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$E2$ and $E4$ transition moments in ^{163}Dy and ^{167}Er†

H. J. Wollersheim* and Th. W. Elze
Institut für Kernphysik, Universität Frankfurt/Main, Germany

L. D. Tolsma
Physics Department, Eindhoven University of Technology, Eindhoven, The Netherlands

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Coulomb excitation by α particles is used to determine reduced $E2$ and $E4$ transition matrix elements between ground-band rotational states in ^{163}Dy and ^{167}Er. The following results are obtained for ^{163}Dy: $\langle 7/2\ 5/2\|M(E2)\|5/2\ 5/2\rangle = 3.83\pm0.10$ e b; $\langle 9/2\ 5/2\|M(E2)\|5/2\ 5/2\rangle = 2.31\pm0.02$ e b; $\langle 11/2\ 5/2\|M(E4)\|5/2\ 5/2\rangle = 0.60^{+0.42}_{-0.60}$ e b^2, and for ^{167}Er: $\langle 9/2\ 7/2\|M(E2)\|7/2\ 7/2\rangle = 4.38\pm0.08$ e b; $\langle 11/2\ 7/2\|M(E2)\|7/2\ 7/2\rangle = 2.24\pm0.01$ e b; $\langle 13/2\ 7/2\|M(E4)\|7/2\ 7/2\rangle = 0.77^{+0.39}_{-0.51}$ e b^2. Quadrupole and hexadecapole moments deduced from these values are compared with those of neighboring even-A nuclei.

[NUCLEAR REACTIONS ^{163}Dy(α,α'), ^{167}Er(α,α'), E=12 MeV, measured $\sigma(E_{\alpha'}, 160°)$. Deduced $E2$ and $E4$ matrix elements.]

I. INTRODUCTION

In recent years much experimental information has become available on quadrupole and hexadecapole moments of even-A nuclei in both the rare earth and actinide regions of the Periodic Table. In most of these studies α particles of sufficiently low energy have been used to Coulomb excite the 2^+ and 4^+ levels of the ground-state rotational band. The reduced $E2$ and $E4$ transition matrix elements, $\langle 2^+\|M(E2)\|0^+\rangle$ and $\langle 4^+\|M(E4)\|0^+\rangle$, are determined by comparing the experimental excitation probabilities of the 2^+ and 4^+ rotational states with theoretical values calculated within the framework of a suitable theory of multiple Coulomb excitation.

The purpose of the present experiment is to extend the precise Coulomb excitation studies performed in this laboratory[1-4] and elsewhere[5-13] to odd-A nuclei. Owing to the higher-level density, generally, more ground-band levels are populated by Coulomb excitation with α particles than in the neighboring even nuclei. Thus one is enabled to determine additional $E2$ matrix elements and verify the validity of the nuclear model used in the analysis.

From the observed excitation probabilities of the various ground-band levels, the intrinsic quadrupole and hexadecapole moments are determined and compared with those of the neighboring even-A isotopes. Preliminary results were reported earlier.[14]

II. EXPERIMENTAL PROCEDURE

The experiments were performed by bombarding thin (10–30 μg/cm^2) enriched (> 91%) targets of ^{163}Dy and ^{167}Er with 12 MeV α-particles from the University of Frankfurt Van de Graaff accelerator. The elastically and inelastically scattered projectiles were detected at $\theta_L = 160°$ with

