$E2$ and $E4$ transition moments in ^{172}Yb, ^{174}Yb, and ^{176}Yb

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Electric quadrupole and hexadecapole transition matrix elements in the ground-state rotational bands of ^{172}Yb, ^{174}Yb, and ^{176}Yb were measured in precise Coulomb excitation experiments with 13 MeV α particles. The results are $\langle 2^+\|M(E2)\|0^+\rangle = 2.456 \pm 0.012$ eb, 2.439 ± 0.012 eb, 2.325 ± 0.018 eb and $\langle 4^+\|M(E4)\|0^+\rangle = 0.22^{+0.12}_{-0.18}$ eb^2, $0.21^{+0.14}_{-0.18}$ eb^2, $0.28^{+0.11}_{-0.20}$ eb^2 for ^{172}Yb, ^{174}Yb, and ^{176}Yb, respectively. Model-dependent charge deformation parameters, β_2 and β_4, are deduced from the measured transition moments.

[NUCLEAR REACTIONS ^{172,174,176}Yb(α,α'), $E = 13$ MeV; measured $\sigma(E_{\alpha'};\theta = 160°)$; deduced $\langle 2^+\|M(E2)\|0^+\rangle$, $\langle 4^+\|M(E4)\|0^+\rangle$. Deduced charge deformation parameters, β_2 and β_4.]

I. INTRODUCTION

The presence of sizable hexadecapole deformations in both rare-earth and actinide nuclei has been well established in a number of experiments. Previous systematic studies of the inelastic scattering of α particles at energies above the Coulomb barrier[1,2] provided experimental evidence for β_4 and β_6 deformations in the lanthanides. More recently, inelastic electron scattering[3] was also applied to measure hexadecapole deformations. A considerable body of experimental information concerning hexadecapole moments has become available from Coulomb excitation experiments performed by various authors[4–11] throughout the rare-earth and actinide regions. Moreover, the interference effect between Coulomb and nuclear interactions was used recently to extract[12,13] both nuclear charge and mass deformation parameters.

Most of the Coulomb excitation studies performed in the rare-earth region were used to determine $E2$ and $E4$ moments in nuclei with $152 \leq A \leq 170$, while little information[6,9,13] is available on heavier-mass lanthanides. Coulomb excitation experiments in this region become increasingly difficult to analyze as A increases because the direct $E4$ transition to the 4^+ rotational state, which serves to determine the $E4$ moment, becomes smaller relative to the two-step $E2$ transition. It was the purpose of this work to perform a precision Coulomb excitation experiment on the even Yb isotopes, thus supplying more complete information about quadrupole and hexadecapole moments in these rare-earth nuclei. Preliminary results were reported earlier.[14]

II. EXPERIMENTAL PROCEDURE

The experiments were performed by bombarding thin (10–30 μg/cm^2), highly enriched (>96%) targets of ^{172}Yb, ^{174}Yb, and ^{176}Yb on 20 μg/cm^2 carbon backings with 13 MeV α particles from the University of Frankfurt Van de Graaff accelerator. Elastically and inelastically scattered α particles were detected at $\theta_{lab} = 160°$ with a cooled Si-surface-barrier detector. The energy resolution was typically 23 keV, full width at half maximum. A

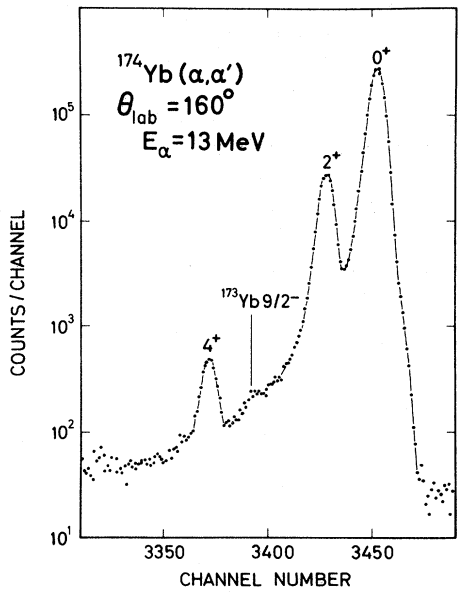

FIG. 1. Elastically and inelastically scattered α particles from ^{174}Yb.

spectrum resulting from the ^{174}Yb$(\alpha, \alpha')^{174}$Yb reaction is shown in Fig. 1.

The beam energy for the present study was selected on the basis of an excitation function for the 2^+ state of ^{174}Yb which was measured with 2% accuracy between 12.0 and 14.5 MeV in 250 keV intervals. Since this excitation function showed no deviation from pure Coulomb excitation at energies below 13.5 MeV, the bombarding energy was chosen to be 13 MeV.

