabilities, reduced transition matrix elements $\langle 2\|M(\mathrm{E2})\|0\rangle$ and $\langle 4\|M(\mathrm{E4})\|0\rangle$ are obtained and deformation parameters may be deduced by assuming a particular model for the charge distribution.

The data obtained in the earlier experiments were interpreted by employing the semiclassical theory [8] of Coulomb excitation. Quantal effects [2,9] which considerably influence the 4^+ excitation probability were computed in second-order perturbation theory and incorporated into the analyses, while the effect of quantal corrections on the 2^+ excitation probability is believed to be negligible. The dominant mode of excitation of the 4^+ level is the two-step E2 transition, however, and the $\langle 4\|M(\mathrm{E4})\|0\rangle$ element is thus very sensitive on the E2 matrix elements. Therefore, exact calculation of both the 2^+ and 4^+ excitation probability appears to be necessary in a self-consistent analysis of Coulomb excitation experiments. It is the purpose of this paper to report the results of such an analysis and to demonstrate that the E2 and E4 matrix elements obtained from exact and second-order analyses agree with 1% and 20%, respectively.

The experiments were performed by bombarding thin, highly enriched targets of Sm, Gd, Dy, and Er isotopes with 12 MeV α-particles from the University of Frankfurt, Van de Graaff accelerator. Inelastically scattered α-particles were detected at $\theta_L = 160°$ with a cooled Si surface-barrier detector. A resolution of 17 keV (FWHM) and a peak-to-background ratio of better than 4×10^4 was achieved. A typical spectrum is shown in fig. 1.

All of our calculations were performed in the limit of the rigid-rotor model. For ^{152}Sm, however, deviations from the pure rotor as recently reported by Diamond et al. [10] are taken into account by means of a "centrifugal-stretching" parameter $\alpha = 1.9 \times 10^{-3}$. Results of the calculations for ^{164}Dy are presented in fig. 2, where the differential cross section for the excitation of the 4^+ level normalized to the case of zero E4 matrix element is plotted versus the reduced matrix element $\langle 4\|M(\mathrm{E4})\|0\rangle$ and compared with experiment.

The semiclassical (SC) calculations were done by using an extended version of the coupled-channel code of Winther and de Boer [11]. The calculations labelled SC(1) included three levels (0^+, 2^+ and 4^+), while in SC(2) the 6^+ state was also included. All non-vanishing E2 and E4 matrix elements between these states were taken into account.

The 4^+ excitation cross sections are markedly reduced from the SC values when quantal effects are

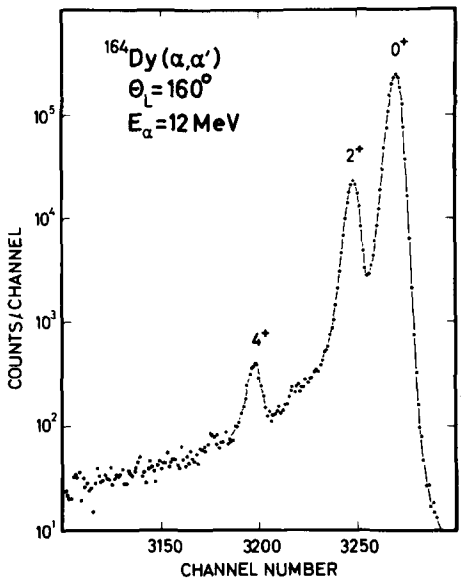

Fig. 1. The ^{164}Dy(α, α') spectrum obtained at $E_\alpha = 12$ MeV and $\theta_L = 160°$.

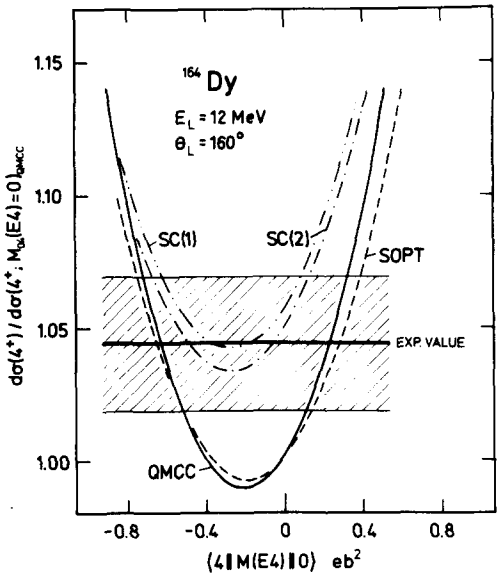

Fig. 2. Comparison of the normalized excitation probabilities of the 4+ level of ^{164}Dy as calculated (1) with a Winther-de Boer coupled-channel code (SC), (2) in quantum mechanical second-order perturbation theory (SOPT), and (3) with a quantum mechanical coupled-channel code (QMCC).

taken into account. The calculations in second-order perturbation theory (SOPT) were performed with the aid of the tables of Alder et al. [9] for both the 2^+ and 4^+ excitation probabilities, while the exact calculations used a quantum mechanical coupled-channel code (QMCC) [6] which included the 0^+, 2^+ and 4^+ levels and all possible matrix elements connecting them.

The general result for all of the nuclei studied here is that the SOPT and QMCC calculations yield 4^+ excitation cross sections that agree within a few percent with each other, with the largest deviation being $\approx 20\%$ for ^{164}Dy. The second-order approximation may therefore be used instead of the time-consuming

Table 1
Reduced transition matrix elements $M_{02} = \langle 2\|M(E2)\|0\rangle$ and $M_{04} = \langle 4\|M(E4)\|0\rangle$ as obtained by semiclassical and quantum-mechanical analyses of the present experiment

	SC(1)		SOPT		QMCC	
	M_{02}(eb)	M_{04}(eb^2)	M_{02}(eb)	M_{04}(eb^2)	M_{02}(eb)	M_{04}(eb^2)
^{152}Sm	1.852 ± 0.017	0.37 ± 0.09[a)] 0.43 ± 0.08	1.861 ± 0.017	0.43 ± 0.08[a)] 0.49 ± 0.08	1.864 ± 0.017	0.46 ± 0.08[a)] 0.51 ± 0.08
^{154}Sm	2.065 ± 0.010	0.43 ± 0.09	2.059 ± 0.010	0.59 ± 0.09	2.072 ± 0.010	0.57 ± 0.09
^{158}Gd	2.228 ± 0.018	$0.18^{+0.14}_{-0.20}$	2.221 ± 0.018	0.42 ± 0.13	2.244 ± 0.018	0.39 ± 0.13
^{164}Dy	2.365 ± 0.012	−0.16 + 0.26[b)]	2.356 ± 0.012	$0.29^{+0.11}_{-0.14}$	2.379 ± 0.012	$0.23^{+0.10}_{-0.12}$
^{166}Er	2.411 ± 0.009	−0.15 + 0.30 [b)]	2.407 ± 0.009	$0.24^{+0.23}_{-0.35}$	2.419 ± 0.009	$0.22^{+0.11}_{-0.16}$

a) Non-rigid rotor. b) No solution, cf. fig. 2.

coupled-channel calculations for the purpose of determining hexadecapole moments in this region of the periodic table. The sign of the matrix element $\langle 4\|M(\mathrm{E4})\|0\rangle$ is not unambiguously determined from the present experiment, but if negative values were used, the extracted hexadecapole deformation parameters would be much larger than expected on theoretical grounds. Therefore, the positive value of $\langle 4\|M(\mathrm{E4})\|0\rangle$ was chosen. In table 1, the reduced E2 and E4 matrix elements obtained from our data are summarized.

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