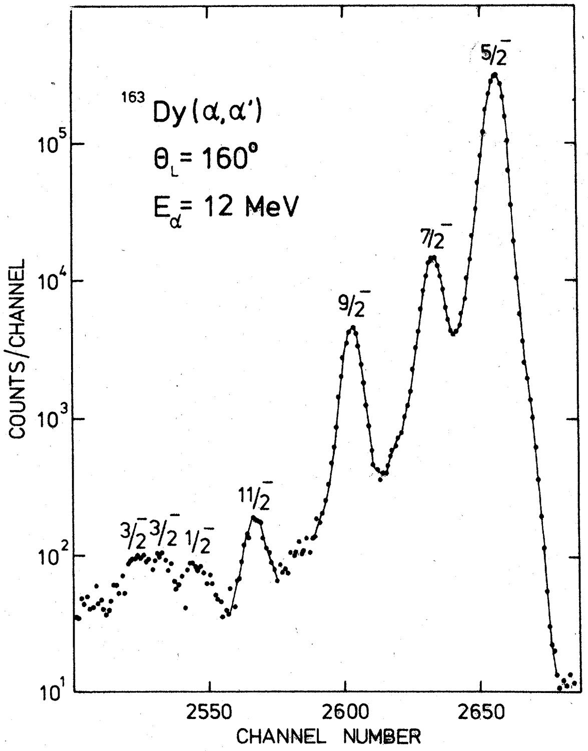

FIG. 1. Spectrum of 12 MeV α-particles scattered from ^{163}Dy.

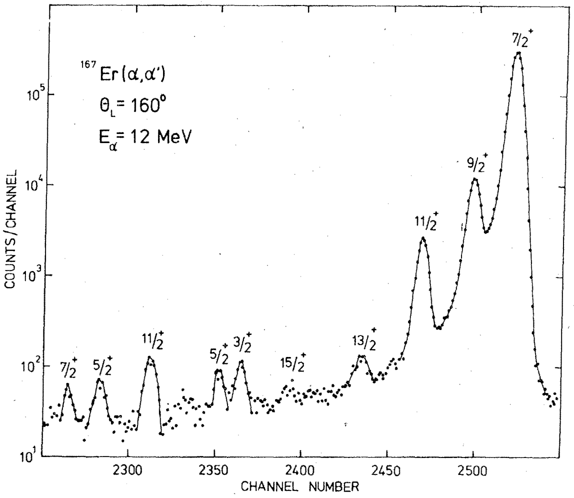

FIG. 2. Spectrum of 12 MeV α-particles scattered from ^{167}Er.

two cooled Si surface-barrier detectors positioned symmetrically to the beam direction. A peak-to-background ratio of better than 30 000:1 and an energy resolution of ~21 keV full width at half maximum (FWHM) have been achieved. Typical spectra are shown in Figs. 1 and 2.

The excitation cross section for the first excited state of the ground band was determined by means of a computer code which separated the respective peak from the elastic group in a self-consistent iterative procedure assuming identical line shapes. The intensities were simultaneously corrected for known impurities in the target material. At higher excitation energies a fourth-order polynomial fit was used to separate the peaks from the background. The excitation probabilities of the $\frac{9}{2}^+$, $\frac{11}{2}^+$, and $\frac{13}{2}^+$ levels in ^{167}Er were determined to an accuracy of 3.5%, 1.3%, and 8.7%, respectively. Similar uncertainties have been observed in ^{163}Dy.

III. ANALYSIS AND DISCUSSION

The reduced $E2$ and $E4$ transition matrix elements were obtained from the measured excitation probabilities by using both the quantum mechanical coupled-channels code JUPIGOR[15] and the semiclassical Winther-de Boer multiple Coulomb excitation code.[16] The calculations show

TABLE I. Results of the Coriolis calculation for ^{167}Er. Coriolis parameters: $E^0_{3/2}$= 500.83 ± 14.11 keV, $E^0_{5/2}$= 733.82 ± 44.11 keV, $E^0_{7/2}$= −143.40 ± 1.59 keV, $\hbar^2/2\theta$= 9.11 ± 0.03 keV, $A_{3/2\ 5/2}$= −10.81 ± 4.69 keV, $A_{5/2\ 7/2}$= −5.11 ± 1.77 keV.