The excitation cross section for the 2^+ rotational state was determined relative to the elastic (0^+) intensity by means of a computer code[15] which separated the 2^+ and 0^+ peaks in a self-consistent iterative manner assuming identical peak shapes. The intensity of the 4^+ state was then obtained by fitting a fourth-order polynomial to the background above and below the 4^+ peak. Lines due to small amounts of contaminants present in the target material were subtracted from the spectrum. The statistical uncertainty associated with the 2^+ intensity was approximately 1.5%, while the 4^+ cross section was accurate to within about 2%.

III. ANALYSIS AND DISCUSSION

In order to extract $E2$ and $E4$ transition matrix elements from the experimental data, Coulomb excitation cross sections were calculated by means of a quantum-mechanical coupled-channels code.[16] These calculations included all nonvanishing $E2$ and $E4$ matrix elements among the 0^+, 2^+, and 4^+ levels of the ground-state rotational band. Since Sayer *et al.*[17] in a recent study of multiple Coulomb excitation employing ^{16}O ions have found that the Yb isotopes in low angular momentum states ($I \leq 4$) are well described by the rigid-rotor model, we have used the rigid-rotor relationships to calculate the reduced $E2$ and $E4$ matrix elements in the ground-state band. In this limit the matrix elements are given by

$$\langle I_f \| M(E\lambda) \| I_i \rangle = [(2I_i+1)(2\lambda+1)/16\pi]^{1/2} \times Q_{\lambda 0} \langle I_i \lambda 00 | I_f 0 \rangle , \quad (1)$$

where $Q_{\lambda 0}$ is the electric multipole moment of order λ in the intrinsic frame,

$$Q_{\lambda 0} = [16\pi/(2\lambda+1)]^{1/2} \times \int \rho(\vec{r}, \beta_2, \beta_4) r^\lambda Y_{\lambda 0}(\theta) d\tau . \quad (2)$$

With Eq. (1) the various transition matrix elements between the ground-band states are expressed in terms of the reduced matrix elements $\langle 2 \| M(E2) \| 0 \rangle$ and $\langle 4 \| M(E4) \| 0 \rangle$, which, in turn, are extracted from experiment by comparing the calculated excitation probabilities of the 2^+ and 4^+ state with the experimentally measured values.

The sign of $\langle 2 \| M(E2) \| 0 \rangle$ was taken to be positive to yield, with the phase convention chosen, a prolate quadrupole shape of the Yb nuclei. The sign of $\langle 4 \| M(E4) \| 0 \rangle$ is not unambiguously determined from the present experiment, however. For a given experimental excitation probability of the 4^+ state, there exist two solutions for the $E4$ matrix element with opposite signs, the positive of which was chosen. If the negative value had been selected, the extracted hexadecapole deformation parameter would have been much larger than expected on theoretical grounds. The reduced $E2$ and $E4$ matrix elements obtained from the present data are summarized in Table I. Also shown in this table are results of Greenberg and Shaw[6] which were reported previously. It is seen that the two sets of matrix elements are in very good agreement.

If more than the 0^+, 2^+, and 4^+ ground-band levels are included in the coupled-channels calculations, the computed excitation cross sections for the 2^+ and 4^+ states are slightly altered. Calculations employing the semiclassical code of Winther and de Boer[18] showed that inclusion of the 6^+ state changes the calculated 4^+ cross section by approximately 0.6%, while contributions due to the excitation of higher states, e.g. β, γ, and octupole vibrational states, are negligibly small. Since the experimental error associated with the 4^+ cross section is considerably larger than the modification due to the 6^+ state, we chose to neglect the 6^+ level, as well as higher states in the quantum-me-

TABLE I. Reduced $E2$ and $E4$ transition matrix elements in Yb nuclei.

	$\langle 2 \| M(E2) \| 0 \rangle$ (e b)		$\langle 4 \| M(E4) \| 0 \rangle$ (e b^2)	
	Present work	Previous work [a]	Present work	Previous work [a]
^{172}Yb	2.456 ± 0.012		$0.22^{+0.12}_{-0.18}$	
^{174}Yb	2.439 ± 0.012	2.433 ± 0.012	$0.21^{+0.14}_{-0.18}$	0.23 ± 0.17
^{176}Yb	2.325 ± 0.018		$0.28^{+0.11}_{-0.20}$	

[a] See Ref. 6.

TABLE II. Quadrupole and hexadecapole deformation parameters.