Spin $I K^\pi (N n_z \Lambda)$	Level energy (keV) Calc.	 Exp.	Amplitude of wave function $\frac{3}{2}^+$ (651)	 $\frac{5}{2}^+$ (642)	 $\frac{7}{2}^+$ (633)
$\frac{7}{2}\ \frac{7}{2}^+$ (633)	−2.0	0.0	0.0009	0.0154	0.9999
$\frac{9}{2}$	81.5	79.3	0.0018	0.0234	0.9997
$\frac{11}{2}$	181.4	177.6	0.0029	0.0304	0.9995
$\frac{13}{2}$	299.4	293.7	0.0042	0.0371	0.9993
$\frac{15}{2}$	435.6	432.4	0.0057	0.0436	0.9990
$\frac{17}{2}$	589.9	592.0	0.0073	0.0500	0.9987
$\frac{19}{2}$	762.3	772.0	0.0093	0.0564	0.9984
$\frac{3}{2}\ \frac{3}{2}^+$ (651)	532.0	532.0	1.0000	⋯	⋯
$\frac{5}{2}$	578.0	574.5	0.9948	0.1021	⋯
$\frac{7}{2}$	638.4	641.7	0.9880	0.1547	−0.0033
$\frac{9}{2}$	716.2		0.9799	0.1991	−0.0064
$\frac{11}{2}$	811.4		0.9711	0.2383	−0.0101
$\frac{5}{2}\ \frac{5}{2}^+$ (642)	816.0	812.5	−0.1021	0.9948	⋯
$\frac{7}{2}$	883.3	874.0	−0.1547	0.9878	−0.0151
$\frac{9}{2}$	969.8	933.0	−0.1992	0.9797	−0.0225
$\frac{11}{2}$	1075.2		−0.2385	0.9707	−0.0289
$\frac{13}{2}$	1200.0	1205.0	−0.2735	0.9612	−0.0345

TABLE II. Reduced matrix elements $\langle I_f K_f \| M(E\lambda) \| I_i K_i \rangle$ in units of $e\mathrm{b}^{\lambda/2}$ for levels populated by Coulomb excitation in the ^{163}Dy(α,α') and ^{167}Er(α,α') reactions.

^{163}Dy

$$\langle \tfrac{7}{2}\,\tfrac{5}{2} \| M(E2) \| \tfrac{5}{2}\,\tfrac{5}{2} \rangle = 3.83 \pm 0.10$$
$$\langle \tfrac{9}{2}\,\tfrac{5}{2} \| M(E2) \| \tfrac{5}{2}\,\tfrac{5}{2} \rangle = 2.31 \pm 0.02$$
$$\langle \tfrac{11}{2}\,\tfrac{5}{2} \| M(E4) \| \tfrac{5}{2}\,\tfrac{5}{2} \rangle = 0.60 \pm^{0.42}_{0.60}$$

^{167}Er

$$\langle \tfrac{9}{2}\,\tfrac{7}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 4.38 \pm 0.08$$
$$\langle \tfrac{11}{2}\,\tfrac{7}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 2.24 \pm 0.01$$
$$\langle \tfrac{13}{2}\,\tfrac{7}{2} \| M(E4) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.77 \pm^{0.39}_{0.51}$$

$$\langle \tfrac{3}{2}\,\tfrac{3}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.49 \pm 0.03$$
$$\langle \tfrac{5}{2}\,\tfrac{3}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.42 \pm 0.02$$
$$\langle \tfrac{5}{2}\,\tfrac{5}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.53 \pm 0.03$$
$$\langle \tfrac{7}{2}\,\tfrac{5}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.44 \pm 0.04$$
$$\langle \tfrac{11}{2}\,\tfrac{11}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle = 0.70 \pm 0.02$$