	Experiment				Theory			
	Homogeneous distribution [a]		Fermi distribution [b]		Oscillator potential [c]		Woods-Saxon potential [d]	
	β_2	β_4	β_2	β_4	β_2	β_4	β_2	β_4
^{172}Yb	$0.284^{+0.010}_{-0.007}$	$-0.006^{+0.027}_{-0.040}$	$0.326^{+0.010}_{-0.008}$	$-0.008^{+0.027}_{-0.043}$	0.300	−0.012	0.28	−0.04
			(0.294)	(−0.007)				
^{174}Yb	$0.280^{+0.009}_{-0.007}$	$-0.007^{+0.030}_{-0.040}$	$0.322^{+0.010}_{-0.009}$	$-0.007^{+0.023}_{-0.051}$	0.295	−0.024	0.28	−0.05
			(0.291)	(−0.008)				
^{176}Yb	$0.263^{+0.011}_{-0.006}$	$0.012^{+0.023}_{-0.044}$	$0.301^{+0.012}_{-0.007}$	$0.011^{+0.025}_{-0.046}$	0.290	−0.037	0.28	−0.05
			(0.272)	(0.009)				

[a] $R_0 = 1.22A^{1/3}$ fm.
[b] $R_0 = 1.10A^{1/3}$ fm, $a = 0.6$ fm. Values in parentheses were obtained with $R_0 = 1.16A^{1/3}$ fm, $a = 0.66$ fm.
[c] See Ref. 19.
[d] See Ref. 20.

chanical calculations that yielded the results given in Table I.

The reduced $E2$ and $E4$ matrix elements were used to extract quadrupole and hexadecapole deformation parameters β_2 and β_4 for the nuclear charge distribution. It is seen from Eqs. (1) and (2) that the reduced $E\lambda$ matrix element is given by

$$\langle \lambda \| \boldsymbol{M}(E\lambda) \| 0 \rangle = \int \rho(\vec{r}, \beta_2, \beta_4) r^{\lambda} Y_{\lambda 0}(\theta) d\tau. \tag{3}$$

The integral was solved numerically for $\lambda = 2$ and 4 assuming two different models for the charge density. For a homogeneous charge distribution $\rho(\vec{r}, \beta_2, \beta_4)$ is given by

$$\rho(\vec{r}, \beta_2, \beta_4) = \begin{cases} \rho_0 & r \leq R(\theta) \\ 0 & r > R(\theta) \end{cases}, \tag{4}$$

where the charge surface

$$R(\theta) = R_0[1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta)]. \tag{5}$$

For a diffuse-surface Fermi distribution the following form was used

$$\rho(\vec{r}, \beta_2, \beta_4) = \rho_0 \left[1 + \exp\left(\frac{r - R(\theta)}{a}\right)\right]^{-1}. \tag{6}$$

Normalization of the integral in Eq. (3) was accomplished by adjusting ρ_0 to give the total nuclear charge $Ze = \int \rho(\vec{r}, \beta_2, \beta_4) d\tau$, while both the radius parameter R_0 and the diffuseness a were held constant. For the Fermi distribution the radius was taken to be $R_0 = 1.1A^{1/3}$ fm and the diffuseness $a = 0.6$ fm. Employing the alternative normalization procedure, viz. adjusting R_0 and holding ρ_0 constant, leaves the results for β_2 and β_4 virtually unchanged. Deformation parameters for the homogeneous charge density were extracted using $R_0 = 1.22A^{1/3}$ fm to facilitate comparison with theory.[19] The quadrupole and hexadecapole deformations obtained from the present experiment are summarized in Table II, along with theoretical values for the equilibrium deformations β_2 and β_4.

Comparing the extracted β_2 and β_4 values with theory we note good agreement of our results for ^{172}Yb and ^{174}Yb with the calculations of Möller[19] for a generalized oscillator potential. The results of Götz *et al.*[20] who used a Woods-Saxon potential for their calculation of equilibrium deformations are also in good agreement with our values, provided the same potential parameters as in Ref. 20 are used to extract the β_λ values from the reduced $E2$ and $E4$ transition matrix elements, i.e., $R_0 = 1.16A^{1/3}$ fm and $a = 0.66$ fm. As can be noted from Table II, the experimental β_4 parameters appear to be systematically smaller than the calculated values. Moreover, the result for β_4 in ^{176}Yb derived from the present experiment is positive, in contrast to the theoretical predictions and previous (α, α') studies[1,21] above the Coulomb barrier. In view of the sizable experimental uncertainties associated with the hexadecapole deformations it is probably premature to draw conclusions from these deviations, however. It would be desirable to perform additional experiments in this region of the Periodic Table to verify the trend of the hexadecapole deformations.

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