that the hexadecapole moment can be determined from the cross section of the third excited state, while its influence on the first and second excited states is negligibly small since the excitation of the latter two levels is governed by $E2$ transitions. For ^{167}Er two $E2$ matrix elements, $\langle \tfrac{9}{2}\,\tfrac{7}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle$ and $\langle \tfrac{11}{2}\,\tfrac{7}{2} \| M(E2) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle$, were determined from the excitation of the $\tfrac{9}{2}$ and $\tfrac{11}{2}$ levels by means of the quantum mechanical coupled-channels code. The ratio of these matrix elements was compared with the theoretical model predictions which are needed to calculate the entire $E2$ matrix. For both nuclei studied here, it appears that the reduced transition probabilities are well described by the rigid-rotor model, without Coriolis mixing. Whether or not band mixing can be neglected was investigated in a three-band Coriolis calculation in which level energies were fitted to the experimental values by varying band-head energies, rotational parameter, and coupling strengths. The result of such a calculation, which included levels of the $K = \tfrac{7}{2}^+$, $\tfrac{5}{2}^+$, and $\tfrac{3}{2}^+$ bands in ^{167}Er, is shown in Table I. As can be seen, the coupling between the ground-state band and the higher bands is rather weak. A previously assigned $K = \tfrac{9}{2}^+$ band based on a level at 592 keV in ^{167}Er was not included in our Coriolis calculations, since Tveter *et al.*[17] have found that the $\tfrac{11}{2}^+$ state at 711 keV is the band head of a γ-vibrational band. This assignment is supported by the strong excitation of the 711 keV level in the present experiment. Similar results are obtained for the $K = \tfrac{1}{2}^-$, $\tfrac{3}{2}^-$, and $\tfrac{5}{2}^-$ bands in ^{163}Dy.

The influence of quantum mechanical effects on the cross sections was evaluated by calculating the excitation probabilities in terms of the $E2$ matrix with both the semiclassical and quantum mechanical coupled-channels code. The quantal effects reduced the differential cross sections of the $\tfrac{9}{2}^+$, $\tfrac{11}{2}^+$, and $\tfrac{13}{2}^+$ levels in ^{167}Er by approximately 1.6%, 1.7%, and 6.5%, respectively. These quantum mechanical corrections were then applied to the differential cross sections calculated with the semiclassical code as a function of the $E4$ matrix. The reduction of the excitation probability of the $\tfrac{13}{2}^+$ state by 6.5% was taken to be independent of the magnitude of the $\langle \tfrac{13}{2}\,\tfrac{7}{2} \| M(E4) \| \tfrac{7}{2}\,\tfrac{7}{2} \rangle$ matrix element. This is justified on the basis of the results shown in Fig. 2 of Ref. 1.

The $E2$ and $E4$ matrix elements obtained from the present study are listed in Table II. The sign

TABLE III. Comparison of the quadrupole and hexadecapole moments of ^{163}Dy and ^{167}Er with those of neighboring even-A nuclei.

	Q_{20} (b)	Q_{40} (b^2)	$\langle K' = K_{\mathrm{g.s.}} \pm 2 \mid M(E2;\pm 2) \mid K_{\mathrm{g.s.}} \rangle$ (eb)
^{162}Dy (Ref. 6)	7.36 ± 0.03	0.64 ± 0.24	
^{163}Dy	7.29 ± 0.13	$1.02 \pm^{0.71}_{1.02}$	
^{164}Dy (Ref. 1)	7.54 ± 0.04	$0.54 \pm^{0.24}_{0.28}$	
^{166}Er (Refs. 1 and 4)	7.67 ± 0.03	$0.52 \pm^{0.26}_{0.38}$	0.256 ± 0.005 ($K' = 2$)
^{167}Er	7.60 ± 0.10	$1.35 \pm^{0.69}_{0.90}$	0.248 ± 0.007 ($K' = \tfrac{11}{2}$) 0.249 ± 0.012 ($K' = \tfrac{3}{2}$)
^{168}Er (Refs. 6 and 18)	7.61 ± 0.06	$0.47 \pm^{0.28}_{0.43}$	0.255 ± 0.005 ($K' = 2$)

of the $E4$ matrix element was taken to be positive, in analogy to the even-A neighboring nuclei. Also shown are interband matrix elements for ^{167}Er which have been determined from the measured cross sections by means of semiclassical calculations assuming identical intrinsic quadrupole moments[4] in the $K=\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{11}{2}^+$ rotational bands. The $E2$ matrix elements obtained are found to be in good agreement with previous measurements.[18]

Intrinsic quadrupole and hexadecapole moments derived from the measured reduced $E2$ and $E4$ matrix elements are compared with those of the even-A neighboring nuclei in Table III. It is seen that there is agreement of these values within the experimental uncertainties. In the case of the Er isotopes, the intrinsic interband matrix elements between the ground-state band and the γ-vibrational bands ($K_\gamma = K_{g.s.} \pm 2$) are also shown. In summary, the measured multipole moments indicate that the shapes of the strongly deformed odd-A nuclei ^{163}Dy and ^{167}Er are similar to those which have been determined for the respective neighboring even-A isotopes.

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*Present address: Gesellschaft für Schwerionenforschung, Darmstadt, Germany.

[1] H. J. Wollersheim, W. Wilcke, Th. W. Elze, and D. Pelte, Phys. Lett. 48B, 323 (1974).

[2] H. J. Wollersheim, W. Wilcke, and Th. W. Elze, Phys. Rev. C 11, 2008 (1975).

[3] H. J. Wollersheim and Th. W. Elze, Nucl. Phys. A278, 87 (1977).

[4] H. J. Wollersheim and Th. W. Elze, Z. Phys. A280, 277 (1977).

[5] F. S. Stephens, R. M. Diamond, N. K. Glendenning, and J. de Boer, Phys. Rev. Lett. 24, 1137 (1970); F. S. Stephens, R. M. Diamond, and J. de Boer, *ibid.* 27, 1151 (1971).

[6] K. A. Erb, J. E. Holden, J. Y. Lee, J. X. Saladin, and T. K. Saylor, Phys. Rev. Lett. 29, 1010 (1972).

[7] W. Ebert, P. Hecking, K. Pelz, S. G. Steadman, and P. Winkler, Z. Phys. 263, 191 (1973).

[8] W. Brückner, J. G. Merdinger, D. Pelte, U. Smilansky, and K. Traxel, Phys. Rev. Lett. 30, 57 (1973).

[9] A. H. Shaw and J. S. Greenberg, Phys. Rev. C 10, 263 (1974).

[10] R. M. Ronningen, R. B. Piercey, R. S. Grantham, J. H. Hamilton, A. V. Ramayya, B. van Nooijen, H. Kawakami, C. Maguire, L. L. Riedinger, and W. K. Dagenhart, Bull. Am. Phys. Soc. 21, 985 (1976).

[11] C. E. Bemis, Jr., F. K. McGowan, J. L. C. Ford, Jr., W. T. Milner, P. H. Stelson, and R. L. Robinson, Phys. Rev. C 8, 1466 (1973); F. K. McGowan, C. E. Bemis, J. L. C. Ford, Jr., W. T. Milner, R. L. Robinson, and P. H. Stelson, Phys. Rev. Lett. 27, 1741 (1971).

[12] C. Baktash and J. X. Saladin, Phys. Rev. C 10, 1136 (1974).

[13] J. H. Hamilton, L. Varnell, R. M. Ronningen, R. V. Ramayya, J. Lange, L. L. Riedinger, R. L. Robinson, and P. H. Stelson, Bull. Am. Phys. Soc. 19, 579 (1974).

[14] H. J. Wollersheim and Th. W. Elze, Verh. Dtsch. Phys. Ges. 11, 915 (1976).

[15] L. D. Tolsma, J. Comput. Phys. 17, 384 (1975).

[16] A. Winther and J. de Boer, in *Coulomb Excitation*, edited by K. Alder and A. Winther (Academic, New York, 1966), p. 303.

[17] A. Tveter and B. Herskind, Nucl. Phys. A134, 599 (1969).

[18] C. Baktash, J. X. Saladin, J. O'Brien, I. Y. Lee, and J. E. Holden, Phys. Rev. C 10, 2265 (1974).